CONNECTED MATHEMATICS® 3

Thinking With Mathematical Models

Linear and Inverse Variation

Lappan, Phillips, Fey, Friel

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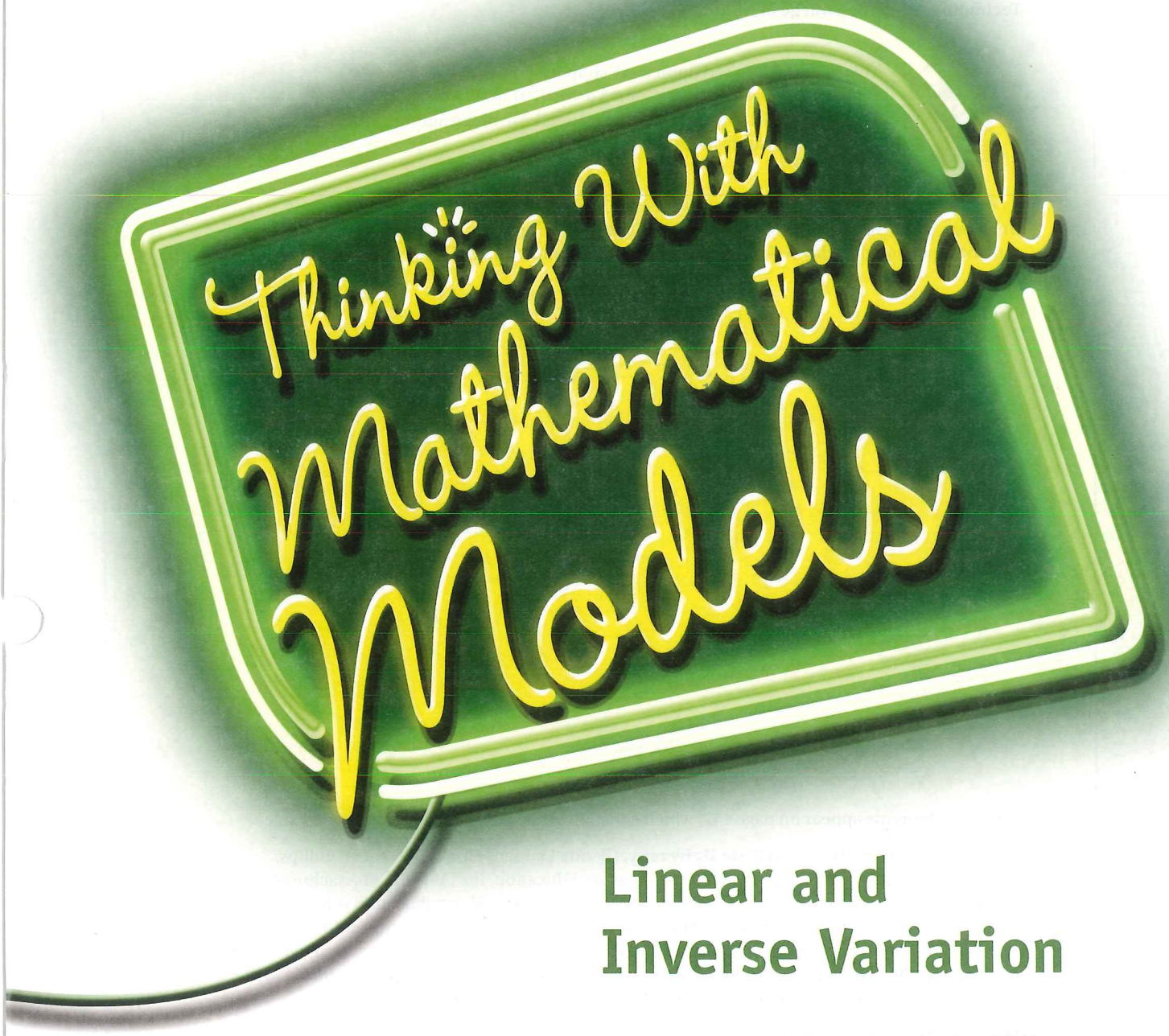

Linear and Inverse Variation

Glenda Lappan, Elizabeth Difanis Phillips,
James T. Fey, Susan N. Friel

Pearson
Boston, Massachusetts

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Thinking With Mathematical Models

Linear and Inverse Variation

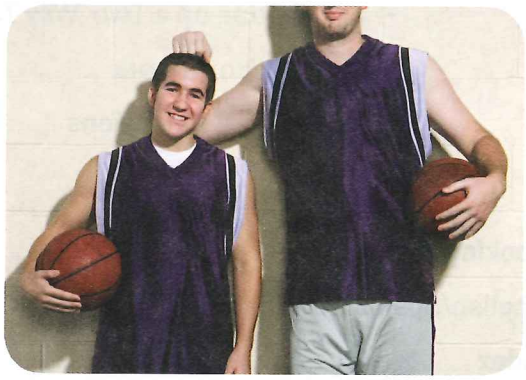
Look for these icons that point to digital resources. Launch Tools PearsonRealize Student Activities

Looking Ahead

How is the thickness of a steel beam or bridge related to its strength? **How** is the length of a beam or bridge related to its strength?

The equation $c = 0.15t + 2.50$ gives the charge c in dollars for renting a paddle boat for t minutes. **How** much time can you buy on a paddle boat if you have \$12?

An all-star basketball player in the National Basketball Association (NBA) is 6 feet 9 inches (206 cm) tall. **How** can you use this measurement to estimate his shoe size, his weight, or his arm span?



In earlier *Connected Mathematics* units, you explored relationships between two variables. You learned how to find linear relationships from tables and graphs and to write their equations. Using the equations, you solved problems. In this unit, you will develop your skills for recognizing and analyzing linear relationships. You will compare linear and nonlinear patterns and learn about inverse variation, a specific nonlinear pattern.

You will conduct experiments, analyze data, and write equations to summarize, or model, the data patterns. You will use your equations to estimate, or predict, values not found in the data set.

The skills you develop in this unit will help you answer questions such as those on the facing page.

Mathematical Highlights

Thinking With Mathematical Models

In *Thinking With Mathematical Models*, you will model relationships with graphs and equations. You will use your models to analyze situations and solve problems.

The Investigations in this Unit will help you understand the following ideas.

- Represent data using graphs, tables, word descriptions and algebraic expressions.
- Recognize linear and nonlinear relationships in tables and graphs.
- Use linear and inverse variation equations to model bivariate data.
- Use residual analysis to measure the fit of linear and inverse variation models.
- Analyze, approximate, and solve linear equations.
- Use linear and inverse variation equations to solve problems and to make predictions and decisions
- Use scatter plots, two-way tables, and correlation coefficients to describe patterns of association in pairs of variables.
- Use standard deviation to measure variability in data distributions.

When you encounter a new problem, it is a good idea to ask yourself questions. In this Unit, you might ask questions such as:

What are the key variables in this situation?

If there is a pattern relating the variables, is it strong enough to allow me to make predictions?

What is the pattern relating the variables?

What kind of equation will express the relationship?

How can I use the equation to answer questions about the relationship?

Common Core State Standards

Mathematical Practices and Habits of Mind

In the *Connected Mathematics* curriculum you will develop an understanding of important mathematical ideas by solving problems and reflecting on the mathematics involved. Every day, you will use "habits of mind" to make sense of problems and apply what you learn to new situations. Some of these habits are described by the *Common Core State Standards for Mathematical Practices* (MP).

MP1 Make sense of problems and persevere in solving them.

When using mathematics to solve a problem, it helps to think carefully about

- data and other facts you are given and what additional information you need to solve the problem;
- strategies you have used to solve similar problems and whether you could solve a related simpler problem first;
- how you could express the problem with equations, diagrams, or graphs;
- whether your answer makes sense.

MP2 Reason abstractly and quantitatively.

When you are asked to solve a problem, it often helps to

- focus first on the key mathematical ideas;
- check that your answer makes sense in the problem setting;
- use what you know about the problem setting to guide your mathematical reasoning.

MP3 Construct viable arguments and critique the reasoning of others.

When you are asked to explain why a conjecture is correct, you can

- show some examples that fit the claim and explain why they fit;
- show how a new result follows logically from known facts and principles.

When you believe a mathematical claim is incorrect, you can

- show one or more counterexamples—cases that don't fit the claim;
- find steps in the argument that do not follow logically from prior claims.

MP4 Model with mathematics.

When you are asked to solve problems, it often helps to

- think carefully about the numbers or geometric shapes that are the most important factors in the problem, then ask yourself how those factors are related to each other;
- express data and relationships in the problem with tables, graphs, diagrams, or equations, and check your result to see if it makes sense.

MP5 Use appropriate tools strategically.

When working on mathematical questions, you should always

- decide which tools are most helpful for solving the problem and why;
- try a different tool when you get stuck.

MP6 Attend to precision.

In every mathematical exploration or problem-solving task, it is important to

- think carefully about the required accuracy of results: is a number estimate or geometric sketch good enough, or is a precise value or drawing needed?
- report your discoveries with clear and correct mathematical language that can be understood by those to whom you are speaking or writing.

MP7 Look for and make use of structure.

In mathematical explorations and problem solving, it is often helpful to

- look for patterns that show how data points, numbers, or geometric shapes are related to each other;
- use patterns to make predictions.

MP8 Look for and express regularity in repeated reasoning.

When results of a repeated calculation show a pattern, it helps to

- express that pattern as a general rule that can be used in similar cases;
- look for shortcuts that will make the calculation simpler in other cases.

You will use all of the Mathematical Practices in this Unit. Sometimes, when you look at a Problem, it is obvious which practice is most helpful. At other times, you will decide on a practice to use during class explorations and discussions. After completing each Problem, ask yourself:

- What mathematics have I learned by solving this Problem?
- What Mathematical Practices were helpful in learning this mathematics?

Investigation 1

Exploring Data Patterns

People in many professions use data and mathematical reasoning to solve problems and make decisions. For example, engineers analyze data from lab tests to determine how much weight a bridge can hold. Market researchers use survey data to predict demand for new products. Stockbrokers use formulas to forecast growth of investments over time.

In several previous *Connected Mathematics* units, you used tables, graphs, and equations to explore and describe relationships between variables. In this Investigation, you will develop your skill in using these tools to organize data from experiments, find patterns, and make predictions.

1.1 Bridge Thickness and Strength

Many bridges are built with frames of steel beams. Steel is very strong, but any beam will bend or break if you put too much weight on it.

Common Core State Standards

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear) . . .

Also 8.F.A.2, 8.SP.A.1, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-BF.A.1a

- How do you think the strength of a beam is related to its thickness?
- What other variables might affect the strength of a bridge?

Problem 1.1

Engineers often use scale models to test their designs. You can do your own experiments to discover mathematical patterns involved in building bridges.

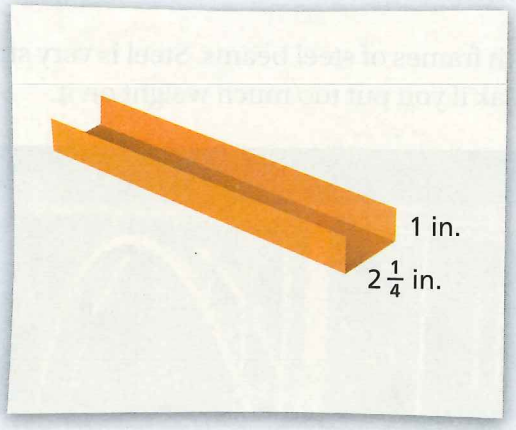
Instructions for a Bridge-Thickness Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- Several 11 inch-by-$4\frac{1}{4}$ inch strips of paper

Instructions:

- Start with one of the paper strips. Make a "bridge" by folding up 1 inch on each long side.



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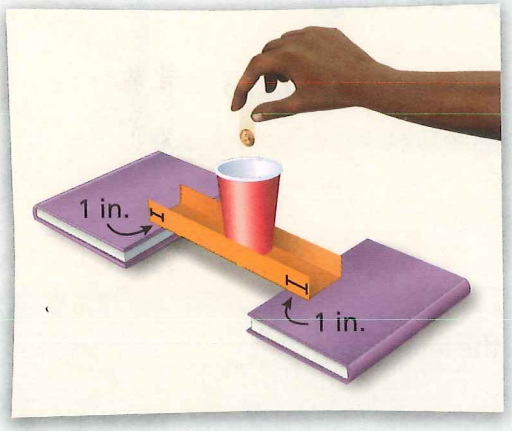
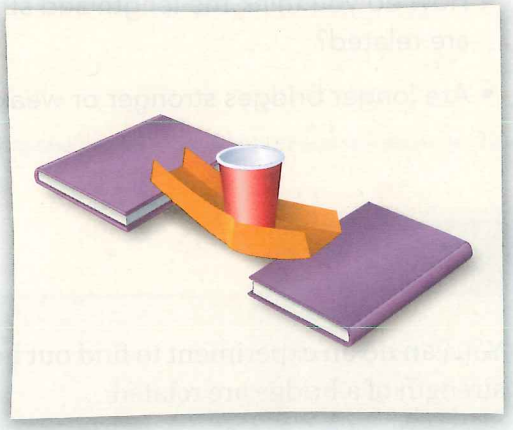
Problem 1.1 continued

- Suspend the bridge between the books. The bridge should overlap each book by 1 inch. Place the cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. This number is the *breaking weight* of the bridge.





- Put two *new* strips of paper together to make a bridge with twice as many layers. Find the breaking weight for this bridge.
- Repeat this experiment to find the breaking weights of bridges made from three, four, and five strips of paper.

A Make a table and a graph of your (bridge layers, breaking weight) data.

B Does the relationship between the number of layers and the breaking weight seem to be linear or nonlinear? How do the graph and the table show this relationship?

C Suppose you could split layers of paper in half. What breaking weight would you predict for a bridge 2.5 layers thick? Explain.

D Predict the breaking weight for a bridge 6 layers thick. Explain your reasoning.

E Test your prediction of strength for the 6-layer bridge. Explain why results from such a test might not exactly match predictions.

ACE Homework starts on page 15.

1.2 Bridge Length and Strength

In the last problem you tested the strength of some paper bridges. You found that bridges with more layers are stronger than bridges with fewer layers.

- How do you think the length and strength of a bridge are related?
- Are longer bridges stronger or weaker than shorter bridges?

Problem 1.2

You can do an experiment to find out how the length and strength of a bridge are related.

Instructions for a Bridge-Length Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- $4\frac{1}{4}$-inch-wide paper strips with lengths 4, 6, 8, 9, and 11 inches

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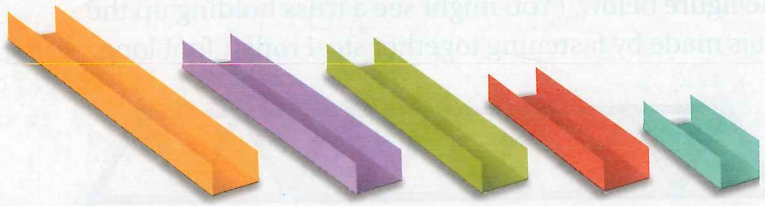
Problem 1.2 continued

Instructions:

- Fold the paper strips to make bridges as shown below.



- Start with the 4-inch bridge. Suspend the bridge between the two books as you did before. The bridge should overlap each book by 1 inch. Place the paper cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. As in the first experiment, this number is the breaking weight of the bridge.
- Repeat the experiment to find breaking weights for the other bridges.

A Make a graph of your data.

B Describe the relationship between bridge length and breaking weight. How is that relationship shown by patterns in your table and graph?

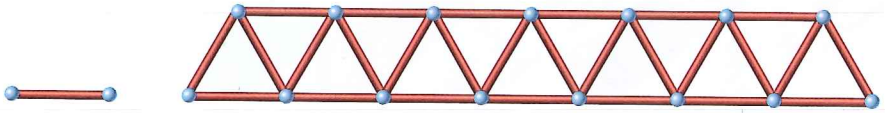
C Use your data to predict the breaking weights for bridges of lengths 3, 5, 10, and 12 inches. Explain how you made your predictions.

D Compare your data from this experiment to the data from the experiment on bridges with different numbers of layers. How is the relationship between the number of layers in a bridge and its breaking weight similar to the relationship between bridge length and breaking weight? How is it different?

ACE Homework starts on page 15.

1.3 Custom Construction Parts
Finding Patterns

Suppose a company called Custom Steel Products (CSP for short) supplies materials to builders. One common structure that CSP makes is called a *truss*, as shown in the figure below. (You might see a truss holding up the roof of a building.) It is made by fastening together steel rods 1 foot long.



1-foot steel rod

7-foot truss made from 27 rods

This truss has an overall length of 7 feet. The manager at CSP needs to know the number of rods in any length of truss a customer might order.

Problem 1.3

Study the drawing above to see if you can figure out what the manager needs to know. It might help to work out several cases and look for a pattern.

A Copy and complete the table below to show the number of rods in trusses of different overall lengths.

Length of Truss (ft)	2	3	4	5	6	7	8
Number of Rods	7	11	■	■	■	27	■

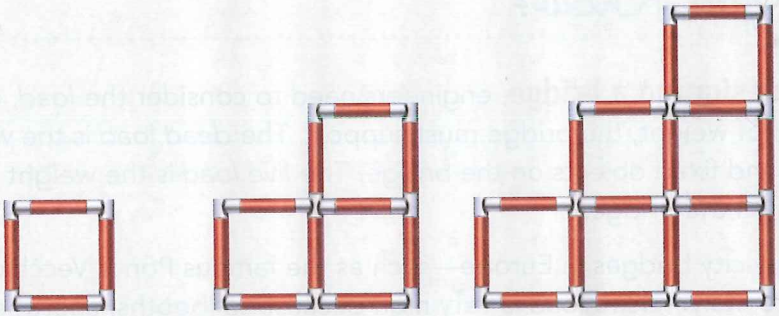
1. Make a graph of the data in your table.
2. Describe the pattern of change in the number of rods used as the truss length increases.

continued on the next page >

Problem 1.3 continued

3. How is the pattern you described shown in the table? How is it shown in the graph?
4. How many steel rods are in a truss 50 feet long overall? Explain how to find this number without drawing the truss.
5. By counting the triangles she could see for any length, Jenna says she figured out a pattern for the number of rods. For overall length 7, she sees 7 triangles and 6 rods connecting these triangles, so she writes $7 \times 3 + 6 = 27$. For length L, she writes $N = 3L + L - 1$. Explain where she gets the $3L$ and the $L - 1$ in her expression.

B Custom Steel Products also makes staircase frames like those shown here.



1 step made from 4 rods

2 steps made from 10 rods

3 steps made from 18 rods

1. Copy and complete the table below to show the number of rods in staircase frames with different numbers of steps.

CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	■	■	■	■	■

2. Make a graph of the data in your table.
3. Describe the pattern of change in the number of rods as the number of steps increases.
4. How is the pattern you described shown in the table? How is it shown in the graph?
5. How many steel rods are in a staircase frame with 12 steps? Explain how you could find this number without drawing the staircase frame.

continued on the next page >

Problem 1.3 continued

C How is the pattern in Question A similar to the pattern in Question B? How is it different? Explain how the similarities and differences are shown in the tables and graphs.

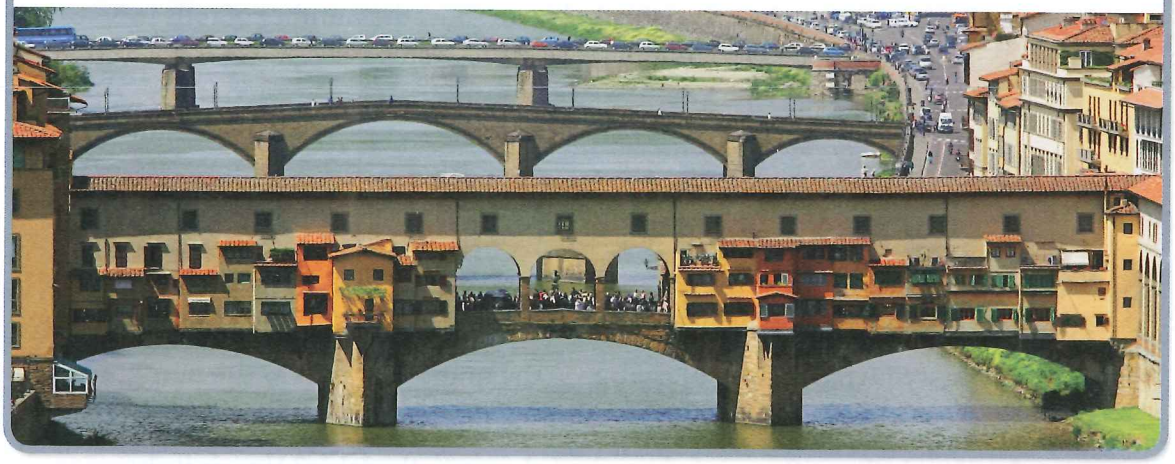
D Compare the patterns of change in this problem with the patterns of change in Problems 1.1 and 1.2. Describe any similarities and differences you find.

ACE Homework starts on page 15.

Did You Know?

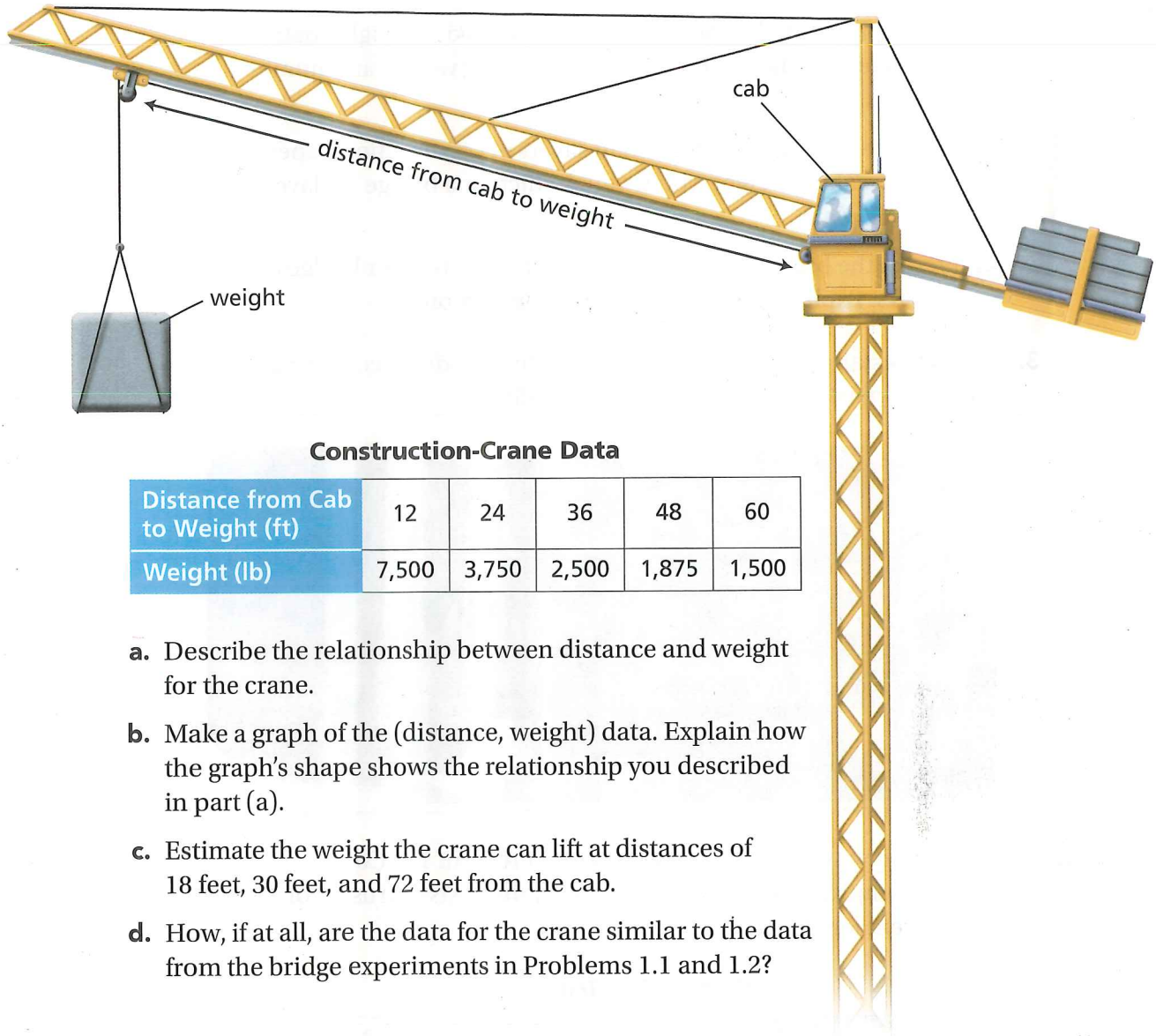
When designing a bridge, engineers need to consider the *load*, or the amount of weight, the bridge must support. The *dead load* is the weight of the bridge and fixed objects on the bridge. The *live load* is the weight of moving objects on the bridge.

On many city bridges in Europe—such as the famous Ponte Vecchio in Florence, Italy—dead load is very high because tollbooths, apartments, and shops are built right onto the bridge surface. Local ordinances can limit the amount of automobile and rail traffic on a bridge to help control live load.



Applications

1. The table shows the maximum weight a crane arm can lift at various distances from its cab.



Construction-Crane Data

Distance from Cab to Weight (ft)	12	24	36	48	60
Weight (lb)	7,500	3,750	2,500	1,875	1,500

a. Describe the relationship between distance and weight for the crane.

b. Make a graph of the (distance, weight) data. Explain how the graph's shape shows the relationship you described in part (a).

c. Estimate the weight the crane can lift at distances of 18 feet, 30 feet, and 72 feet from the cab.

d. How, if at all, are the data for the crane similar to the data from the bridge experiments in Problems 1.1 and 1.2?

2. A group of students conducted the bridge-thickness experiment with construction paper. The table below contains their results.

Bridge-Thickness Experiment

Number of Layers	1	2	3	4	5	6
Breaking Weight (pennies)	12	20	29	42	52	61

a. Make a graph of the (number of layers, breaking weight) data. Describe the relationship between breaking weight and number of layers.

b. Suppose it is possible to use half-layers of construction paper. What breaking weight would you predict for a bridge 3.5 layers thick? Explain.

c. Predict the breaking weight for a construction-paper bridge of 8 layers. Explain how you made your prediction.

3. A truss or staircase frame from Custom Steel Products costs \$2.25 for each rod, plus \$50 for shipping and handling.

a. Refer to your data from Question A of Problem 1.3. Copy and complete the table below to show the costs of trusses of different lengths.

Cost of CSP Truss

Truss Length (ft)	1	2	3	4	5	6	7	8
Number of Rods	3	7	■	■	■	■	27	■
Cost of Truss	■	■	■	■	■	■	■	■

b. Make a graph of the (truss length, cost) data.

c. Describe the relationship between truss length and cost.

d. Refer to your data from Question B of Problem 1.3. Copy and complete the table below to show the costs of staircase frames with different numbers of steps.

Cost of CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	■	■	■	■	■
Cost of Frame	■	■	■	■	■	■	■	■

e. Make a graph of the (number of steps, cost) data.

f. Describe the relationship between number of steps and cost.

4. During the medal ceremonies at a track meet, the top athletes stand on platforms made from stacked wooden boxes. The number of boxes depends on the number of medal winners.

1 medalist
1 box

2 medalists
3 boxes

3 medalists
6 boxes

a. Copy and complete the table below.

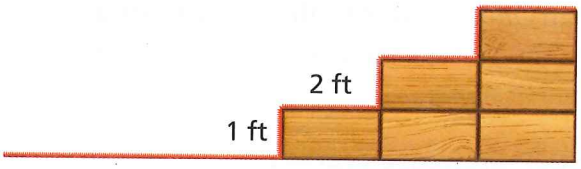
Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	■	■	■	■	■

b. Make a graph of the (number of medalists, number of boxes) data.

c. Describe the pattern of change shown in the table and graph.

d. Each box is 1 foot high and 2 feet wide. A red carpet starts 10 feet from the base of the platform and covers all the risers and steps.



Copy and complete the table below.

Carpet for Platforms

Number of Steps	1	2	3	4	5	6	7	8
Carpet Length (ft)	■	■	■	■	■	■	■	■

e. Make a graph of the (number of steps, carpet length) data.

f. Describe the pattern of change in the carpet length as the number of steps increases. Compare this pattern to the pattern in the (number of medalists, number of boxes) data.

5. Parts (a)–(f) refer to relationships between variables you have studied in this Investigation. Tell whether each is *linear* or *nonlinear*.

a. Cost depends on truss length (ACE Exercise 3).

b. Cost depends on the number of rods in a staircase frame (ACE Exercise 3).

c. Bridge strength depends on bridge thickness (Problem 1.1).

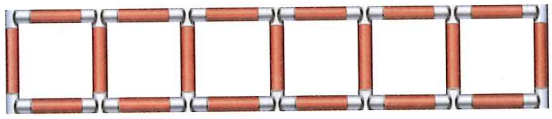
d. Bridge strength depends on bridge length (Problem 1.2).

e. Number of rods depends on truss length (Problem 1.3).

f. Number of rods depends on the number of steps in a staircase frame (Problem 1.3).

g. Compare the patterns of change for all the nonlinear relationships in parts (a)–(f).

6. CSP also sells ladder bridges made from 1-foot steel rods arranged to form a row of squares. Below is a sketch of a 6-foot ladder bridge.



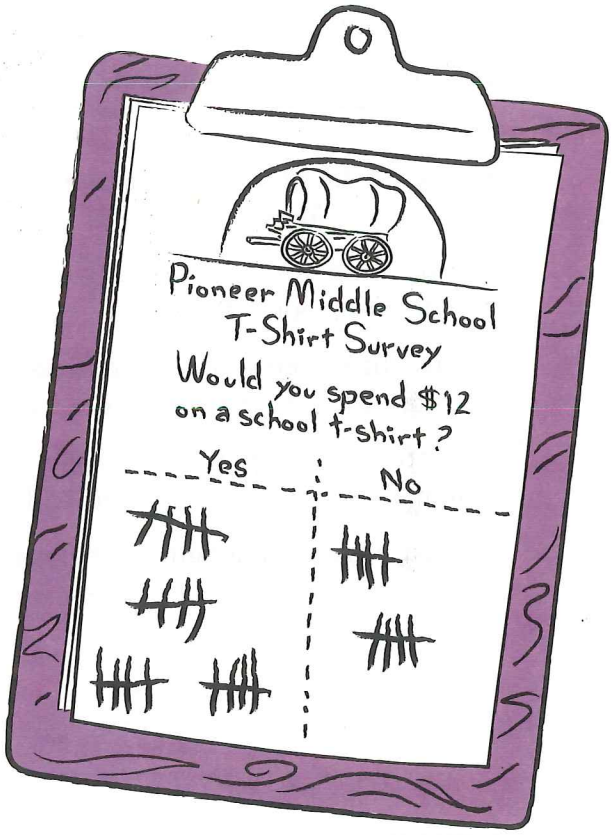
6-foot ladder bridge made from 19 rods

   a. Make a table and a graph showing how the number of rods in a ladder bridge is related to the length of the bridge.

   b. Compare the pattern of change for the ladder bridges with those for the trusses and staircase frames in Problem 1.3.

Connections

A survey of one class at Pioneer Middle School found that 20 out of 30 students would spend $12 for a school T-shirt. Use this information for Exercises 7 and 8.



7. **Multiple Choice** Suppose there are 600 students in the school. Based on the survey, how many students do you predict would spend $12 for a school T-shirt?

   A. 20 **B.** 200

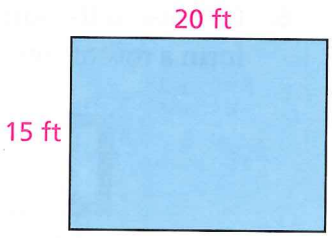
   C. 300 **D.** 400

8. **Multiple Choice** Suppose there are 450 students in the school. Based on the survey, how many students do you predict would spend $12 for a school T-shirt?

   F. 20 **G.** 200

   H. 300 **J.** 400

9. At the right is a drawing of a rectangle with an area of 300 square feet.



a. Make drawings of at least three other rectangles with an area of 300 square feet.

b. What is the width of a rectangle with an area of 300 square feet if its length is 1 foot? If its length is 2 feet? If its length is 3 feet?

c. What is the width of a rectangle with an area of 300 square feet and a length of ℓ feet?

d. How does the width of a rectangle change if the length increases, but the area remains 300 square feet?

e. Make a graph of (width, length) pairs for rectangles with an area of 300 square feet. Explain how your graph illustrates your answer for part (d).

10. The rectangle pictured in Exercise 9 has a perimeter of 70 feet.

a. Make drawings of at least three other rectangles with a perimeter of 70 feet.

b. What is the width of a rectangle with a perimeter of 70 feet if its length is 1 foot? 2 feet? ℓ feet?

c. What is the width of a rectangle with a perimeter of 70 feet if its length is $\frac{1}{2}$ foot? $\frac{3}{2}$ feet?

d. Give the dimensions of rectangles with a perimeter of 70 feet and length-to-width ratios of 3 to 4, 4 to 5, and 1 to 1.

e. Suppose the length of a rectangle increases, but the perimeter remains 70 feet. How does the width change?

f. Make a graph of (length, width) pairs that give a perimeter of 70 feet. How does your graph illustrate your answer for part (e)?

11. The 24 students in Ms. Cleary's homeroom are surveyed. They are asked which of several prices they would pay for a ticket to the school fashion show. The table shows the results.

Ticket-Price Survey

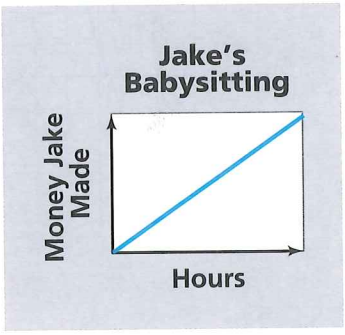
Ticket Price	$1.00	$1.50	$2.00	$2.50	$3.00	$3.50	$4.00	$4.50
Probable Sales	20	20	18	15	12	10	8	7

a. There are 480 students in the school. Use the data from Ms. Cleary's class to predict ticket sales for the entire school for each price.

b. Use your results from part (a). For each price, find the school's projected income from ticket sales.

c. Which price should the school charge if it wants to earn the maximum possible income?

12. At the right is a graph of the amount of money Jake earned while babysitting for several hours.



a. Put scales on the axes that make sense. Explain why you chose your scales.

b. What would the equation of the graph be, based on the scale you chose in part (a)?

c. If the line on this graph were steeper, what would it tell about the money Jake is making? Write an equation for such a line.

13. In each pair of equations below, solve the first and graph the second.

a. $0 = 3x + 6$ $\qquad$ $y = 3x + 6$

b. $0 = x - 2$ $\qquad$ $y = x - 2$

c. $0 = 3x + 10$ $\qquad$ $y = 3x + 10$

d. In each pair, how is the solution related to the graph?

For Exercises 14–17, tell which graph matches the equation or the set of criteria.

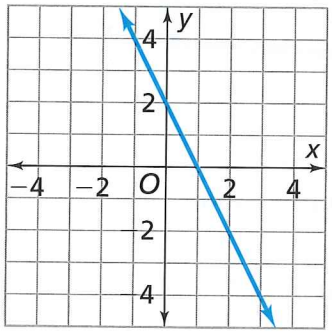
14. $y = 3x + 1$

15. $y = -2x + 2$

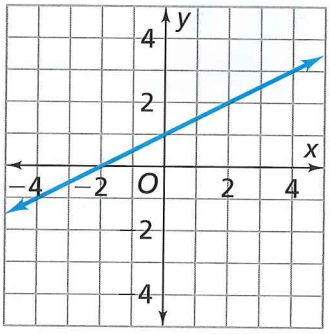
16. $y = x - 3$

17. y-intercept $= 1$; slope $= \frac{1}{2}$

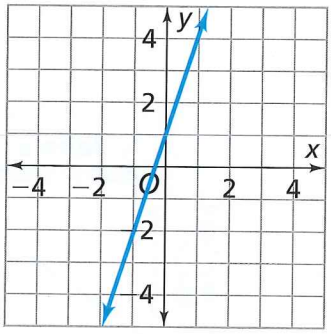
Graph A



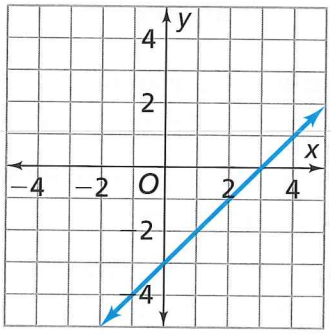
Graph B



Graph C



Graph D



In Exercises 18 and 19, each pouch holds the same number of coins. The coins all have the same value. Find the number of coins in each pouch. Explain your method.

18.

19.

20. Refer to Exercises 18 and 19.

a. For each exercise, write an equation to represent the situation. Let x represent the number of coins in a pouch.

b. Solve each equation. Explain the steps in your solutions.

c. Compare your strategies with those you used in Exercises 18 and 19.

In Exercises 21–28, solve each equation for x.

21. $3x + 4 = 10$

22. $6x + 3 = 4x + 11$

23. $6x - 3 = 11$

24. $-3x + 5 = 7$

25. $4x - \frac{1}{2} = 8$

26. $\frac{x}{2} - 4 = -5$

27. $3x + 3 = -2x - 12$

28. $\frac{x}{4} - 4 = \frac{3x}{4} - 6$

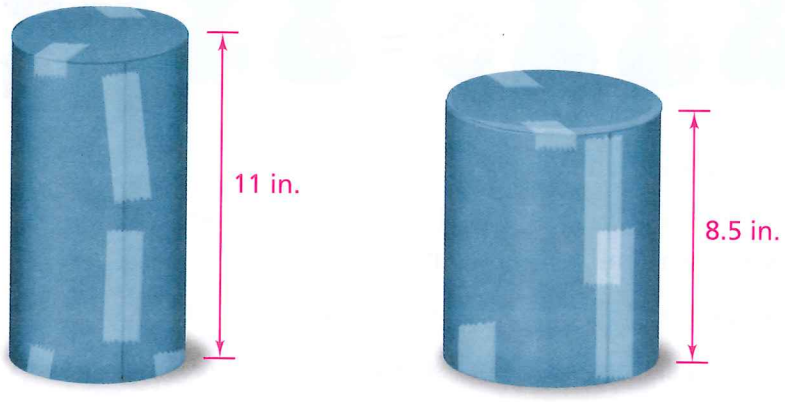
For Exercises 29–31, tell whether the statement is *true* or *false*. Explain your reasoning.

29. $6(12 - 5) > 50$ **30.** $3 \cdot 5 - 4 > 6$ **31.** $10 - 5 \cdot 4 > 0$

32. For this exercise, you will need two 8.5-inch by 11-inch sheets of paper and some scrap paper.

a. Roll one sheet of paper to make a cylinder 11 inches high. Overlap the edges very slightly and tape them together. Make bases for the cylinder by tracing the circles on the ends of the cylinder, cutting out the tracings, and taping them in place.

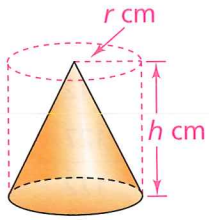
b. Roll the other sheet of paper to make a cylinder 8.5 inches high. Make bases as you did in part (a).



c. Do the cylinders appear to have the same surface area (including the bases)? If not, which has the greater surface area?

d. Suppose you start with two identical rectangular sheets of paper that are *not* 8.5 by 11 inches. You make two cylinders as you did before. Which cylinder will have the greater surface area, the taller cylinder or the shorter one? How do you know?

33. The volume of the cone in the drawing below is $\frac{1}{3}(28\pi)$ cm^3. Recall that the formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$. What are some possible values of radius and height for the cone?



Extensions

34. Study the patterns in this table. Note that the numbers in the x column may not be consecutive after $x = 6$.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	■	■	■	■
■	■	■	1,024	■
■	■	■	2,048	■
■	■	1,728	■	■
n	■	■	■	■

a. Use the patterns in the first several rows to find the missing values.

b. Are any of the patterns linear? Explain.

35. The table below gives data for a group of middle school students.

Data for Some Middle School Students

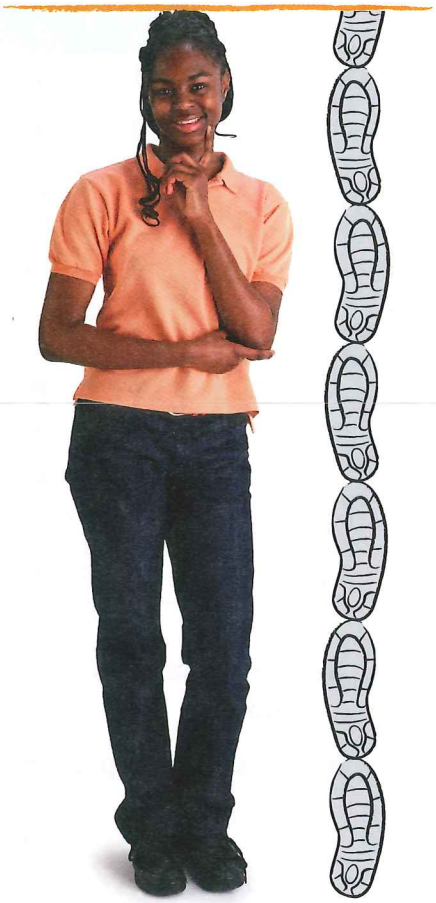
Student	Name Length	Height (cm)	Foot Length (cm)
Thomas Petes	11	126	23
Michelle Hughes	14	117	21
Shoshana White	13	112	17
Deborah Locke	12	127	21
Tonya Stewart	12	172	32
Richard Mudd	11	135	22
Tony Tung	8	130	20
Janice Vick	10	134	21
Bobby King	9	156	29
Kathleen Boylan	14	164	28

a. Make graphs of the (name length, height) data, the (name length, foot length) data, and the (height, foot length) data.

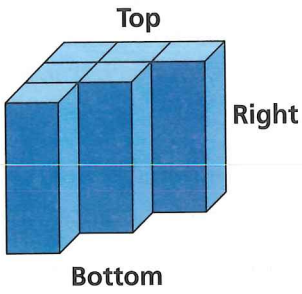
b. Look at the graphs you made in part (a). Which seem to show linear relationships? Explain.

c. Estimate the average height-to-foot-length ratio. How many foot-lengths tall is the typical student in the table?

d. Which student has the greatest height-to-foot-length ratio? Which student has the least height-to-foot-length ratio?



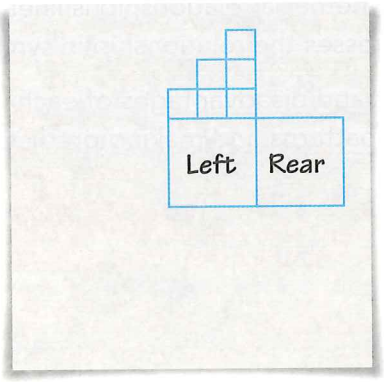
36. A staircase is a type of prism. This is easier to see if the staircase is viewed from a different perspective. In the prism shown here, each of the small squares on the top has an area of 1 square unit.



a. Sketch the base of the prism. What is the area of the base?

b. Rashid tries to draw a flat pattern that will fold up to form the staircase prism. Below is the start of his drawing. Finish Rashid's drawing and give the surface area of the entire staircase.

Hint: You may want to draw your pattern on grid paper and then cut it out and fold it to check.



c. Suppose the prism has six stairs instead of three. Assume each stair is the same width as those in the prism above. Is the surface area of this six-stair prism twice that of the three-stair prism? Explain.

Mathematical Reflections 1

In this Investigation, you used tables and graphs to represent relationships between variables and to make predictions. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **You** can represent a relationship between variables with a table, a graph, a description in words, or an equation.
   a. **How** can you decide whether a relationship is linear by studying the pattern in a data table?
   b. **How** can you decide whether a relationship is linear by studying the pattern in a graph?
   c. **How** can you decide whether a relationship is linear by studying the words used to describe the variables?
   d. **How** can you decide whether a relationship is linear by studying the equation that expresses the relationship in symbolic form?
2. **What** are the advantages and disadvantages of each representation in finding patterns and making predictions?

Common Core Mathematical Practices

As you worked on the problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Jayden described his thoughts in the following way:

For Problem 1.1, we noticed from our table and graph that the data look linear. The graph shows this the best. The data points are in a nearly straight line.

In the table, the rate of change for different thicknesses varies from 7 pennies (change from 1 to 2 layers) to 10 pennies (change from 3 to 4 layers), with the average rate of change being about 8 pennies for each additional layer.

Some variability occurs because this is an experiment. We predict that 6 layers would hold about 50 pennies if we use the rate of change of 8 pennies to predict the increase.

Common Core Standards for Mathematical Practice

MP4 Model with mathematics

- What other Mathematical Practices can you identify in Jayden's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 2

Linear Models and Equations

In Investigation 1, you used tables, graphs, and equations to study relationships between variables. You found that the strength of a paper bridge depends on both its number of layers and its length. You found that the number of steel pieces needed to build a truss depends on the length of the truss. The number of pieces in a staircase frame depends on the number of steps.

If there is exactly one value of the dependent variable related to each value of the independent variable, mathematicians call the relationship a **function.** For example, the relationship between bridge breaking weight and length is a function. The relationship between the number of steel pieces used and the length of a truss is a function.

Common Core State Standards

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8a Understand that solutions to a system of two linear equations in two variables corresponds to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or with verbal descriptions).

Also **8.EE.C.8c, 8.F.A.3, 8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, A-SSE.A.1a, A-CED.A.1, A-REI.C.6, F-IF.A.1, F-IF.C.7a, F-BF.A.1a, F-LE.A.2, F-LE.B.5, S-ID.B.6, S-ID.B.6a, S-ID.B.6b, S-ID.B.6c**

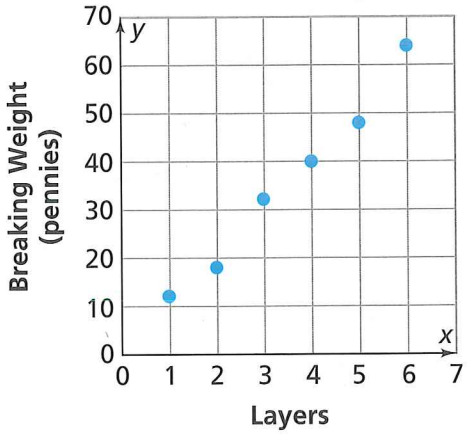
It is often helpful to express functions with equations or formulas. The functions and their equations are called **mathematical models** of the relationships between variables. Equations tell you how to calculate the value of the dependent variable when you know the value of the independent variable. In this Investigation, you will develop skills in writing and using linear equations to model relationships between variables.

2.1 Modeling Linear Data Patterns

Organizing and displaying the data from experiments such as the tests of bridge strength helps you see patterns and make predictions. For linear data, you can usually find a graph and an equation to express the approximate relationship between the variables.

The table and graph below show sample data from Investigation 1.

Bridge Thickness (layers)	Breaking Weight (pennies)
1	12
2	18
3	32
4	40
5	48
6	64



You can see that the points do not lie exactly on a line, but you can draw a line that is a good match for the data pattern. Drawing such a line gives you a model for the data.

- What line would you draw as a model for the data pattern?
- How would you find an equation for this linear function?

Problem 2.1

A The lines in Figure 1 and Figure 2 below represent two different equations or models for the data. The line in Figure 1 connects the points at the left and right ends of the plot. The line in Figure 2 passes among the points but hits none exactly.

1. Which of the two lines seems to fit the data better? Explain your choice.
2. Can you sketch a line that is a better fit than these two?

Figure 1

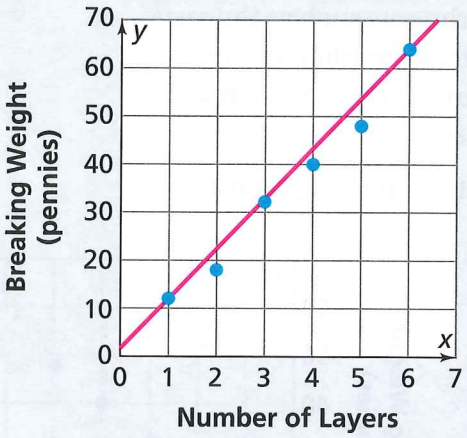
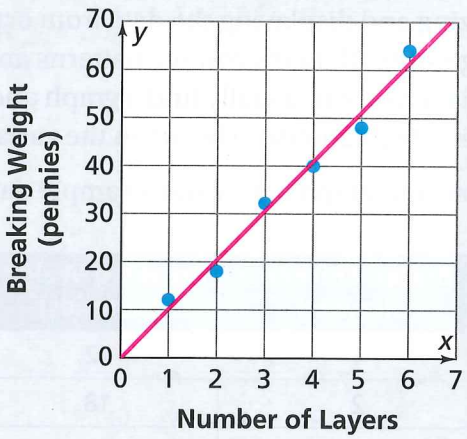


Figure 2



B To find out how accurate each model is, you can calculate errors in the predictions made by the model. Those errors are the differences between the actual data and what each model predicts. Each such error is called a **residual**. Copy and complete the table below for Figure 1 and Figure 2.

Number of Layers	1	2	3	4	5	6
Breaking Weight (pennies)						
Actual	12	18	32	40	48	64
Predicted by Model	■	■	■	■	■	■
Residual (actual − predicted)	■	■	■	■	■	■

1. The first line goes through points (1, 12) and (6, 64). The equation for this line is $y = 10.4x + 1.6$. How would you describe the errors of prediction, or residuals, for this linear model?

continued on the next page >

Problem 2.1 continued

2. Sally thinks the equation of the second modeling line is $y = 10x$. Do you agree with Sally? Explain.
3. How would you describe the errors of prediction, or residuals, for Sally's linear model?
4. Do the residuals suggest that one of the models is better than the other? Explain.

C You can find linear models for many situations.

The Student Paint Crew gives weekend and vacation jobs painting houses and apartments to high school and college students. The time a job takes depends on the area to be painted.

Prior jobs give some data relating job area (in units of 1,000 square feet) and time to paint (in hours). The table below shows some of the data.

Area (1,000 sq ft)	1	3	5	8	10
Time (hours)	3	8	12	20	25

1. Plot the given (area, time) data on a graph.
2. Draw a line to match the data pattern.
3. Find the equation of your modeling line.
4. Find the residuals for the model you develop. Explain what they tell you about the accuracy of the linear model.

ACE Homework starts on page 45.

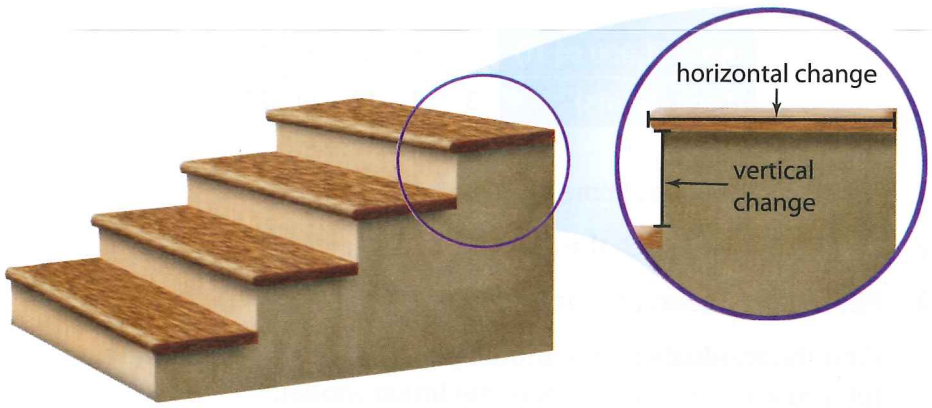
2.2 Up and Down the Staircase

Exploring Slope

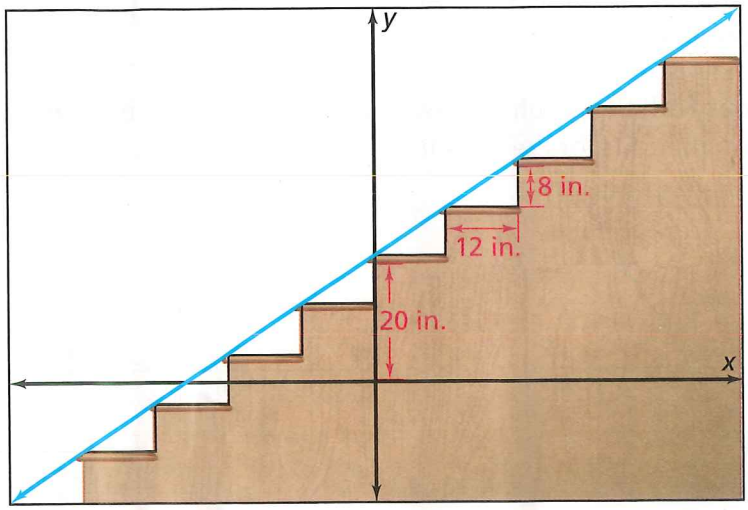
Linear functions are often used as models for patterns in data plots. In *Moving Straight Ahead*, you learned several facts about equations representing linear functions.

- Any linear function can be expressed by an equation in the form $y = mx + b$.
- The value of the coefficient m tells the rate at which the values of y increase (or decrease) as the values of x increase by 1. Since m tells you the change in y for every one-unit change in x, it can also be called the *unit rate*. A unit rate is a rate in which the second number is 1, or 1 of a quantity.
- The value of m also tells the steepness and direction (upward or downward) of the graph of the function.
- The value of b tells the point at which the graph of the function crosses the y-axis. That point has coordinates $(0, b)$ and is called the **y-intercept.**

In any problem that calls for a linear model, the goal is to find the values of m and b for an equation with a graph that fits the data pattern well. To measure the steepness of a linear equation graph, it helps to imagine a staircase that lies underneath the line.



The steepness of a staircase is commonly measured by comparing two numbers, the *rise* and the *run*. The rise is the vertical change from one step to the next, and the run is the horizontal change from one step to the next.



The steepness of the line is the ratio of rise to run. This ratio is the **slope** of the line.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

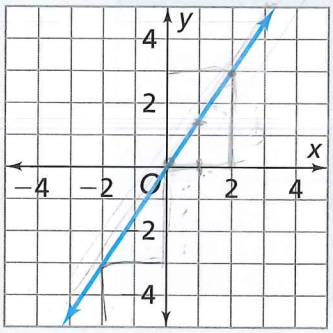
In the diagram of the staircase, the slope of the line is $\frac{2}{3}$. The y-intercept is $(0, 20)$. So the equation for the linear function is $y = \frac{2}{3}x + 20$.

Problem 2.2

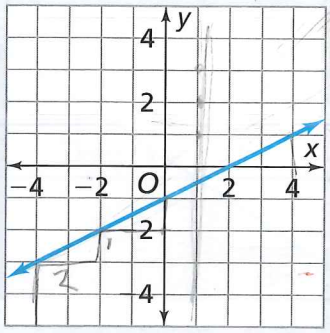
Use the data given in each question to find the equation of the linear function relating y and x.

A For the functions with the graphs below, find the slope and y-intercept. Then write the equations for the lines in the form $y = mx + b$.

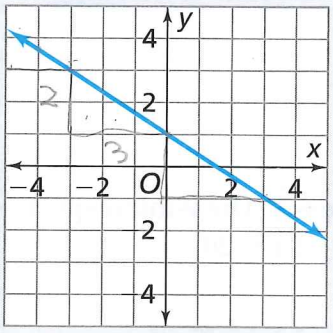
1.



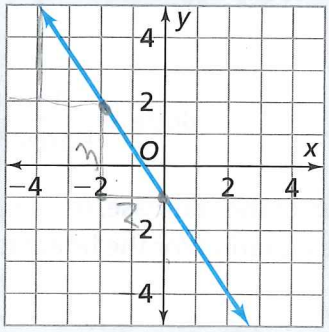
2.



3.



4.



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Problem 2.2 continued

B **1.** Find equations for the linear functions that give these tables. Write them in the form $y = mx + b$.

a.

x	−2	−1	0	1	2
y	−1	1	3	5	7

b.

x	−6	−2	2	6	10
y	−4	−2	0	2	4

2. For each table, find the unit rate of change of y compared to x.

3. Does the line represented by this table have a slope that is greater than or less than the equations you found in part 1(a) and part 1(b)?

x	−1	0	1	2	3
y	4	1	−2	−5	−8

C The points $(4, 2)$ and $(-1, 7)$ lie on a line. $m = \frac{\Delta y}{\Delta x} = \frac{5}{-5}$

1. What is the slope of the line?

2. Find two more points that lie on this line. Describe your method.

3. Yvonne and Jackie observed that any two points on a line can be used to find the slope. Are they correct? Explain why or why not.

D Kevin said that the line with equation $y = 2x$ passes through the points $(0, 0)$ and $(1, 2)$. He also said the line with equation $y = -3x$ passes through the points $(0, 0)$ and $(1, -3)$. In general, lines with equations of the form $y = mx$ always pass through the points $(0, 0)$ and $(1, m)$. Is he correct? Explain.

E What is the slope of a horizontal line? Of a vertical line?

ACE Homework starts on page 45.

2.3 Tree Top Fun

Equations for Linear Functions

Tree Top Fun (TTF, for short) runs adventure sites with zip lines, swings, rope ladders, bridges, and trapezes. The company uses mathematical models to relate the number of customers, prices, costs, income, and profit at its many locations.

Problem 2.3

When finding an equation, it may help to calculate values of the dependent variable for some specific values of the independent variable. Then you can look for a pattern in those calculations. You can use the information given in words, tables of data, and graphs.

A Use what you know about linear equations to work out models for the Tree Top Fun business. Find an equation for each of the linear functions described below.

1. The standard charge per customer at TTF is \$25. Write an equation that relates the daily income I to the number n of customers.
2. Each TTF site has operating costs of \$500 per day. Write an equation that relates daily profit P to the number n of customers.
3. One TTF site bought a new rope bridge for \$4,500. TTF will make monthly payments of \$350 until the bill is paid. Write an equation for the unpaid balance B after m monthly payments.

continued on the next page >

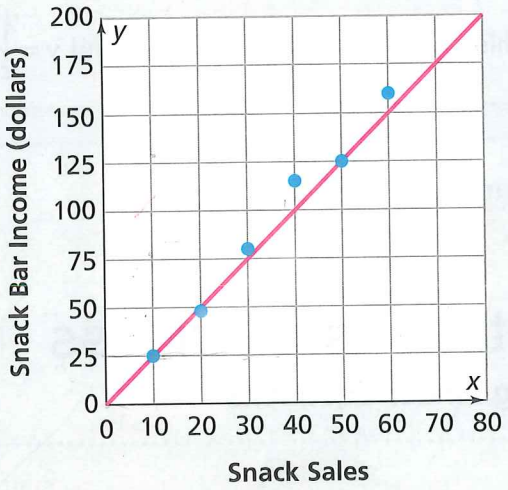
Problem 2.3 continued

B One operator of a Tree Top Fun franchise suggested the group admission fees in the table below.

Number in Group	1	2	3	4	5	10	15	20
Admission (dollars)	75	90	105	120	135	210	285	360

1. Explain how you know the relationship between the admission fee for a group and the number of people in the group is linear.
2. What are the slope and y-intercept of the graph of the data?
3. What equation relates admission fee A to the number n in the group?

C The owners of Tree Top Adventures opened a snack bar at one site. The graph below shows the income from snack sales for six different days. What is the equation of the linear model on the graph?



D Suppose you are asked to write an equation of the form $y = mx + b$ to represent a linear function. What is your strategy for each situation?

1. You are given a description of the function in words.
2. You are given two or more (x, y) values or a table of (x, y) values.
3. You are given a graph showing points with coordinates.

continued on the next page >

Problem 2.3 continued

E A state mathematics test asked students to find equations for linear functions. Two students, Dana and Chris, gave the answers below.

1. To find an equation for the line with slope -3 that passes through the point $(4, 3)$, Dana wrote the following steps. Is he correct? Explain.

> $y = -3x + b$, so $3 = -3(4) + b$
>
> This means $b = 15$ and $y = -3x + 15$.

2. To find an equation for the line that passes through points $(4, 5)$ and $(6, 9)$, Chris wrote the following steps. Is she correct? Explain.

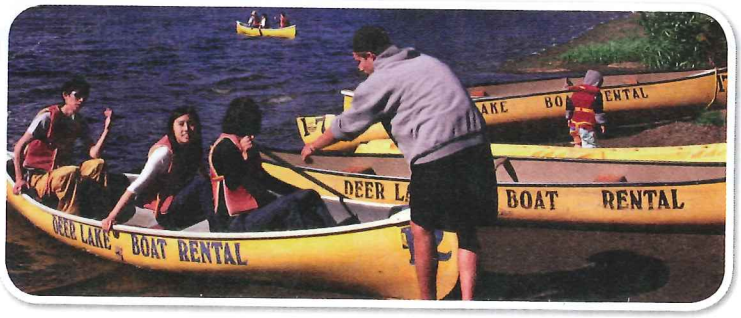
> $m = \frac{6-4}{9-5}$, so $y = \frac{1}{2}x + b$
>
> This means $5 = \frac{1}{2}(4) + b$, $b = 3$, and $y = \frac{1}{2}x + 3$.

ACE Homework starts on page 45.

2.4 Boat Rental Business

Solving Linear Equations

Sandy's Boat House rents canoes at a cost advertised as \$9 per hour for trips on the Red Cedar River. The owner actually gives customers a better deal. She was once a mathematics teacher, and she uses the equation $c = 0.15t + 2.50$ to find the charge c in dollars for renting a canoe for t minutes.



Problem 2.4

When Rashida and Serena applied for jobs at Sandy's, the owner gave them the following test questions to see if they could calculate charges correctly.

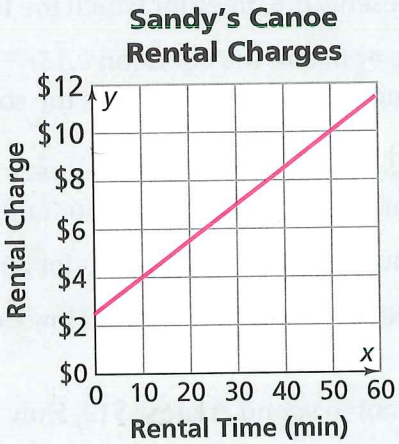
A 1. Explain what the numbers in the equation $c = 0.15t + 2.50$ tell you about the situation.

2. How much does it cost to rent a canoe for 25 minutes?

3. A customer is charged \$9.25. How long did he use the canoe?

4. A customer has \$6 to spend. How long can she use a canoe?

B The owner gave Rashida a graph of $c = 0.15t + 2.50$ and asked her how it could be used to estimate answers to Question A. How could Rashida respond?



C The owner asked Serena to explain how she could use the table below to estimate answers to Question A. How could Serena respond?

Canoe Rental Time (min)	10	20	30	40	50	60
Rental Charge (dollars)	4.00	5.50	7.00	8.50	10.00	11.50

continued on the next page >

Problem 2.4 continued

D The owner next asked Serena and Rashida to work together to find exact answers, not estimates, for Question A, parts (3) and (4).

1. For part (3) of Question A, the girls solved the linear equation $0.15t + 2.50 = 9.25$. They reasoned as follows:

   - If $0.15t + 2.50 = 9.25$, then $0.15t = 6.75$.
   - If $0.15t = 6.75$, then $t = 45$.
   - To check the answer, substitute 45 for t: $0.15(45) + 2.50 = 9.25$.

   Are Serena and Rashida correct? How do you know?

2. For Question A, part (4), Rashida said, "The customer can use the canoe for 23.3 minutes if she has \$6." Serena said there are other possibilities—for example, 20 minutes or 15 minutes. Rashida said you can find the answer by solving the **inequality** $0.15t + 2.50 \leq 6$. This inequality represents the times for which the rental costs at most \$6.

   Use the table, graph, and the equation $0.15t + 2.50 = 6$ to find all times for which the inequality is true. Express the solution as an inequality.

E River Fun Boats rents paddle boats. The equation $c = 4 + 0.10t$ gives the charge in dollars c for renting a paddle boat for t minutes.

1. What is the charge to rent a paddle boat for 20 minutes?
2. A customer at River Fun is charged \$9. How long did the customer use a paddle boat?
3. Suppose you want to spend at most \$12. How long could you use a paddle boat?

ACE Homework starts on page 45.

2.5 Amusement Park or Movies

Intersecting Linear Models

A company owns two attractions in a resort area—the Big Fun amusement park and the Get Reel movie multiplex. At each attraction, the number of visitors on a given day is related to the probability of rain. The company wants to be able to predict Saturday attendance at each attraction in order to assign its workers efficiently.

This table gives attendance and rain-forecast data for several recent Saturdays.

Saturday Resort Attendance

Probability of Rain (%)	0	20	40	60	80	100
Big Fun Attendance	1,000	850	700	550	400	250
Get Reel Attendance	300	340	380	420	460	500

- What equations model the relationships of Big Fun and Get Reel attendance to the probability of rain?
- For what probability of rain will one attraction be more popular than the other?

Problem 2.5

A Use the table to find linear functions relating the probability of rain p to the following quantities.

1. Saturday attendance F at Big Fun
2. Saturday attendance R at Get Reel

Saturday Resort Attendance

Probability of Rain (%)	0	20	40	60	80	100
Big Fun Attendance	1,000	850	700	550	400	250
Get Reel Attendance	300	340	380	420	460	500

B Use your functions from Question A to answer these questions. Show your calculations and explain your reasoning.

1. Suppose there is a 50% probability of rain this Saturday. What is the expected attendance at each attraction?
2. Suppose 475 people visited Big Fun one Saturday. Estimate the probability of rain on that day.
3. What probability of rain gives a predicted Saturday attendance of at least 360 people at Get Reel?
4. Is there a probability of rain for which the predicted attendance is the same at both attractions?
5. For what probability of rain is attendance at Big Fun likely to be greater than at Get Reel?
6. For what probability of rain is attendance at Big Fun likely to be less than at Get Reel?

Homework starts on page 45.

Applications

1. Below are some results from the bridge experiment in a CMP class.

Bridge-Thickness Experiment

Number of Layers	2	4	6	8
Breaking Weight (pennies)	15	30	50	65

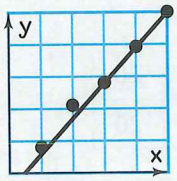
a. Plot the (number of layers, breaking weight) data. Draw a line that models the data.

b. Find an equation for the line you drew.

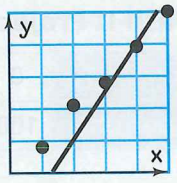
c. Use your equation to predict the breaking weights of paper bridges 3, 5, and 7 layers thick.

2. The two graphs below show the same data points. Which line models the data better? Explain.

Student 1

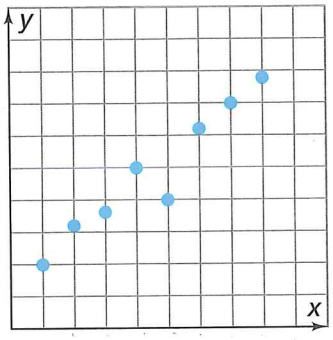


Student 2

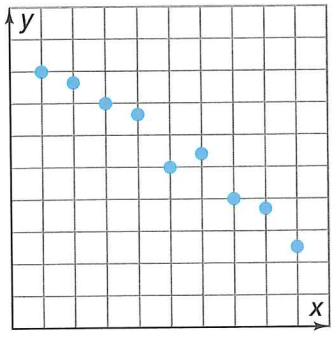


3. Copy each graph onto grid paper. Draw a line that fits each set of data as closely as possible. Describe the strategies you used.

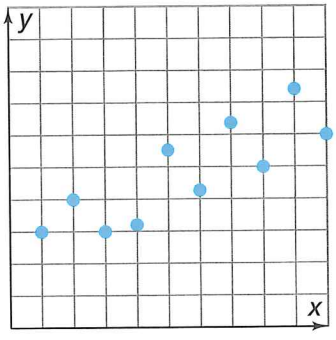
Graph A



Graph B



Graph C



4. This table gives the average weights of Chihuahuas from birth to 16 weeks.

Average Weights for Chihuahuas

Age (wk)	0	2	4	6	8	10	12	14	16
Weight (oz)	4	9	13	17.5	21.5	25	30	34	39

SOURCE: *The Complete Chihuahua Encyclopedia*

a. Graph the (age, weight) data. Draw a line that models the data pattern.

b. Write an equation of the form $y = mx + b$ for your line. Explain what the values of m and b tell you about this situation.

c. Use your equation to predict the average weight of Chihuahuas for odd-numbered ages from 1 to 15 weeks.

d. What average weight does your linear model predict for a Chihuahua that is 72 weeks old? Explain why this prediction is likely to be inaccurate.

5. The U-Wash-It car wash did market research to determine how much to charge for a car wash. The company made this table based on its findings.

U-Wash-It Projections

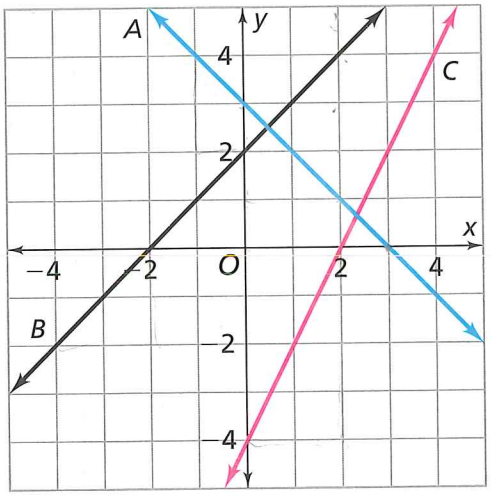
Price per Wash ($)	0	5	10	15	20
Customers Expected per Day	100	80	65	45	20

a. Graph the (price, expected customers) data. Draw a line that models the data pattern.

b. Write an equation in the form $y = mx + b$ for your graph. Explain what the values of m and b tell you about this situation.

c. Use your equation to find the number of customers expected for prices of \$2.50, \$7.50, and \$12.50.

6. Here is a graph of three lines.



a. Complete the table.

Line	Constant Rate of Change	*y*-intercept	*x*-intercept
A			
B			
C			

b. Here are the equations of the three lines. Match each line with its equation.

equation D: $y = 2 + x$ | equation E: $y = -4 + 2x$ | equation F: $y = 3 - x$

line A | line B | line C

7. Two points determine a line.

a. Which of these points are on the line that passes through (0, 3) and (2, 5)?

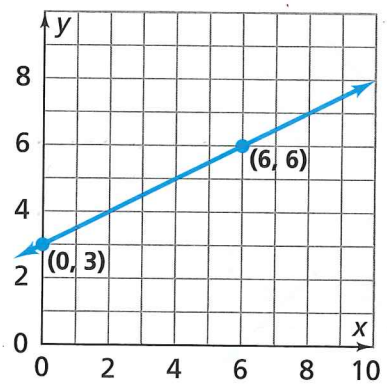
(4, 7) (4, 8) (4, 10)

b. Which of these points are on the line that passes through (−2, 10) and (1, 4)?

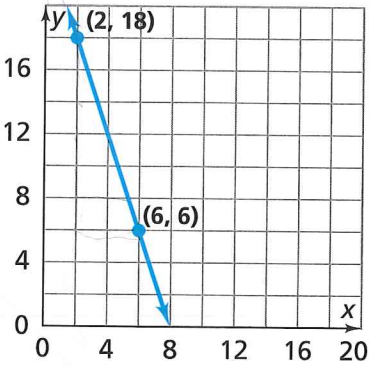
(2, 0) (2, 2) (2, 10)

8. Find the slope, y-intercept, and equation of each line.

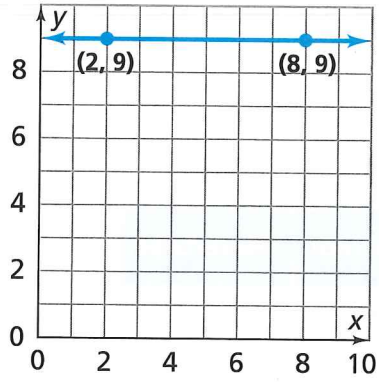
a.



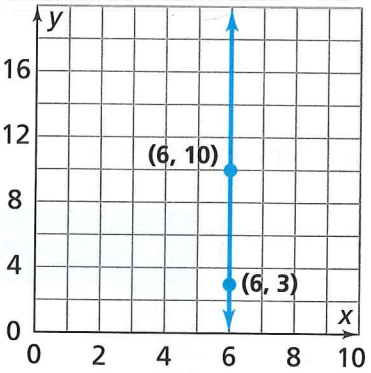
b.



c.



d.



Assume that the relationships in Exercises 9–12 are linear.

9. Kaya buys a \$20 phone card. She is charged \$.15 per minute for calls. What equation gives the value v left on her card after she makes t minutes of calls?

10. A typical American baby weighs about 8 pounds at birth and gains about 1.5 pounds per month for the first year of life. What equation relates weight w in pounds to age a in months?

1 month **6 months** **1 year**

11. Dakota lives 1,500 meters from school. She leaves for school, walking at a speed of 60 meters per minute. Write an equation for her distance d in meters from school after she walks for t minutes.

12. A car can average 140 miles on 5 gallons of gasoline. Write an equation for the distance d in miles the car can travel on g gallons of gas.

13. Write a linear equation relating x and y for each table.

a.

x	0	3	6	10
y	2	8	14	22

b.

x	0	3	6	10
y	20	8	−4	−20

c.

x	2	4	6	8
y	5	8	11	14

d.

x	0	3	6	9
y	20	11	2	−7

For Exercises 14–19, find an equation for the line that satisfies the conditions.

14. slope 4.2; y-intercept (0, 3.4)

15. slope $\frac{2}{3}$; y-intercept (0, 5)

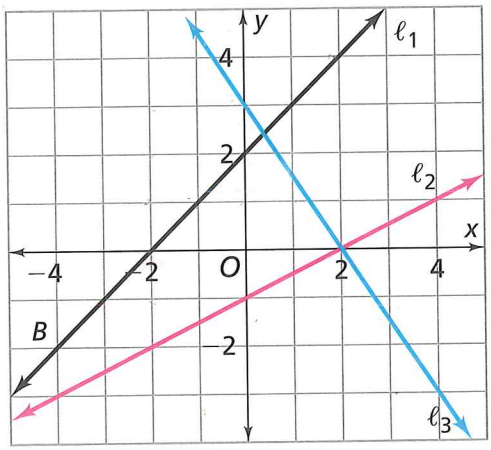
16. slope 2; passing through (4, 12)

17. passing through (0, 15) and (5, 3)

18. passing through (−2, 2) and (5, −4)

19. parallel to the line with equation $y = 15 - 2x$ and passing through (3, 0)

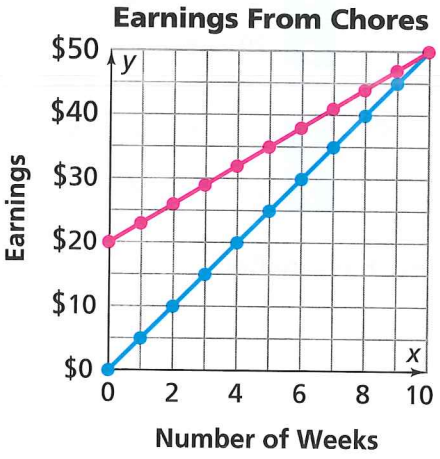
20. Write an equation for each line in the graph below.



21. Anchee and Jonah earn weekly allowances for doing chores over the summer.

- Anchee's father pays her \$5 each week.
- Jonah's mother paid him \$20 at the beginning of the summer and now pays him \$3 each week.

The relationships between number of weeks and dollars earned are shown in this graph.



a. Which line represents Jonah's earnings? Which line represents Anchee's earnings? Explain.

b. Write two linear equations in the form $y = mx + b$ to show the relationships between Anchee's earnings and the number of weeks she works and between Jonah's earnings and the number of weeks he works.

c. In each equation, what do the values of m and b tell you about the relationship between the number of weeks and the dollars earned?

d. What do the values of m and b tell you about each line?

For Exercises 22–25, do the following.

(a) Solve the equation and show your steps.

(b) Graph the related linear function. (For example, for $5.5x + 32 = 57$, graph $y = 5.5x + 32$.)

(c) Label the point on the graph that gives the solution.

22. $5.5x + 32 = 57$

23. $-24 = 4x - 12$

24. $5x - 51 = 24$

25. $74 = 53 - 7x$

26. At Water Works Amusement Park, the daily profit from the concession stands depends on the number of park visitors. The equation $P = 2.50v - 500$ gives the estimated profit P in dollars if v people visit the park. In parts (a)–(c), use a graph to estimate the answer. Then find the answer by writing and solving an equation or inequality.

a. For what number of visitors will the profit be about \$2,000?

b. One day 200 people visit the park. What is the approximate concession-stand profit for that day?

c. For what number of visitors will the profit be at least \$500?

27. The following formulas give the fare f, in dollars, charged by two bus companies for trips of d miles.

- Transcontinental: $f = 0.15d + 12$
- Intercity Express: $f = 5 + 0.20d$

In parts (a)–(c), use a graph to estimate the answer. Then find the answer by writing and solving an equation or inequality.

a. For Transcontinental, how many miles is a trip that costs \$100?

b. For Intercity Express, how far can a person travel for a fare that is at most \$100?

c. Is there a distance for which the fare for the two bus lines is the same? If so, give the distance and the fare.

In Exercises 28–30, solve each equation. Show the steps in your solutions.

28. $5x + 7 = 3x - 5$ **29.** $7 + 3x = 5x - 5$ **30.** $2.5x - 8 = 5x + 12$

In Exercises 31–34, find at least three values of x for which the inequality is true.

31. $4x \leq 12$ **32.** $3x < 18$

33. $4x + 5 \leq 13$ **34.** $3x - 9 \leq 18$

35. Every Friday, the mechanic for Columbus Public Schools records the miles driven and gallons of gas used by each school bus. One week, the mechanic records the data below.

Data for Columbus Bus Fleet

Bus Number	1	2	3	4	5	6	7	8
Gas Used (gal)	5	8	12	15	18	20	22	25
Miles Driven	80	100	180	225	280	290	320	375

a. Write a linear equation that models the relationship between miles driven d and gallons of gas used g.

b. Use your equation to predict the number of miles a school bus could travel on 10 gallons of gas.

c. Use your equation to predict the number of gallons of gas required to drive a school bus 250 miles.

d. What do the values of m and b in your equation $d = mg + b$ tell about the fuel efficiency of the school bus fleet?

36. One of the most popular items at a farmers' market is sweet corn. This table shows relationships among the price of the corn, the supply of corn (how much corn the market has), and the demand for the corn (how much corn people want to buy).

Sweet Corn Supply and Demand

Price per Dozen ($)	1.00	1.50	2.00	2.50	3.00	3.50
Demand (dozens)	200	175	140	120	80	60
Supply (dozens)	40	75	125	175	210	260

a. Why do you think the demand for corn decreases as the price goes up?

b. Why do you think the supply of corn increases as the price goes up?

c. Write a linear equation that models the relationship between demand d and price p.

d. Write a linear equation that models the relationship between supply s and price p.

e. Use graphs to estimate the price for which the supply equals the demand. Then find the price by solving symbolically.

Connections

37. Tell whether each table represents a linear relationship. Explain your reasoning.

a.

x	2	4	6	8	10	12	14
y	0	1	2	3	4	5	6

b.

x	1	2	3	4	5	6	7
y	0	3	8	15	24	35	48

c.

x	1	4	6	7	10	12	16
y	2	−1	−3	−4	−7	−9	−13

38. For parts (a)–(d), copy the table. Then use the equation to complete the table. Tell whether the relationship is linear. Explain your reasoning.

a. $y = -3x - 8$

x	−5	−2	1	4
y	■	■	■	■

b. $y = 4(x - 7) + 6$

x	−3	0	3	6
y	■	■	■	■

c. $y = x(3x + 2)$

x	−3	0	3	6
y	■	■	■	■

d. $y = 4 - 3x$

x	−3	0	3	10
y	■	■	■	■

Copy each pair of numbers in Exercises 39–44. Insert $<$, $>$, or $=$ to make a true statement.

39. $-5 \blacksquare 3$

40. $\frac{2}{3} \blacksquare \frac{1}{2}$

41. $\frac{9}{12} \blacksquare \frac{3}{4}$

42. $3.009 \blacksquare 3.1$

43. $-\frac{2}{3} \blacksquare -\frac{1}{2}$

44. $-4.25 \blacksquare -2.45$

45. Madeline sets the scale factor on a copy machine at 150%. She then uses the machine to copy a polygon. Write an equation that relates the perimeter of the polygon after the enlargement, a, to the perimeter before the enlargement, b.

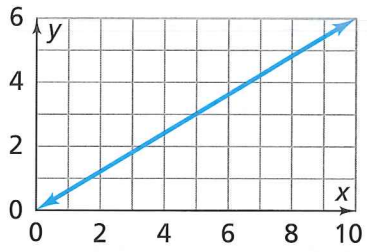
For Exercises 46–54, evaluate the expression without using a calculator.

46. $-15 + (-7)$ **47.** $-7 - 15$ **48.** $-7 - (-15)$

49. $-15 + 7$ **50.** $-20 \div 5$ **51.** $-20 \div (-5)$

52. $20 \div (-4)$ **53.** $-20 \div (-2.5)$ **54.** $-20 \bullet (-2.5)$

55. You can express the slope of a line in different ways. The slope of the line below is $\frac{6}{10}$, or 0.6. You can also say the slope is 60% because the rise is 60% of the run.



These numbers represent slopes of lines.

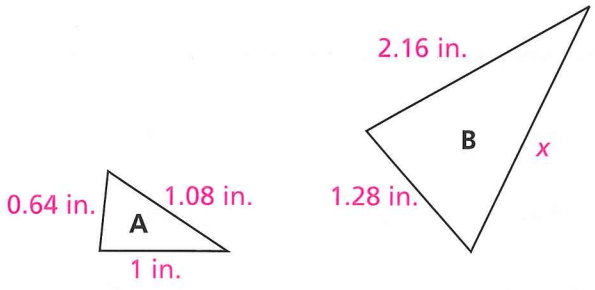
$\frac{-4}{-2}$ 60% $\frac{4}{4}$ 1.5 150% 200%

a. Which numbers represent the same slope?

b. Which number represents the greatest slope?

c. Which number represents the least slope?

56. The figures below are *similar*. (They have the same shape but are different sizes.)



a. Find the value of x.

b. What is the scale factor from Triangle A to Triangle B?

c. What is the scale factor from Triangle B to Triangle A?

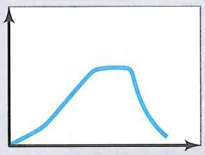
d. How are the scale factors in parts (b) and (c) related?

57. Read the following stories and look at the graphs.

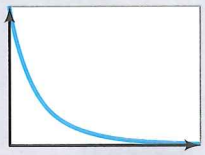
a. Match each story with a graph. Tell how you would label the axes of the graph. Explain how each part of the story is represented in the graph.

- **Story 1** A parachutist is taken up in a plane. After she jumps, the wind blows her off course. She ends up tangled in the branches of a tree.
- **Story 2** Ella puts some money in the bank. She leaves it there to earn interest for several years. Then one day, she withdraws half the money in the account.
- **Story 3** Gerry has a big pile of gravel to spread on his driveway. On the first day, he moves half the gravel from the pile to his driveway. The next day he is tired and moves only half of what is left. The third day he again moves half of what is left in the pile. He continues in this way until the pile has almost disappeared.

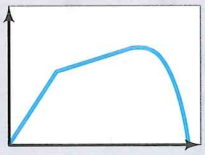
Graph A



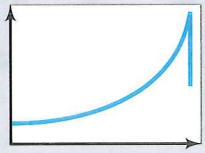
Graph B



Graph C



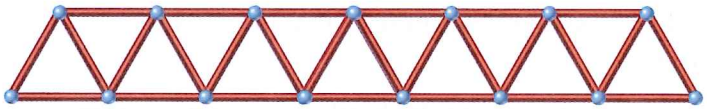
Graph D



b. One of the graphs does not match a story. Make up your own story for that graph.

Extensions

58. Recall that Custom Steel Products builds trusses from steel pieces. Here is a 7-foot truss.



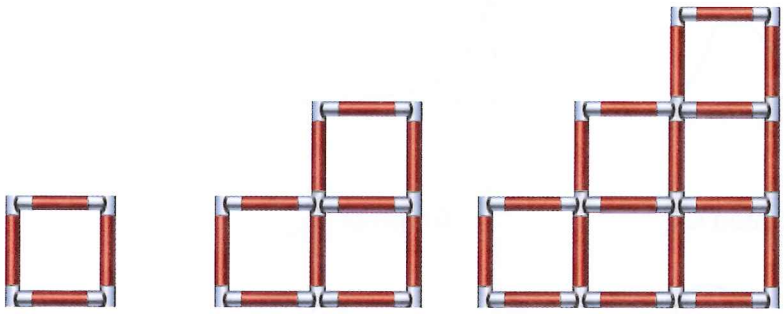
7-foot truss made from 27 rods

a. Which of these formulas represents the relationship between truss length L and number of pieces r?

$r = 3L$ $\quad r = L + (L - 1) + 2L$ $\quad r = 4(L - 1) + 3$ $\quad r = 4L - 1$

b. How might you have reasoned to come up with each formula?

59. **Multiple Choice** Recall that Custom Steel Products uses steel pieces to make staircase frames. Here are staircase frames with 1, 2, and 3 steps.



1 step made from 4 rods

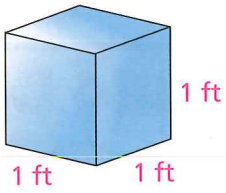
2 steps made from 10 rods

3 steps made from 18 rods

Which of these formulas represents the relationship between the number of steps n and number of pieces r?

A. $r = n^2 + 3n$ **B.** $r = n(n + 3)$ **C.** $r = n^2 + 3$ **D.** $r = (n + 3)n$

Custom Steel Products builds cubes out of square steel plates measuring 1 foot on a side. Below is a 1-foot cube. Use this information for Exercises 61–63.



60. How many square plates are needed to make a 1-foot cube?

61. **Multiple Choice** Suppose CSP wants to triple the dimensions of the cube. How many times greater than the original area will the surface area of this larger cube be?

A. 2 **B.** 3 **C.** 4 **D.** 9

62. **Multiple Choice** Suppose CSP triples the dimensions of the original cube. How many times the volume of the original cube is the volume of the new cube?

F. 8 **G.** 9 **H.** 27 **J.** 81

63. A bridge-painting company uses the formula $C = 5{,}000 + 150L$ to estimate painting costs. C is the cost in dollars, and L is the length of the bridge in feet. To make a profit, the company increases a cost estimate by 20% to arrive at a bid price. For example, if the cost estimate is \$10,000, the bid price will be \$12,000.

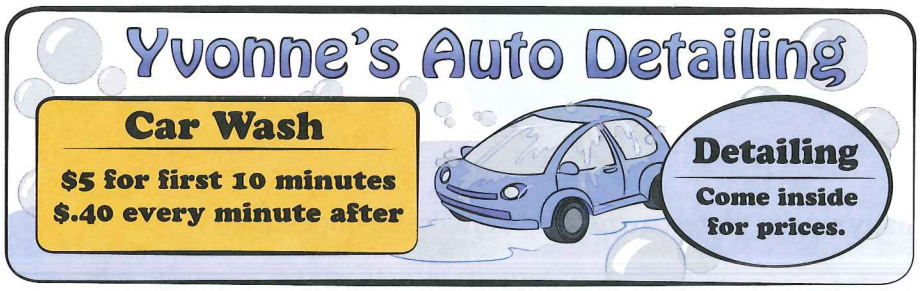
a. Find bid prices for bridges 100 feet, 200 feet, and 400 feet long.

b. Write a formula relating the final bid price to bridge length.

c. Use your formula to find bid prices for bridges 150 feet, 300 feet, and 450 feet long.

d. How would your formula change if the markup for profit was 15% instead of 20%?

64. At Yvonne's Auto Detailing, car washes cost \$5 for any time up to 10 minutes, plus \$.40 per minute after that. The managers at Yvonne's are trying to agree on a formula for calculating the cost c for a t-minute car wash.



a. Sid thinks $c = 0.4t + 5$ is correct. Is he right?

b. Tina proposes the formula $c = 0.4(t - 10) + 5$. Is she right?

c. Jamal says Tina's formula can be simplified to $c = 0.4t + 1$. Is he right?

Write an equation for each relationship.

65. The Bluebird Taxi Company charges \$3.00 for the first 2 miles of any trip and \$2.40 for each mile after that. How is the taxi fare related to the distance of a trip?

66. An airport offers free parking for 30 minutes and then charges \$2.00 for each hour after that. How is the price for parking related to the total time a car is parked?

67. The Regal Cinema makes \$6.50 on each ticket sold. The cinema has operating expenses of \$750 per day, as well. How is daily profit related to number of tickets sold?

68. Rush Computer Repair sends technicians to businesses to fix computers. Technicians charge a fixed fee of \$50, plus \$50 per hour. How is total cost for a repair related to the time the repair takes?

Mathematical Reflections 2

In this Investigation, you learned how to find linear models for data patterns. You also developed skill in writing linear equations, practiced translating verbal descriptions into linear equations, and extended your knowledge of solving linear equations. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Why** is it helpful to use a linear model for a set of data?
2. **When** does it make sense to choose a linear function to model a set of data?
3. **How** would you find the equation for a linear function in the following situations?
   a. **You** are given a description of the variables in words.
   b. **You** are given a table of values for the variables.
   c. **You** are given a graph of sample data points.
4. **What** strategies can you use to solve a linear equation such as $500 = 245 + 5x$?
5. **What** kind of mathematical sentences express "at least" and "at most" questions about linear functions?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Elena described her thoughts in the following way:

> *In Problem 2.1, we learned about computing residuals as a way to evaluate different models (lines) for a set of data. I was confused by this idea. Jamie and I added rows to the table that was included with the problem. Tables can really help organize ideas. When we used one model, we computed the differences of the actual values and the values predicted by the model line.*
>
> *For Model 1, we got residuals 0, −4, −1, −3, −6, and 0, or a total of −14. For Model 2, the total of the residuals was 4. We think Model 2 might be a better model to use.*
>
> **Common Core Standards for Mathematical Practice**
>
> **MP7** Look for and make use of structure.

- What other Mathematical Practices can you identify in Elena's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 3

Inverse Variation

In Investigation 1, you explored the relationship of strength, number of layers, and length of a bridge. You found that the relationship between strength and number of layers was approximately linear. You also found that the relationship between strength and length was not linear. In this investigation, you will explore other nonlinear functions.

3.1 Rectangles With Fixed Area

In recent years, the populations of many small towns have declined as residents move to large cities for jobs. The town of Roseville has a plan to attract new residents.

Roseville offers free land to "homesteaders" who are willing to build houses. Each lot is rectangular and has an area of 21,780 square feet. The lengths and widths of the lots vary.

The town planners want a quick way to check lot sizes for the new homesteaders.

Common Core State Standards

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

Also 8.F.A.1, 8.F.B.5, 8.SP.A.1, A-CED.A.2, A-CED.A.4, F-IF.C.8, F-BF.A.1a, S-ID.B.6a

- What function relates the length and width of rectangles with area 21,780 square feet?
- What patterns appear in tables and graphs of that function?

Problem 3.1

In Problem 3.1, you will look at patterns in length and width values for rectangles with fixed area.

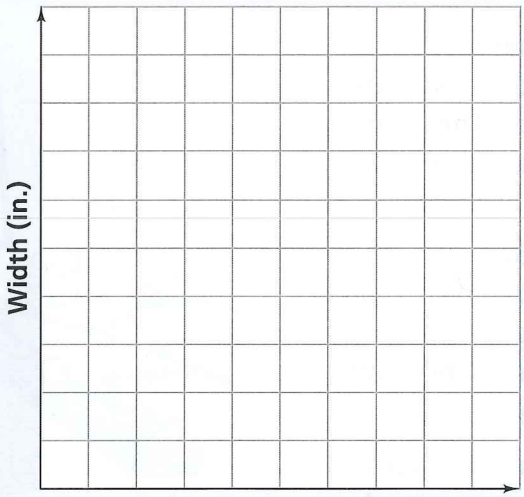
A **1.** Copy and complete this table.

Rectangles With Area 24 in.^{2}

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	■	■	■	■	■	■	■	■

2. Plot your (length, width) data from part (1) on a graph like the one below. Then draw a line or curve that models the pattern in the data.

Rectangles With Area 24 in.^{2}



3. Describe the pattern of change in the width of a rectangle as the length increases. Is the relationship between length and width linear?

4. Write an equation relating width w to length ℓ for rectangles with an area of 24 square inches.

continued on the next page >

Problem 3.1 *continued*

B Now consider rectangles with area of 32 square inches.

1. Write an equation for the relationship between the width w and the length ℓ.
2. Make a table and a graph of values for w and ℓ. Show lengths from 1 to 10 inches.

C Compare tables, graphs, and rules for the functions relating width and length of rectangles with area 24 square inches and area 32 square inches. Then use your results to answer the Roseville planners' questions about lots with area 21,780 square feet.

Homework starts on page 69.

3.2 Distance, Speed, and Time

The relationship between length and width for rectangles with a fixed area is not linear. As ℓ increases, w decreases, but not at a constant rate. The graph is a curve showing how ℓ and w relate to each other.

The relationship between ℓ and w is an example of an important relationship called an inverse variation. Two nonzero variables x and y are related by an **inverse variation** if

$$y = \frac{k}{x} \quad \text{or} \quad xy = k$$

where k is a constant other than 0.

Here are some questions about inverse variation that you can keep in mind as you work on the following problems about distance, speed, and time of travel.

- How does the value of one variable change as the value of the other changes?
- How is that pattern of change shown in a table of data and on a graph?
- What equation shows how the two variables are related?

Problem 3.2

Inverse variation occurs in many situations. You have probably thought about how the length of a trip depends on speed.

A Mr. Cordova lives in Detroit, Michigan. He often travels to Baltimore, Maryland, to visit his grandfather. The trip is about 500 miles each way. Here are Mr. Cordova's notes for his trips to Baltimore last year.

Date	Notes	Travel Time
February 15	Traveled by plane.	1.5 hours
May 22	Drove.	10 hours
July 3	Drove. Stopped for repairs.	14 hours
November 23	Flew. Flight delayed.	4 hours
December 23	Took overnight train.	18 hours

1. Calculate the average speed in miles per hour for each trip. Record your results in a table like this.

Mr. Cordova's Baltimore Trips

Travel Time (h)					
Average Speed (mi/h)					

2. Plot the data on a graph. Draw a line or curve to model the data. Describe the change in average speed as travel time increases.
3. Write an equation for the relationship between time t and speed s.
4. Is the relationship between distance and time an inverse variation? Explain why or why not.

continued on the next page >

Problem 3.2 continued

B The Cordova family is planning a trip of 300 miles to Mackinac Island, near the upper peninsula of Michigan. Mr. Cordova does some calculations to see how the travel time will change if the family drives at different average speeds.

Travel Times for Different Speeds

Average Speed (mi/h)	30	40	50	60	70
Travel Time (h)	10	7.5	6	5	4.3

1. Describe the change in travel time as the average speed increases.
2. What equation relates travel time t to average speed s?
3. How is the pattern relating travel time to average speed shown in a graph of (s, t) data?
4. Is the relationship between travel time and average speed an inverse variation? Explain why or why not.

C Suppose Mr. Cordova decides to aim for an average speed of 50 miles per hour for the trip to Mackinac Island.

1. Make a table and a graph to show how the distance traveled will increase as time passes. Show times from when the family leaves home to when they reach their destination.
2. Describe the pattern of change in distance as time passes. Explain how that pattern is shown by values in your table and points on your graph.
3. Mr. Cordova's sister plans to go to Mackinac Island, also. She can drive from Detroit to the island in 5 hours. Use your table and graph from part (1) to compare Mr. Cordova's average speed to his sister's average speed. Who drives faster?
4. Write equations relating distance traveled d to time t for Mr. Cordova and his sister. How do these equations support your answer to part (3)?
5. Is the equation relating distance and time an inverse variation? Explain why or why not.

ACE Homework starts on page 69.

3.3 Planning a Field Trip

Finding Individual Cost

The science teachers at Everett Middle School want to take their eighth-graders on a field trip to a nature center. It costs $750 to rent the center facilities.

The school budget does not provide funds to rent the nature center, so students must pay a fee. The trip will cost $3 per student if all 250 go. The teachers know, however, that it is unlikely that all students will go. They want a way to find the cost per student for any number of students.

- What kind of relationship between number of students and cost should the teachers expect?
- How can that relationship be expressed with an equation and a graph?

Problem 3.3

To identify the relationship between the number of students and the cost, begin with sample calculations. Then look for a pattern in your results.

A **1.** Copy and complete the table below.

Number of Students	25	50	75	100	125	150	175	200	250
Cost per Student (dollars)	■	■	■	■	■	■	■	■	■

2. Describe the pattern relating the cost per student to the number of students who visit the nature center.

3. Write an equation relating cost per student c to number of students n.

4. Sketch a graph showing how the cost per student changes as the number of students increases.

continued on the next page >

Problem 3.3 continued

B 1. Find the change in the cost per student as the number of students increases in the following ways.

   a. from 10 to 20 b. from 100 to 110 c. from 200 to 210

2. Is the function relating number of students and cost linear? Explain.

3. Do equal increases in numbers of students cause equal decreases in cost per student? Explain.

C How will doubling the number of students change the cost per student? To test your ideas about that question, find answers to these related questions.

1. Find the change in per-student cost as the number of students increases in the following ways.

   a. from 20 to 40 b. from 40 to 80 c. from 80 to 160

2. Describe any patterns you see in your answers to part (1).

3. How does your equation from Question A help to explain the effect of doubling the number of students?

D The science teachers decide to charge \$5 per student for the field trip. They will use any extra money to buy science equipment for the school.

1. Write an equation for the amount A the teachers will collect if n students go on the trip.

2. Sketch a graph of the relationship.

3. Does the graph show a linear relationship or an inverse variation? Explain.

Everett Middle School
Nature Center Field Trip

	\$5 Paid in Advance	\$5 to be Paid Later
Jaden	✓	
Stephen		✓
Maria	✓	
Leonardo	✓	

ACE Homework starts on page 69.

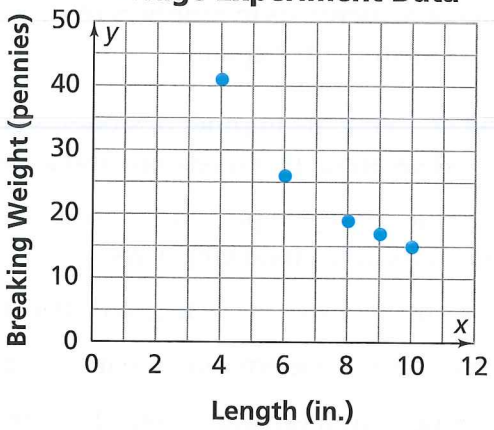
3.4 Modeling Data Patterns

In many real-world problems it is impossible to find an equation that fits given data exactly. For example, consider the table and graph below. They show the bridge experiment data collected by a group of students.

Bridge Experiment Data

Length (in.)	Breaking Weight (pennies)
4	41
6	26
8	19
9	17
10	15

Bridge Experiment Data



- Do the data suggest the relationship between length and breaking weight is linear or an inverse variation?
- What equation would model the relationship well?

Problem 3.4

Use the data from the table and the graph.

A What do you see in the table and the graph that suggests an inverse variation relationship between breaking weight w and bridge length ℓ?

B What value of k would make the equation $k = w\ell$ true for a length of 4 inches? For a length of 6 inches?

C What equation is a good model for the function relating weight w to length ℓ?

D What breaking weights does your model predict for bridges of length 3, 5, 7, and 11 inches?

ACE Homework starts on page 69.

Applications

1. Consider rectangles with an area of 16 square inches.

   a. Copy and complete the table.

Rectangles With Area 16 in.^{2}

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	■	■	■	■	■	■	■	■

   b. Make a graph of the data.

   c. Describe the pattern of change in width as length increases.

   d. Write an equation that shows how the width w depends on the length ℓ. Is the relationship linear?

2. Consider rectangles with an area of 20 square inches.

   a. Make a table of length and width data for at least five rectangles.

   b. Make a graph of your data.

   c. Write an equation that shows how the width w depends on the length ℓ. Is the relationship linear?

   d. Compare and contrast the graphs in this exercise and those in Exercise 1.

   e. Compare and contrast the equations in this exercise and those in Exercise 1.

3. A student collected these data from the bridge-length experiment.

Bridge-Length Experiment

Length (in.)	4	6	8	9	10
Breaking Weight (pennies)	24	16	13	11	9

   a. Find an inverse variation equation that models the data.

   b. Explain how your equation shows that breaking weight decreases as length increases. Is this decrease reasonable for the situation? Explain.

For Exercises 4–7, tell whether the relation between x and y is an inverse variation. If it is, write an equation for the relationship.

4.

x	1	2	3	4	5	6	7	8	9	10
y	10	9	8	7	6	5	4	3	2	1

5.

x	1	2	3	4	5	6	7	8	9	10
y	48	24	16	12	9.6	8	6.8	6	5.3	4.8

6.

x	2	3	5	8	10	15	20	25	30	40
y	50	33	20	12.5	10	6.7	5	4	3.3	2.5

7.

x	0	1	2	3	4	5	6	7	8	9
y	100	81	64	49	36	25	16	9	4	1

8. The marathon is a 26.2-mile race. The best marathon runners can complete the race in a bit more than 2 hours.

a. Make a table and graph that show how the average running speed for a marathon changes as the time to complete the race increases. Show times from 2 to 8 hours in one-hour intervals.

b. Write an equation for the relationship between time t and average running speed s for a marathon.

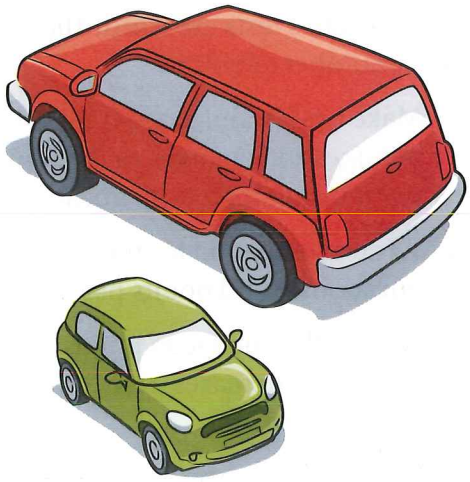
c. Tell how the average running speed changes as the time increases from 2 hours to 3 hours, from 3 hours to 4 hours, and from 4 hours to 5 hours.

d. How do the answers for part (c) show that the relationship between average running speed and time is not linear?

9. Testers drove eight vehicles 200 miles on a track at the same speed. The table below shows the amount of fuel each car used.

Fuel-Efficiency Test

Vehicle Type	Fuel Used (gal)
Large Truck	20
Large SUV	18
Limousine	16
Large Sedan	12
Small Truck	10
Sports Car	12
Compact Car	7
Sub-Compact Car	5



a. Find the fuel efficiency in miles per gallon for each vehicle.

b. Make a graph of the (fuel used, miles per gallon) data. Describe the pattern of change shown in the graph.

c. Write a formula for calculating the fuel efficiency based on the fuel used for a 200-mile test drive.

d. Use your formula to find how fuel efficiency changes as the number of gallons of fuel increases from 5 to 10, from 10 to 15, and from 15 to 20.

e. How do the answers for part (d) show that the relationship between fuel used and fuel efficiency is not linear?

10. The route for one day of a charity bike ride covers 50 miles. Individual participants ride this distance at different average speeds.

a. Make a table and a graph that show how the riding time changes as the average speed increases. Show speeds from 4 to 20 miles per hour in intervals of 4 miles per hour.

b. Write an equation for the relationship between the riding time t and average speed s.

c. Tell how the riding time changes as the average speed increases from 4 to 8 miles per hour, from 8 to 12 miles per hours, and from 12 to 16 miles per hour.

d. How do the answers for part (c) show that the relationship between average speed and time is not linear?

11. Students in Mr. Einstein's science class complain about the length of his tests. He argues that a test with more questions is better for students because each question is worth fewer points. All of Mr. Einstein's tests are worth 100 points. Each question is worth the same number of points.

a. Make a table and a graph that show how the number of points per question changes as the number of questions increases. Show point values for 2 to 20 questions in intervals of 2.

b. Write an equation for the relationship between the number of questions n and points per question p.

c. What is the change in points per question if the number of questions increases from 2 to 4? From 4 to 6? From 6 to 8? From 8 to 10?

d. How do the answers for part (c) show that the relationship between the number of questions and points per question is not linear?

Connections

12. Here are some possible descriptions of a line.

Descriptions of a Line

- *slope positive, 0, or negative*
- *y-intercept positive, 0, or negative*
- *crossing the x-axis to the right of the origin*
- *passing through the origin at (0, 0)*
- *crossing the x-axis to the left of the origin*
- *never crossing the x-axis*

For each equation below, list all of the properties that describe the graph of that equation.

a. $y = x$

b. $y = 2x + 1$

c. $y = -5$

d. $y = 4 - 3x$

e. $y = -3 - x$

13. Write equations and sketch the graphs of lines with the following properties.

a. slope of 3.5, y-intercept at $(0, 4)$

b. slope $\frac{3}{2}$, passing through $(-2, 0)$

c. passing through the points $(2, 7)$ and $(6, 15)$

d. slope $-\frac{15}{5}$, passing through the point $(-2.5, 4.5)$

14. Suppose the town of Roseville is giving away lots with a perimeter of 500 feet, rather than with an area of 21,780 square feet.

a. Copy and complete this table.

Rectangles With a Perimeter of 500 ft

Length (ft)	50	100	150	200	225
Width (ft)					

b. Make a graph of the (length, width) data. Draw a line or curve that models the data pattern.

c. Describe the pattern of change in width as length increases.

d. Write an equation for the relationship between length and width. Explain why it is or is not a linear function.

A number b is the additive inverse of the number a if $a + b = 0$. For example, -5 is the additive inverse of 5 because $5 + (-5) = 0$. For Exercises 15–20, find the additive inverse of each number.

15. 2

16. $-\frac{6}{2}$

17. 2.5

18. -2.11

19. $\frac{7}{3}$

20. $\frac{3}{7}$

21. On a number line, graph each number from Exercises 15–20 and its additive inverse. Describe any patterns you see.

A number b is the multiplicative inverse of the number a if $ab = 1$. For example, $\frac{3}{2}$ is the multiplicative inverse of $\frac{2}{3}$ because $(\frac{3}{2} \cdot \frac{2}{3}) = 1$. For Exercises 22–27, find the multiplicative inverse of each number.

22. 2 **23.** -2 **24.** 0.5

25. 4 **26.** $\frac{3}{4}$ **27.** $\frac{5}{3}$

28. On a number line, graph each number in Exercises 22–27 and its multiplicative inverse. Describe any patterns you see.

Jamar takes a 10-point history quiz each week. Here are his scores on the first five quizzes: 8, 9, 6, 7, 10. Use this information for Exercises 29–30.

29. Multiple Choice What is Jamar's average quiz score?

A. 6 **B.** 7 **C.** 8 **D.** 9

30. Jamar misses the next quiz and gets a 0.

a. What is his average after six quizzes?

b. After 20 quizzes, Jamar's average is 8. He gets a 0 on the 21st quiz. What is his average after 21 quizzes?

c. Why did a score of 0 have a different effect on the average when it was the sixth score than it did when it was the 21st score?

In Exercises 31 and 32, solve each equation using a symbolic method. Then describe how the solution can be found using a graph and a table.

31. $5x - 28 = -3$ **32.** $10 - 3x = 7x - 10$

For Exercises 33–35, find the equation of the line with the given properties.

33. slope $\frac{1}{2}$, y-intercept (0, 5)

34. slope 3, passing through the point (2, 2)

35. passing through the points (5, 2) and (1, 10)

36. Find the equation for the line shown below.

37. Suppose a car travels at a speed of 60 miles per hour. The function $d = 60t$ relates time t in hours and distance d in miles. This function is an example of *direct variation*. A relationship between variables x and y is a direct variation if it can be expressed as $y = kx$, where k is a constant.

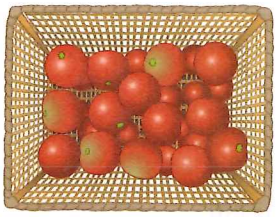
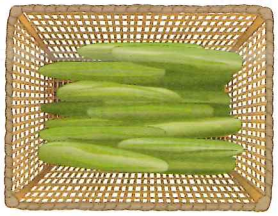
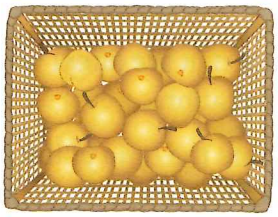
a. Describe two functions in this unit that are direct variations. Give the rule for each function as an equation.

b. For each function from part (a), find the ratio of the dependent variable to the independent variable. How is the ratio related to k in the general function?

c. Suppose the relationship between x and y is a direct variation. How do the y-values change as the x-values increase? How does this pattern of change appear in a graph of the relationship?

d. Compare direct variation and inverse variation. Be sure to discuss the graphs and equations of each.

For Exercises 38–40, tell which store offers the better buy. Explain your choice.

	Gus's Groceries		The Super Market
38.	Tomatoes are 6 for $4.00		TOMATOES ARE 8 FOR $4.60
39.	Cucumbers are 4 for $1.75		CUCUMBERS ARE 5 FOR $2.00
40.	Apples are 6 for $3.00		APPLES ARE 5 FOR $2.89

41. Suppose 6 cans of tomato juice cost $3.20. Find the cost of the following numbers of cans.

a. 1 can **b.** 10 cans **c.** n cans

Extensions

42. The drama club members at Henson Middle School are planning their spring show. They decide to charge $4.50 per ticket. They estimate their expenses for the show at $150.

a. Write a function for the relationship between the number of tickets sold and the club's total profit.

b. Make a table to show how the profit changes as the ticket sales increase from 0 to 500 in intervals of 50.

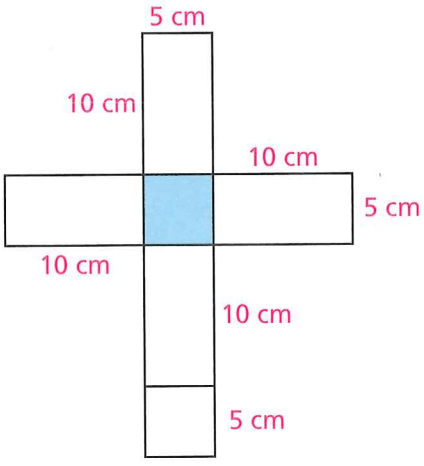
c. Make a graph of the (tickets sold, total profit) data.

d. Add a column (or row) to your table to show the per-ticket profit for each number of tickets sold. For example, for 200 tickets, the total profit is $750, so the per-ticket profit is $750 ÷ 200, or $3.75.

e. Make a graph of the (tickets sold, per-ticket profit) data.

f. How are the patterns of change for the (tickets sold, total profit) data and (tickets sold, per-ticket profit) data similar? How are they different? How are the similarities and differences shown in the tables and graphs of each function?

43. The net below folds to make a rectangular prism.



a. What is the volume of the prism?

b. Suppose the dimensions of the shaded face of the prism are doubled. The other dimensions are adjusted so the volume remains the same. What are the new dimensions of the prism?

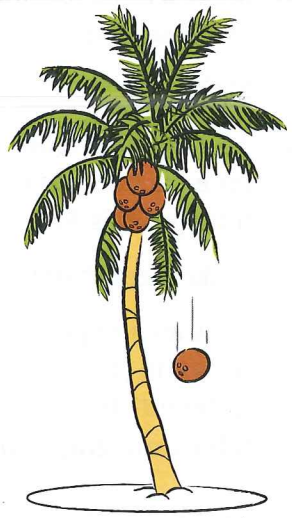
c. Which prism has the smaller surface area, the original prism or the prism from part (b)? Explain.

44. Ms. Singh drives 40 miles to her sister's house. Her average speed is 20 miles per hour. On her way home, her average speed is 40 miles per hour. What is her average speed for the round trip?

For Exercises 45–47, find the value of c for which both ordered pairs satisfy the same inverse variation. Then write an equation for the relationship.

45. $(3, 16), (12, c)$ **46.** $(3, 9), (4, c)$ **47.** $(3, 4), (4, c)$

48. Multiple Choice The acceleration of a falling object is related to the object's mass and the force of gravity acting on it. For a fixed force F, the relationship between mass m and acceleration a is an inverse variation. Which equation describes the relationship of F, m, and a?



A. $F = ma$ **B.** $m = Fa$

C. $\frac{m}{F} = a$ **D.** $\frac{m}{a} = F$

49. Multiple Choice Suppose the time t in the equation $d = rt$ is held constant. What happens to the distance d as the rate r increases?

F. d decreases. **G.** d increases.

H. d stays constant. **J.** There is not enough information.

50. Multiple Choice Suppose the distance d in the equation $d = rt$ is held constant. What happens to the time t as the rate r increases?

A. t decreases. **B.** t increases.

C. t stays constant. **D.** There is not enough information.

Mathematical Reflections 3

In this Investigation, you explored several examples of inverse variation and looked for patterns in the tables, graphs, and equations of these relationships. These questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Suppose** the relationship between variables x and y is an inverse variation.
   a. **How** do the values of y change as the values of x increase?
   b. **Describe** the trend in a graph of (x, y) values.
   c. **Describe** the equation that relates the values of x and y.
2. **How** is an inverse variation similar to a linear relationship? How is it different?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Nick described his thoughts in the following way:

> *In Question C of Problem 3.3, we looked at what happens to the cost per student when we double the number of students. Bill said that the per-student cost is cut in half. The cost per student when 20 students go is $37.50. If 40 students go, the cost per student would be $18.75.*
>
> *When the number of students doubles from 40 to 80, the cost is cut in half again. This makes sense because you spread the cost among twice as many people, so each person pays half as much.*
>
> *Then Jen asked, "What would happen if we tripled the number of students?" Bill explained that the cost per student would be one third as much. The cost per student when 20 students go is $37.50, so the cost per student if 60 students go is one third of $37.50, which is $12.50.*

Common Core Standards for Mathematical Practice

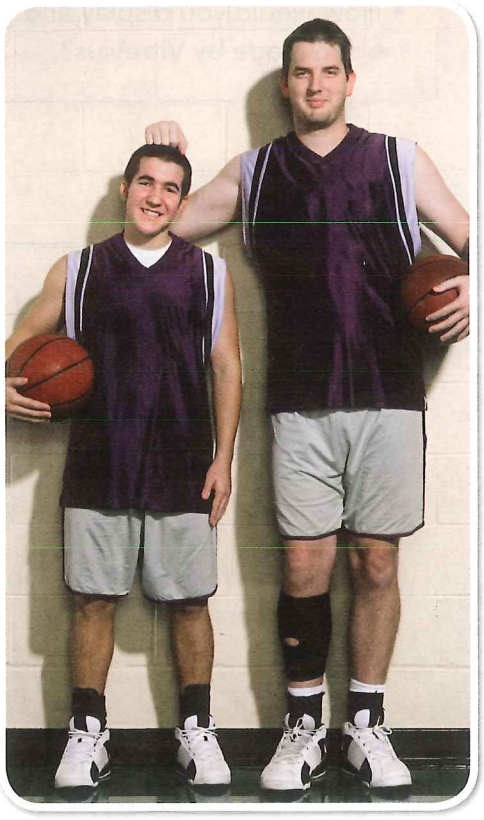
MP3 Construct viable arguments and critique the reasoning of others.

- What other Mathematical Practices can you identify in Nick's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 4

Variability and Associations in Numerical Data

When playing basketball, it helps to be tall and to have long arms. The average player in the National Basketball Association is more than 6 feet 7 inches tall.



- How rare do you think it is for a man to be as tall as those average NBA players?
- Do you think height and arm span are closely related variables for NBA players?
- Do you think height and arm span are closely related variables for students in your class?

Working on the Problems in this Investigation will help you understand how to measure variability and associations of data values.

Common Core State Standards

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 . . . For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

Also 8.EE.B.5, 8.EE.C.7, 8.F.A.1, 8.F.A.3, 8.F.B.4, 8.F.B.5, S-ID.A.2, S-ID.B.6, S-ID.B.6b, S-ID.C.7, S-ID.C.8, and S-ID.C.9

4.1 Vitruvian Man

Relating Body Measurements

More than 2,000 years ago, a Roman architect and writer named Vitruvius found patterns by relating two body measurements. He claimed a person's arm span is equal to his or her height.

- Do you think the relationship between arm span and height applies to the students in your class?
- How would you display and analyze data collected to test the claim made by Vitruvius?

Problem 4.1

The table shows the height and arm span of students in a CMP class.

Height (in.)	56	57	57	58	59	60	60	60	62	64	64	66	67	67	67	68
Arm span (in.)	54	57	54	61	56	58	59	60	62	63	62	62	65	67	69	67

Do you think the data support the claim that arm span and height are about equal?

A Analyze the data to test your ideas.

1. Plot the (height, arm span) data on a coordinate graph. The resulting graph is called a **scatter plot**.
2. Do you think the scatter plot supports the claim that arm span and height are about equal for most people?

continued on the next page >

Problem 4.1 continued

3. If each student in the class had arm span s equal to height h, what equation would relate the two variables?

   a. Graph the equation on your scatter plot.

   b. Which data points (if any) does your line pass through? Explain how arm span and height are related in those points.

   c. Choose several data points that are not on your line. Explain how arm span and height are related in each case. How do you describe the relationship shown on the graph?

B The tallest person in recorded history was Robert Pershing Wadlow. At age 22, he was 8 feet 11.1 inches (272 cm) tall. His arm span was 9 feet 5.75 inches (289 cm).

1. Where would you plot the point (height, arm span) for Robert Wadlow? Would the point be *on*, *above*, or *below* the line you drew in Question A, part (3)?

2. Does the data point for Robert Wadlow support the claim that arm span and height are roughly equal?

C The accuracy of fit for a linear model is measured by calculating errors from the model. These errors, called **residuals,** are the differences between the actual data and what the model predicts.

1. Find the arm span residuals (actual arm span − predicted arm span) using the model $s = h$ for the CMP class data.

Height (inches)	56	57	57	58	59	60	60	60	62	64	64	66	67	67	67	68
Arm span (inches)																
Actual	54	57	54	61	56	58	59	60	62	63	62	62	65	67	69	67
Predicted by Model	56	57	57	58	59	60	60	60	62	64	64	66	67	67	67	68
Residual	−2	0	−3													

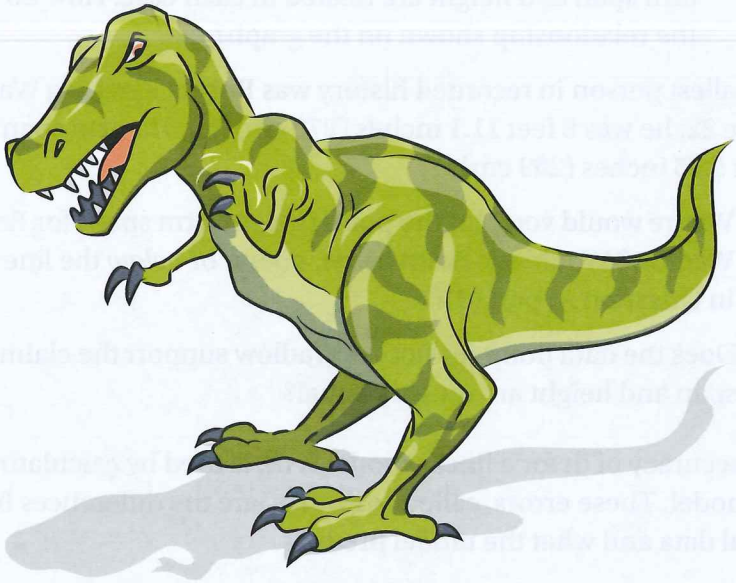
2. To see if there is any pattern in the residuals it helps to plot the (height, residual) data. Use such a plot and data in the table to describe the pattern (if any) in the residuals. Then explain why you think the equation $s = h$ is or is not an accurate model for predicting arm span from height.

continued on the next page >

Problem 4.1 continued

D The dinosaur *Tyrannosaurus rex* grew to 20 feet in height with an arm span of about 10 feet.

1. Do you think the *T. rex* data point fits the pattern that arm span and height are roughly equal? Explain.
2. If you plot the data point, would it be *on*, *above*, or *below* the line you drew in Question A, part (3)?



ACE Homework starts on page 96.

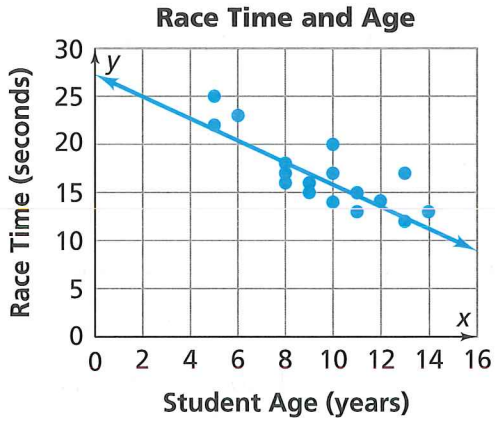
4.2 Older and Faster

Negative Correlation

Magnolia Elementary is a school with students who are 5 to 14 years old. One field day, all students were timed in a 100-meter race. The table shows data for some of the students.

Student Age (years)	5	5	6	8	8	8	9	9	10	10	10	11	11	12	13	13	14
Race Time (seconds)	25	22	23	18	16	17	15	16	17	20	14	15	13	14	17	12	13

The graph below shows the data from the table and a line that models the data.



- How would you describe the relationship between age and race time?
- Would you say the relationship is *strong* or *weak*?
- Are the data points close to the line or spread out?

Problem 4.2

Use the Race Time and Age graph.

A The line drawn on the graph models the relationship between age and race time.

1. What is the approximate slope of the line?
2. How does the slope help you understand the relationship between age and race time?
3. Do you think it makes sense to predict a race time for a 7-year-old student using the line? If so, what do you predict for a 7-year-old? How confident are you in your prediction?
4. Do you think it makes sense to predict a race time for a 21-year-old person using the line? If so, what do you predict for a 21-year-old? How confident are you in your prediction?

continued on the next page >

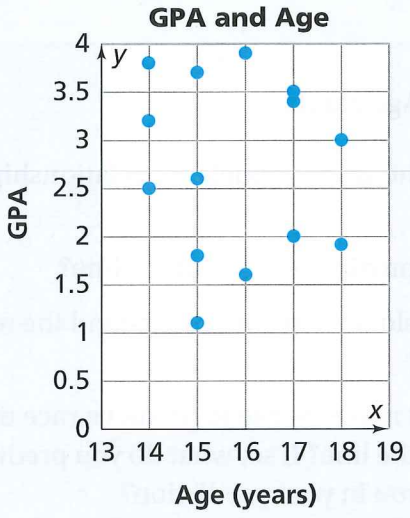
Problem 4.2 continued

B Some data points are very close to the line while others are far from it. The points far from the line don't seem to fit the model.

1. Find two points that don't seem to fit the model. What are their coordinates (age, race time)?
2. Why do you think the points don't match the overall pattern? Explain. Think about the relationship between race time and age.
3. In Problem 4.1, you used a line to model (height, arm span).
   a. If a 6-foot-9-inch NBA basketball player has a 7-foot-5-inch arm span, would that data point fit the model?
   b. Would you plot the data point, *on*, *above*, or *below* the $s = h$ line? Explain.

C The table and graph show age and grade point average (GPA) for 14 students at Magnolia High School.

Student Age (years)	14	14	14	15	15	15	15	16	16	17	17	17	18	18
GPA	2.5	3.2	3.8	1.8	2.6	3.7	1.2	1.6	3.9	2.0	3.4	3.5	1.9	3.0



1. Are age and GPA strongly related for these students? Explain.
2. How is your answer to part (1) supported by the table?
3. How is your answer to part (1) supported by the scatter plot?

ACE Homework starts on page 96.

4.3 Correlation Coefficients and Outliers

Roller coasters are popular rides at amusement parks. A recent survey counted 1,797 roller coaster rides in the world. 734 of them are in North America. Roller coasters differ in maximum drop, maximum height, track length, ride time, and coaster type (wood or steel).

- Which roller coaster variables do you think are strongly related to the top speed on the ride?

Problem 4.3

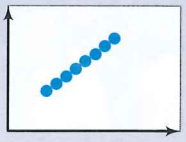
Statisticians measure the strength of a linear relationship between two variables using a number called the **correlation coefficient.** This number is a decimal between -1 and 1. When the points lie close to a straight line, the correlation coefficient is close to -1 or 1.

continued on the next page >

Problem 4.3 continued

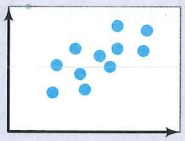
- When points cluster close to a line with positive slope, the correlation coefficient is almost 1, and with negative slope, the correlation coefficient is almost −1.
- Points that do not cluster close to any line have a correlation coefficient of almost 0.
- Positive association has correlation coefficients greater than 0 while negative association has correlation coefficients less than 0.

A **1.** The graph below has a correlation coefficient of 1.0. What do you think a correlation coefficient of 1.0 means?

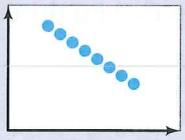


2. Which of the six scatter plots below (a)-(f) has a correlation coefficient of −1.0? What do you think a correlation coefficient of −1.0 means?

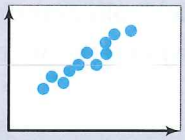
a.



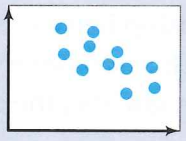
b.



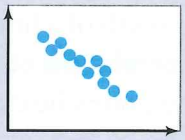
c.



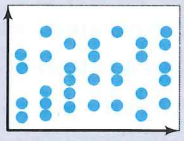
d.



e.



f.



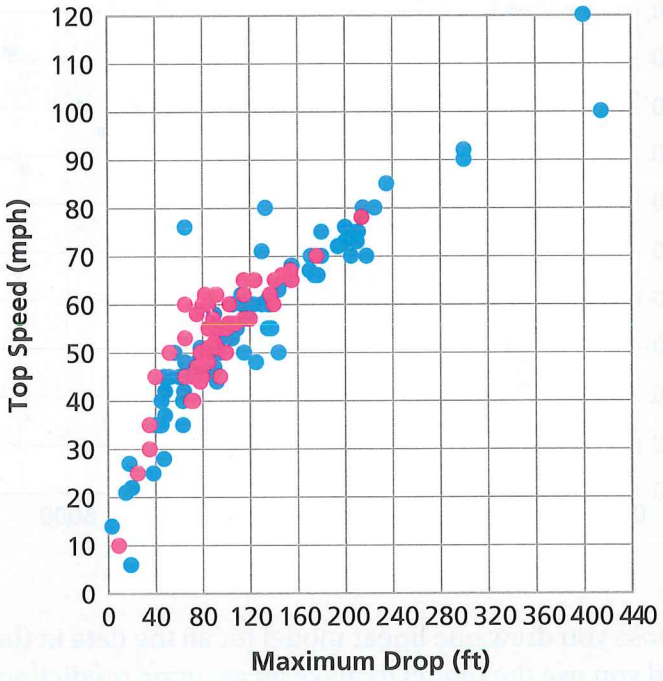
3. Match correlation coefficients −0.8, −0.4, 0.0, 0.4, and 0.8 with the other five scatter plots. Explain your reasoning.

continued on the next page >

Problem 4.3 continued

When you inspect a scatter plot, often you are looking for a strong association between the variables.

B The scatter plot below shows the relationship between the top speed of a roller coaster and its maximum drop. The pink dots represent wood-frame roller coasters. The blue dots represent steel-frame coasters.



1. Suppose you drew one linear model for all the data in the graph. Could you use the model to make an accurate prediction about the top speed of the roller coaster with a given maximum drop? Explain.
2. Estimate the correlation coefficient for the top speed and the maximum drop. Is the correlation coefficient closest to -1, -0.5, 0, 0.5, or 1?
3. Is the maximum drop of a roller coaster likely to be one of the causes of the top speed of the coaster? Why or why not?

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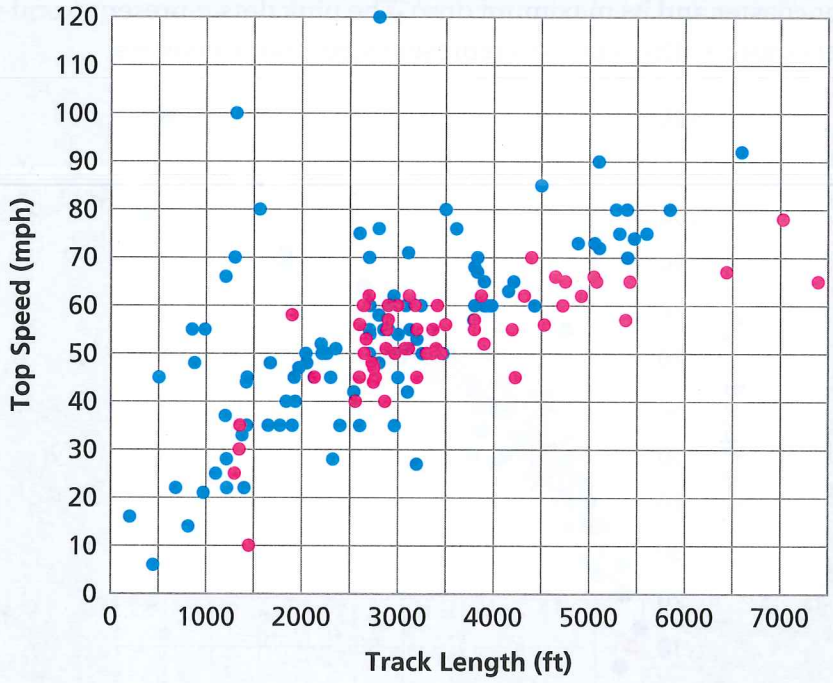
Problem 4.3 continued

C The scatter plot below shows the relationship between the top speed of a roller coaster and its track length. The pink dots represent wood-frame roller coasters. The blue dots represent steel-frame coasters.



1. Suppose you drew one linear model for all the data in the graph. Could you use the model to make an accurate prediction about the top speed of the roller coaster with a given track length? Explain.
2. Estimate the correlation coefficient for the top speed and track length. Is the correlation coefficient closest to -1, -0.5, 0, 0.5, or 1?
3. Is the track length of a roller coaster likely to be one of the causes of the top speed of the coaster? Why or why not?
4. Computer and calculator data analysis tools can take data pairs like those plotted above and calculate exact correlation coefficients. Use the tool that you have available to find the correlation coefficient for the sample of (track length, top speed) data in the table.

Track Length (ft)	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000
Top Speed (mph)	5	20	45	50	40	50	55	60	85	65

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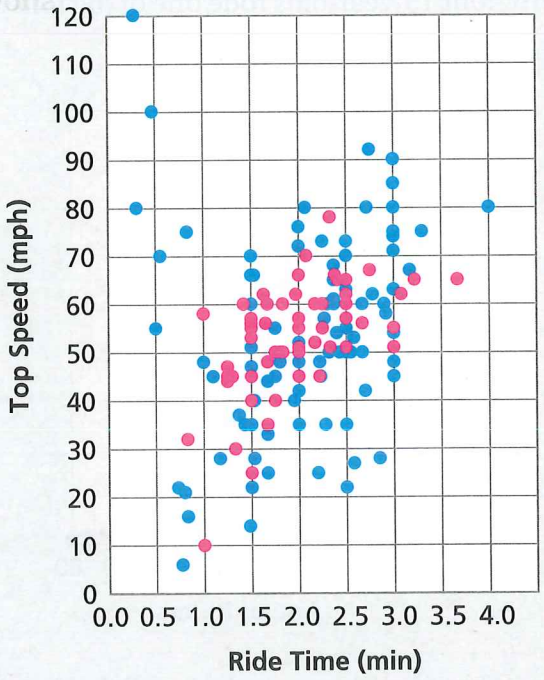
Problem 4.3 continued

D The scatter plot below shows the relationship between the top speed of a roller coaster and the ride time. The pink dots represent wood-frame roller coasters. The blue dots represent steel-frame coasters.



1. Suppose you drew one linear model for all the data in the graph. Could you use the model to make an accurate prediction about the top speed of the roller coaster with a given ride time? Explain.

2. Estimate the correlation coefficient for the top speed and ride time. Is the correlation coefficient closest to -1, -0.5, 0, 0.5, or 1?

3. Suppose most of the points on a scatter plot cluster near a line, with only a few that don't fit the pattern. The points that lie outside a cluster are called **outliers.** Use the graph above. Find each point. Then decide whether the point is an outlier. If it is, explain why you think it is an outlier.

   a. (1.75, 50) **b.** (0.30, 80) **c.** (3.35, 75)

   d. (0.28, 120) **e.** (0.80, 21) **f.** (1.0, 10)

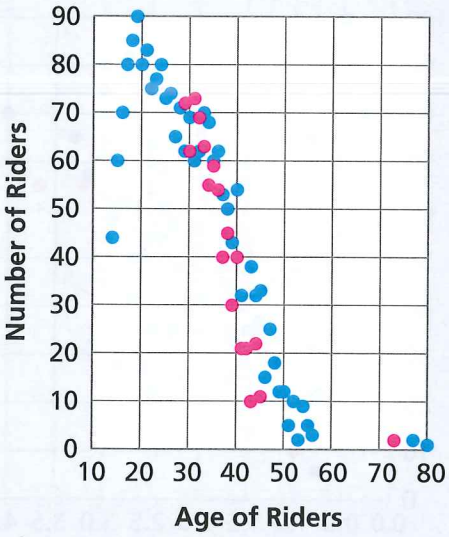
   g. Use the scatter plot in Question C. Find two outliers on that graph and estimate their coordinates (track length, top speed).

continued on the next page >

Problem 4.3 continued

E The scatter plot shows the number of roller coaster riders and their ages on a given day. The pink dots represent wood-frame roller coasters. The blue dots represent steel-frame coasters.

On that day, forty-four 15-year-olds rode one of the roller coasters. The data point is (15, 44).



1. Suppose you drew one linear model for all the data in the graph. Could you use the model to make an accurate prediction about the number of riders on the roller coaster with a given age? Explain.
2. Is the age of riders on a roller coaster likely to be one of the causes of the number of riders on the coaster? Why or why not?
3. Estimate the correlation coefficient for the number of riders and age of riders. Is the correlation coefficient closest to -1, -0.5, 0, 0.5, or 1?
4. Are any of the data points outliers? If so, estimate the coordinates of those points.

F Is it possible to have a correlation coefficient close to -1 or 1 with only a few outliers? Explain your thinking.

ACE Homework starts on page 96.

4.4 Measuring Variability
Standard Deviation

A height of 6 feet 7 inches is unusual for an adult man.

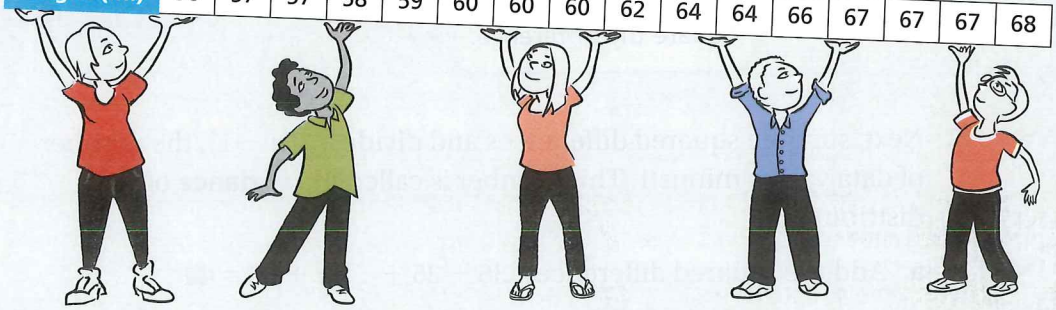
- What height would make an eighth-grade boy or girl above average?

You can use range and interquartile range to describe how data values in a sample vary. You can also use the mean absolute deviation (MAD) to measure the spread of data values. This problem reviews those measures and introduces a measure of spread called *standard deviation*.

Problem 4.4

The table shows the heights of several CMP students. You used this information in Problem 4.1.

Height (in.)	56	57	57	58	59	60	60	60	62	64	64	66	67	67	67	68



A Make a line plot to show the distribution of the data.

B Calculate the summary statistics below, and explain what each number says about the distribution of heights.

1. Range
2. Mean
3. Mean Absolute Deviation (MAD)

continued on the next page >

Problem 4.4 continued

C Like the MAD, you calculate the **standard deviation** of a data set from the differences between data values and the mean. To calculate the standard deviation for the height data, complete each part below.

1. Find the difference of each data value and the mean. In the table below, for example, Jayne's height is 56 inches and the mean is 62 inches. The difference is $(56 - 62) = -6$. Copy the table and complete the middle row with the differences.

2. Square each difference. For example, for Jayne's height, $(-6)^2 = 36$. Complete the third row with the squares of the differences.

Jayne's height

Height (inches)	56	57	57	58	59	60	60	60	62	64	64	66	67	67	67	68
Height − Mean	−6	−5	−5	−4	■	■	■	■	■	■	■	■	■	■	■	■
Squares of differences	36	25	■	■	■	■	■	■	■	■	■	■	■	■	■	■

Square the difference.

3. Next, sum the squared differences and divide by $(n - 1)$, the number of data values minus 1. This number is called the **variance** of the distribution.

   a. Add the squared differences: $36 + 25 + \ldots + 36 =$ ■

   b. Divide by $(n - 1)$, the number of students minus one: $16 - 1 = 15$

   c. The variance is ■.

4. The square root of a number n is written in symbols as $\sqrt{n}$. It is the positive number you multiply by itself to equal n. For example, $\sqrt{25} = 5$ and $\sqrt{6.25} = 2.5$.

   Take the square root of the variance. This number is the standard deviation of the distribution of heights.

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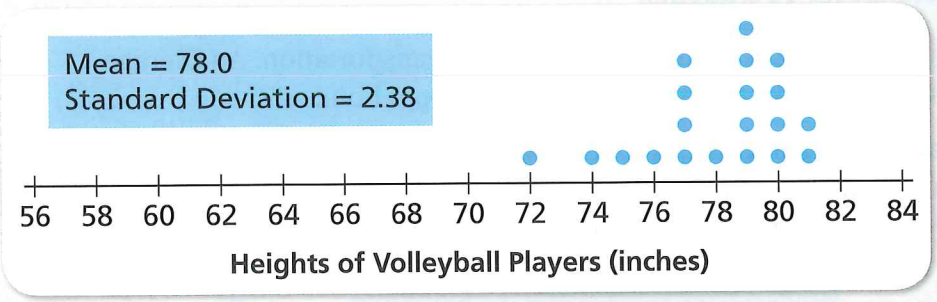
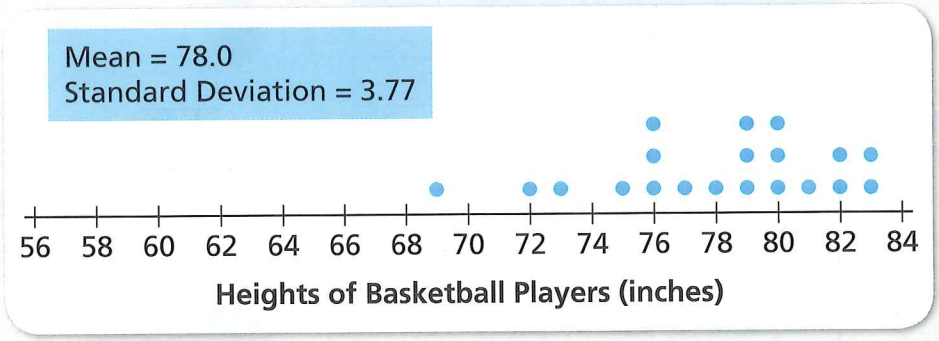
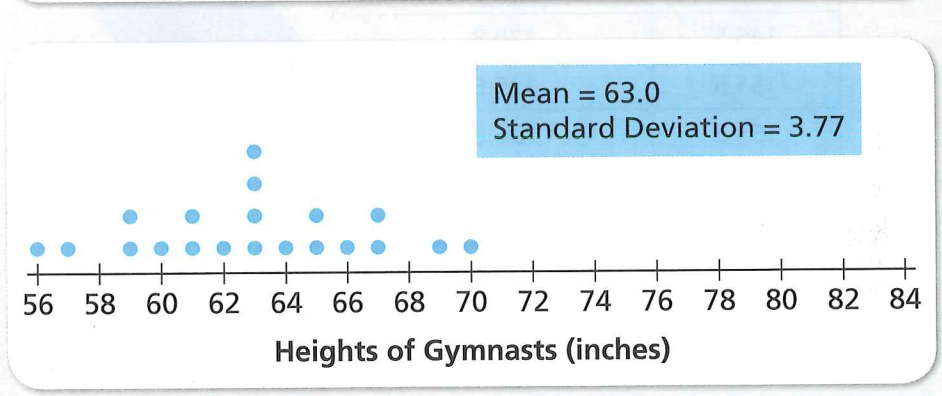
Problem 4.4 continued

D Each dot plot shows the distribution, mean, and standard deviation of heights of 20 athletes. The 20 athletes are a random sample.







1. Compare the heights of volleyball players with the heights of basketball players. What can you say about the similarities and differences using the dot plots?
2. Compare the gymnasts with the basketball players. What can you say about the similarities and differences using the dot plots?

ACE Homework starts on page 96.

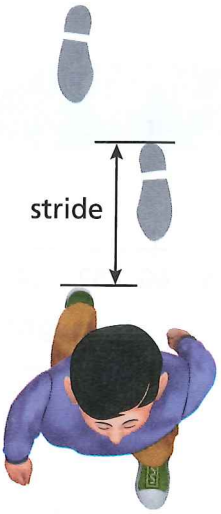
Applications

For Exercises 1 and 2, use the table below. It shows the height and stride distance for 10 students.

For humans, walking is the most basic form of transportation. An average person is able to walk at a pace of about 3 miles per hour.

The distance a person covers in one step depends on his or her stride. To measure stride distance, measure from the heel of the first foot to the heel of that same foot on the next step.

Height (cm)	Stride Distance (cm)
150.8	125.2
149.5	124.2
151.2	125.2
153.1	126.8
150.6	124.4
149.9	123.8
146.5	121.8
146.5	120.8
151.5	125.6
153.5	126.8



1. **a.** What is the median height of these students? Explain how you found the median.

   b. What is the median stride distance of these students? Explain how you found the median.

   c. What is the ratio of median height to median stride distance? Explain.

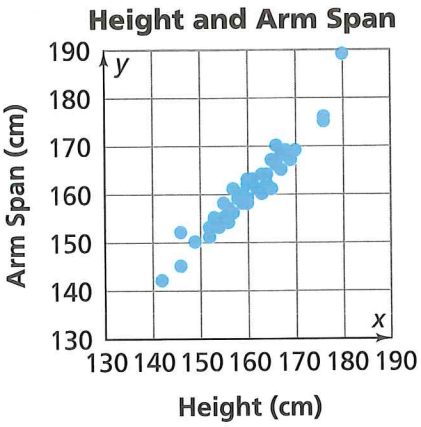
2. **a.** Draw a coordinate graph with height (in centimeters) on the horizontal axis and stride distance (in centimeters) on the vertical axis. To choose a scale for each axis, look at the greatest and least values of each measure.

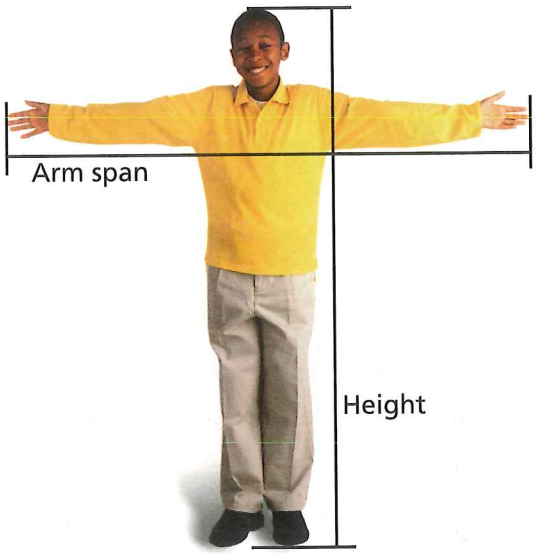
b. Explain how you can use your graph to determine whether the shortest student also has the shortest stride distance.

c. Describe how to estimate the heights of people with each stride distance.

i. 1.50 meters **ii.** 0.90 meters **iii.** 1.10 meters

3. In Problem 4.1 you explored the relationship between arm span and height. The scatter plot below shows data for another group of middle school students.





a. Draw a model line from (130, 130) to (190, 190) on your scatter plot.

b. Use the line to describe the relationship between height and arm span.

c. Write an equation for the line using h for height and a for arm span.

d. What is true about the relationship between height and arm span for points in each part of the graph?

i. points on the model line

ii. points above the model line

iii. points below the model line

4. a. Make a scatter plot from the table. To keep track of engine type, use two different colors as you plot the points. Use one color for jet engines and one color for propeller engines.

Airplane Comparisons

Plane	Engine type	Body length (m)	Wingspan (m)
Boeing 707	jet	47	44
Boeing 747	jet	71	60
Ilyushin IL-86	jet	60	48
McDonnell Douglas DC-8	jet	57	45
Antonov An-124	jet	69	73
British Aerospace 146	jet	29	26
Lockheed C-5 Galaxy	jet	76	68
Antonov An-225	jet	84	88
Airbus A300	jet	54	45
Airbus A310	jet	46	44
Airbus A320	jet	38	34
Boeing 737	jet	33	29
Boeing 757	jet	47	38
Boeing 767	jet	49	48
Lockheed Tristar L-1011	jet	54	47
McDonnell Douglas DC-10	jet	56	50
Douglas DC-4 C-54 Skymaster	propeller	29	36
Douglas DC-6	propeller	32	36
Lockheed L-188 Electra	propeller	32	30
Vickers Viscount	propeller	26	29
Antonov An-12	propeller	33	38
de Havilland DHC Dash-7	propeller	25	28
Lockheed C-130 Hercules/L-100	propeller	34	40
British Aerospace 748/ATP	propeller	26	31
Convair 240	propeller	24	32
Curtiss C-46 Commando	propeller	23	33
Douglas DC-3	propeller	20	29
Grumman Gulfstream I/I-C	propeller	19	24
Ilyushin IL-14	propeller	22	32
Martin 4-0-4	propeller	23	28
Saab 340	propeller	20	21

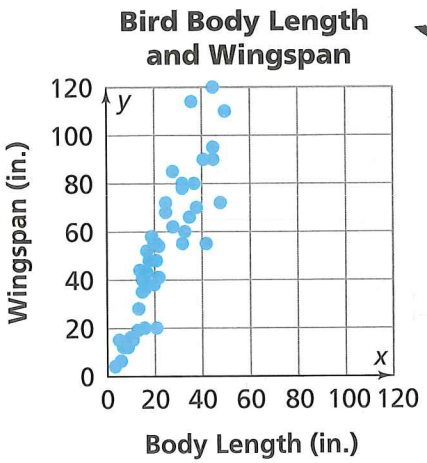
SOURCE: *Airport Airplanes*

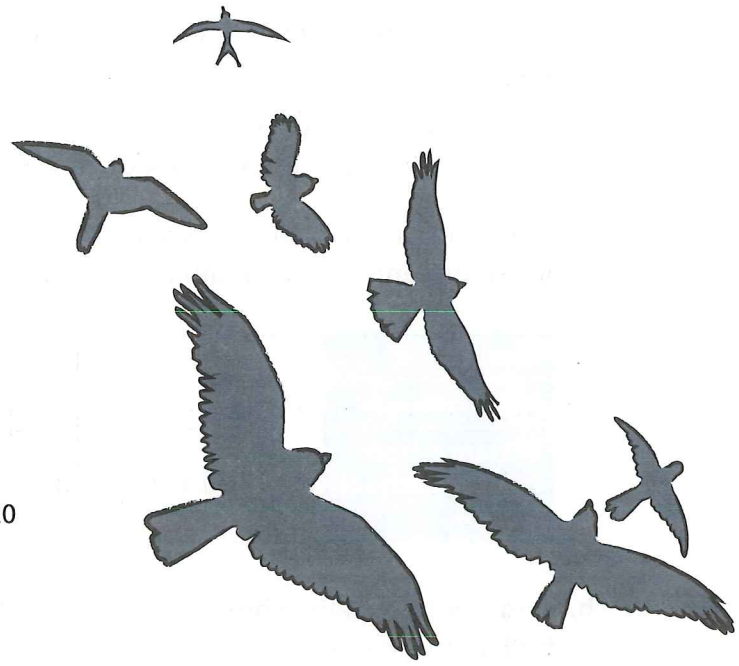
b. Use your results from Exercise 3. Does your equation for the relationship between height and arm span also describe the relationship between body length and wingspan for airplanes? Explain.

c. Predict the wingspan of an airplane with a body length of 40 meters.

d. Predict the body length of an airplane with a wingspan of 60 meters.

5. The scatter plot below shows the relationship between body length and wingspan for different birds.





a. Use your results from Exercise 3. Does your equation for the relationship between height and arm span also describe the relationship between body length and wingspan for birds? Explain.

b. Find a line that fits the overall pattern of points in the scatter plot. What is the equation of your line?

c. Predict the wingspan of a bird with a body length of 60 inches. Explain your reasoning.

6. a. The table shows math and science test scores for 10 students. Make a scatter plot of the data.

Student	1	2	3	4	5	6	7	8	9	10
Math	67	51	87	36	56	44	72	63	45	93
Science	71	69	85	35	60	47	74	63	46	96

b. Describe the relationship between the math and science scores.

c. If the data are linear, sketch a line that fits the data.

d. Identify any data values that you think are outliers. Explain why they are outliers.

e. Estimate a correlation coefficient for the data. Is it closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

7. a. The table shows math scores and distances from home to school for 10 students. Make a scatter plot of the data.

Student	1	2	3	4	5	6	7	8	9	10
Math Score	67	51	87	36	56	44	72	63	45	93
Distance from home to school (miles)	0.6	1.7	0.3	2.2	3.1	0.25	2.6	1.5	0.75	2.1

b. Describe the relationship between the math score and distance from home to school.

c. Estimate a correlation coefficient for the data. Is it closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

8. a. The table shows the number of servers and average time to fill an order at fast-food restaurants. Make a scatter plot of the data.

Number of Servers	3	4	5	6	7
Average Time to Fill an Order (min)	0.6	1.7	0.3	2.2	3.1

b. Describe the relationship between the number of servers and average time to fill an order.

c. Identify any data values that you think are outliers. Explain why they are outliers.

d. Estimate a correlation coefficient for the data. Is it closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

9. a. The table shows the number of absences from school and math scores. Make a scatter plot of the data.

b. Describe the relationship between the number of absences and math scores.

c. If the data are linear, write an equation for a line that models the data.

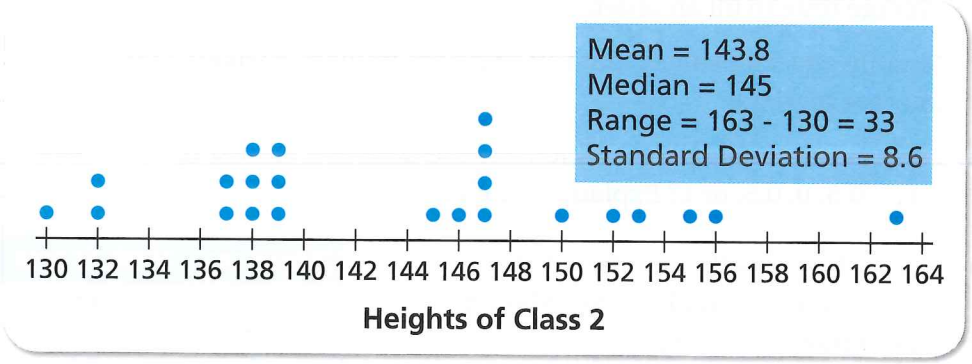
d. Estimate a correlation coefficient for the data. Is it closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

Absences	Math Scores
3	67
5	49
1	96
1	82
3	79
7	37
5	71
3	55
0	100
8	34
7	46
2	69
10	32
0	94
6	53
6	41
2	90
0	92
5	60
7	50
11	10
1	80

Students collected height measurements from two eighth-grade classes. The measurements are in centimeters.

10. Use the dot plot, data, and summary statistics of height measurements from Class 2 below. Describe the distribution of heights in this class.

Class 2: 130, 132, 132, 137, 137, 138, 138, 138, 139, 139, 139, 145, 146, 147, 147, 147, 147, 150, 152, 153, 155, 156, 163



Heights of Class 2

11. a. The data below show heights from another class of eighth graders. Make a dot plot of the data below.

Class 1: 130, 132, 134, 135, 136, 136, 137, 138, 138, 138, 139, 139, 139, 140, 140, 141, 142, 142, 142, 142, 143, 147, 148

b. Calculate the mean, median, range, and standard deviation of the distribution.

c. Use information from parts (a) and (b) to describe the distribution of heights.

d. Compare the distribution of heights in this class to that of the class in Exercise 10.

e. Could you use either distribution to predict the typical height for eighth-graders? Explain your thinking.

12. Use data sets A, B, and C.

$$\text{Set A} = \{9, 10, 11, 7, 13\}$$

$$\text{Set B} = \{10, 10, 10, 10, 10\}$$

$$\text{Set C} = \{1, 1, 10, 19, 19\}$$

a. Calculate the mean of each data set.

b. Calculate the standard deviation of each data set.

c. Explain how you could identify the data set with the greatest standard deviation before doing any calculations.

13. The table shows the monthly salaries of 20 people.

Number of People	5	8	5	2
Salary (dollars)	3,500	4,000	4,200	4,300

a. Calculate the mean of the salaries.

b. Calculate the standard deviation of the salaries.

Connections

14. The table shows height, arm span, and the ratio of arm span to height.

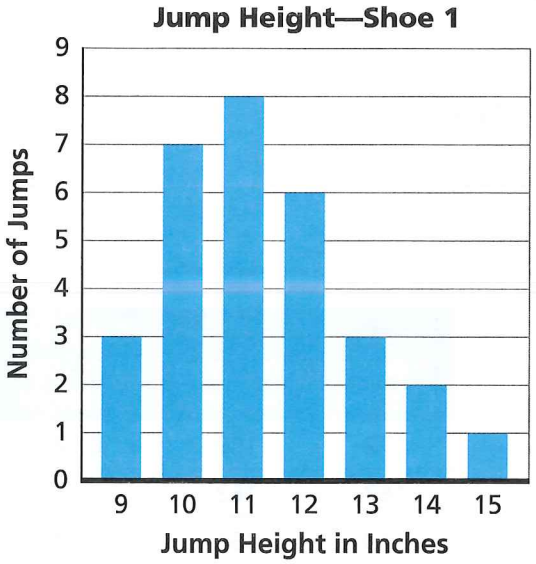
a. Recall Problem 4.1 and the line $s = h$. Where would you find a point with a ratio greater than 1 (*on*, *above*, or *below* the line)? What does it mean when the ratio is greater than 1?

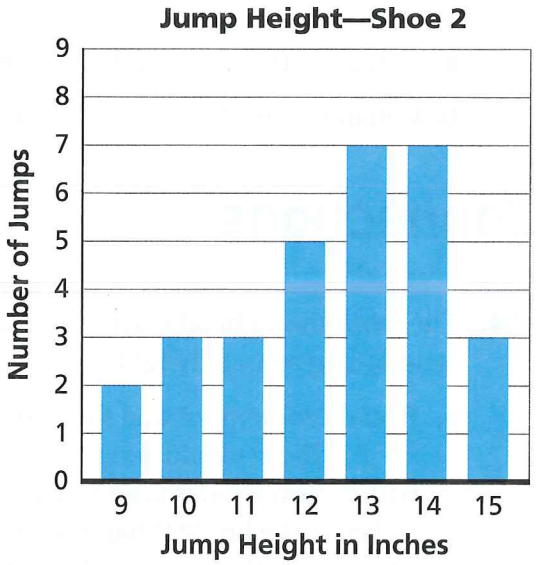
b. For the line $s = h$, where would you find a point with a ratio equal to 1 (*on*, *above*, or *below* the line)? What does it mean when the ratio equals 1?

c. For the line $s = h$, where would you find a point with a ratio less than 1 (*on*, *above*, or *below* the line)? What does it mean when the ratio is less than 1?

Height (inches)	Arm Span (inches)	Ratio of Arm Span to Height
172	169	0.98
167	163	0.98
163	164	1.01
162	164	1.01
163	159	0.97
164	158	0.96
161	159	0.99
161	155	0.96
159	161	1.01
156	156	1.00
154	162	1.05
154	157	1.02
154	156	1.01
155	150	0.97
155	154	0.99
177	174	0.98
171	172	1.01
149	144	0.97
143	148	1.03
142	142	1.00

15. Multiple Choice In testing two new sneakers, the shoe designers judged performance by measuring the heights of jumps. Now a shoe designer needs to choose the better sneaker. Which measure is best for deciding between the two sneakers? Use the graph for each sneaker.





A. Use the mode. The most frequent height jumped for Shoe 1 was 11 inches, and the most frequent height jumped for Shoe 2 was 13 or 14 inches.

B. Use the mean. The average jump height for Shoe 1 was 11 inches. For Shoe 2, it was 12.5 inches.

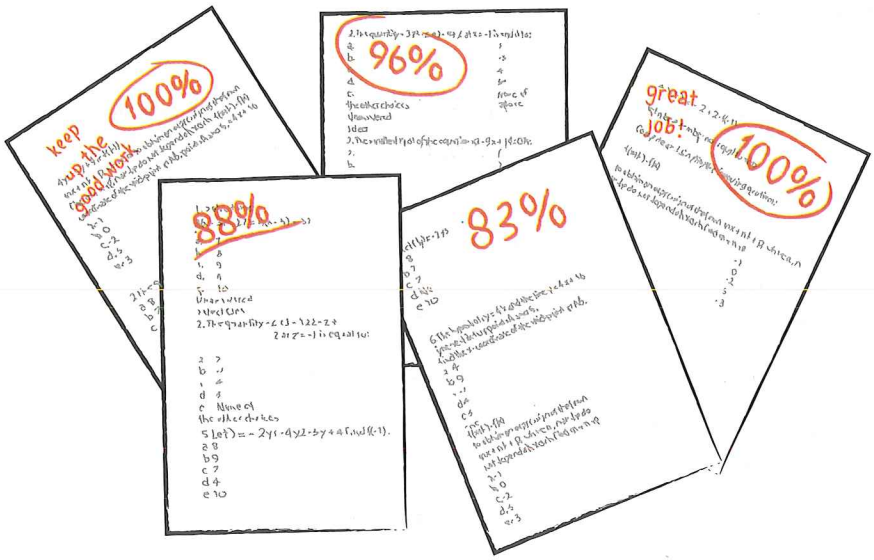
C. Use clusters. Overall, 70% of the students jumped 10 inches to 12 inches in Shoe 1, and the data vary from 9 inches to 15 inches. About 63% of the students jumped 12 inches to 14 inches in Shoe 2, and the data vary from 9 inches to 15 inches.

D. None of the above.

16. a. What is the shape of a distribution when the mean is greater than the median?

b. What is the shape of a distribution when the mean is less than the median?

c. What is the shape of a distribution when the mean and the median are about the same value?



17. Multiple Choice Del Kenya's test scores are 100, 83, 88, 96, and 100. His teacher told the class that they could choose whichever measure of center they wanted her to use to determine final grades. Which measure do you think Del Kenya should choose?

F. Mean **G.** Median **H.** Mode **J.** Range

18. Multiple Choice Five packages with a mean weight of 6.7 pounds were shipped by the Send-It-Quick Mail House. If the mean weight for four of these packages is 7.2 pounds, what is the weight of the fifth package?

A. 3.35 lb **B.** 4.7 lb **C.** 6.95 lb **D.** 8.7 lb

19. Multiple Choice In Mr. Mamer's math class, there are three times as many girls as boys. The girls' mean grade on a recent quiz was 90, and the boys' mean grade was 86. What was the mean grade for the entire class?

F. 88 **G.** 44 **H.** 89 **J.** 95

20. Some numbered cards are put in a hat, and one is drawn at random. There is an even number of cards, no two of which are alike. How many cards might be in the hat to give the probability equal to the following values of choosing a number greater than the median?

a. $\frac{1}{2}$ **b.** $\frac{1}{3}$ **c.** 0

Extensions

21. If you know the number of chirps a cricket makes in a certain period of time, you can estimate the temperature in degrees Fahrenheit or Celsius.

a. Count the number of chirps in one minute, divide by 4, and add 40 to get the temperature in degrees Fahrenheit. Write a formula using F for temperature and s for chirps per minute.

b. Graph your formula. Use a temperature scale from 0 to 212° F.

c. Use your graph to estimate the number of chirps at each temperature.

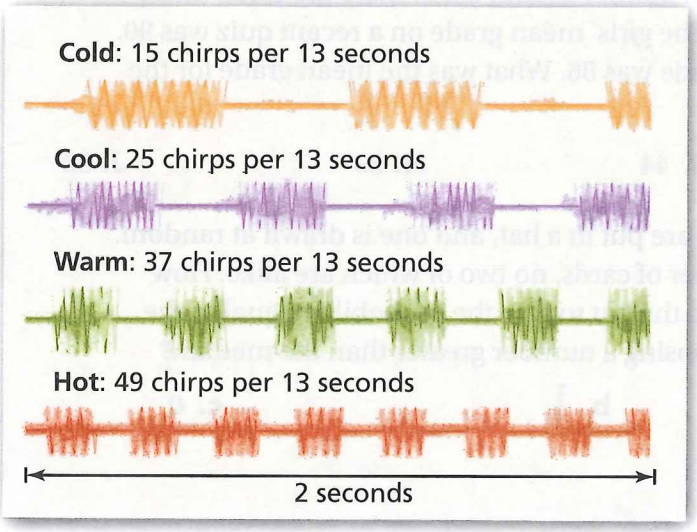
i. 0° F **ii.** 50° F **iii.** 100° F **iv.** 212° F

22. **a.** The chirp frequency of a different kind of cricket allows you to estimate temperatures in degrees Celsius rather than in degrees Fahrenheit. Graph the data in the table.

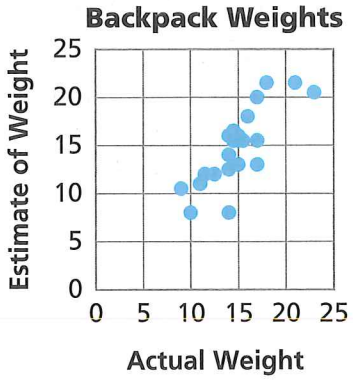
Frequency	195	123	212	176	162	140	119	161	118	175	161	171	164	174	144
Temperature (°C)	31.4	22	34.1	29.1	27	24	20.9	27.8	20.8	28.5	26.4	28.1	27	28.6	24.6

b. Find a formula that lets you predict the temperature in degrees Celsius from the number of chirps.

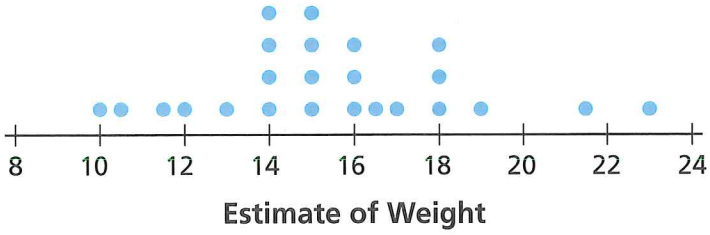
c. Use your formula from part (b) to draw a line on the graph using the points plotted in part (a). How well does the line fit the data?



23. A newspaper article said students carry heavy backpacks. One middle school class decided to check whether the claim was true. Each student estimated the weight of his or her backpack and then weighed it. The scatter plot shows the estimated and actual backpack weights for each student. The dot plots show the distributions for each variable with their mean.

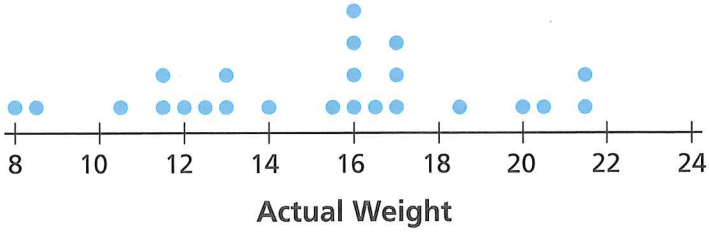


a. Use the statistics in the box to describe the spread of the estimated backpack weights.



Mean = 15.53
Median = 15
Range = 13
SD = 3.14

b. Use the statistics in the box to describe the spread of the actual backpack weights.



Mean = 15.15
Median = 16
Range = 12.5
SD = 3.79

c. Estimate a correlation coefficient for the scatter plot. Is it closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

24. A group of students estimated, and then counted, the number of seeds in several pumpkins. The two tables show the same data sorted differently: One shows the data sorted by actual count and the other by estimate.

Number of Pumpkin Seeds Sorted by Actual Count

Actual	Estimate
309	630
325	621
336	1,423
354	1,200
365	1,200
367	621
381	801
384	604
387	1,900
387	1,100
408	605
410	622
423	759
441	655
442	300
446	621
455	722
462	556
467	621
479	900
486	680
492	1,000
494	564
498	1,458
505	720
506	624
507	200
512	500
523	350
545	2,000
553	202
568	766
606	521
607	1,200

Number of Pumpkin Seeds Sorted by Estimate

Actual	Estimate
507	200
553	202
442	300
523	350
512	500
606	521
462	556
494	564
384	604
408	605
325	621
367	621
446	621
467	621
410	622
506	624
309	630
441	655
486	680
505	720
455	722
423	759
568	766
381	801
479	900
492	1,000
387	1,100
354	1,200
365	1,200
607	1,200
336	1,423
498	1,458
387	1,900
545	2,000

a. How do the actual counts vary? Find the median, the least, and the greatest counts.

b. How do the estimates vary? Find the median, the least, and the greatest estimates.

c. Make a scatter plot of the data. Draw a line on the graph to connect the points (0, 0), (250, 250), (500, 500), and (2250, 2250). What is true about the estimates and actual counts for points near the line?

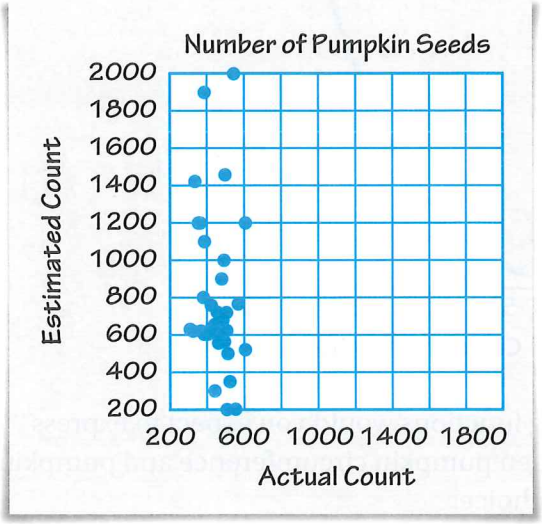
d. What is true about the estimates and actual counts for points *above* the line you graphed in part (c)?

e. What is true about the estimates and actual counts for points *below* the line you graphed in part (c)?

f. In general, did the students make good estimates? Use the median and the range of the data to explain your reasoning.

g. Would a correlation coefficient be closest to -1, -0.5, 0, 0.5, or 1? Explain your choice.

h. One student graphed the data on the scatter plot below. It shows the data bunched together. How could you change the scale(s) on the graph to show the data points better?



25. Multiple Choice Janelle made a scatter plot that shows the relationship between her MP3 music downloads and the unused space on her music player. Which statement would you expect to be true?

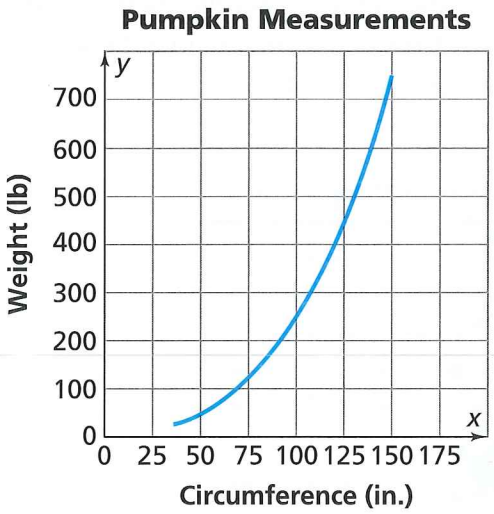
A. As the number of MP3s downloaded increases, the amount of unused space increases.

B. As the number of MP3s downloaded increases, the amount of unused space stays the same.

C. As the number of MP3s downloaded increases, the amount of unused space decreases.

D. As the number of MP3s downloaded decreases, the amount of space used decreases.

26. a. The graph shows a model of the relationship between pumpkin circumference and pumpkin weight. How does the graph suggest that the linear equation $w = c$ would not be a very accurate model for the relationship of weight and circumference?



b. Which of the following functions would you expect to express the relationship between pumpkin circumference and pumpkin weight? Explain your choice.

$w = kc$ $\quad w = kc^2$ $\quad w = kc^3$ $\quad w = \frac{k}{c}$

Mathematical Reflections 4

In this Investigation, you studied data tables and graphs to discover and test the relationship between variables. These questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **Think** about the pattern of points you see in a scatter plot.
   a. **What** pattern would you expect when two variables are related by a linear model with positive slope?
   b. **What** pattern would you expect when two variables are related by a linear model with negative slope?
   c. **What** would you expect to see in a scatter plot when two variables are unrelated?
2. **You** assessed the accuracy of linear models.
   a. **What** do outliers on a scatter plot indicate?
   b. **What** can you learn from the errors of prediction or residuals?
   c. **What** do you know about a linear model from the correlation coefficient?
3. **What** does the standard deviation tell you about a set of data?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Sophie described her thoughts in the following way:

In Problem 4.1A, we had fun experimenting with the graph and looking for patterns. When we plotted the points on the scatter plot, we saw a linear pattern that looks like height and arm span are definitely related.

We know that data vary, but we think we can say that the equation $s = h$ is a good way to model this graph. Then we sketched in the line $s = h$.

We saw that some data points are on the line and then others are above or below. If they are above, it means the arm span is greater than the height. If they are below, it means the height is greater than the arm span.

Common Core Standards for Mathematical Practice

MP4 Model with mathematics

- What other Mathematical Practices can you identify in Sophie's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Investigation 5

Variability and Associations in Categorical Data

Early roller coasters had wooden frames. Now, most roller coasters have steel frames, even though wood is still popular. A recent roller coaster census counted 174 wood-frame coasters, with 129 in North America.

How can people compare wood- and steel-frame roller coasters? Wood and steel are types of frames, not numbers. *Type of roller coaster* is what statisticians call a **categorical variable** that has values *wood* and *steel.*

A study comparing popularity of dogs would investigate the categorical variable *breed of dog* with values *standard poodle, Irish setter, German pointer, Chihuahua, whippet,* and so on.

Common Core State Standards

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Also 8.EE.B.5, 8.EE.C.7, S.ID.B.5 and S.ID.C.9

5.1 Wood or Steel? That's the Question

Relationships in Categorical Data

To plan a new amusement park, a team of coaster designers asked customers, "Do you prefer wood or steel frames in roller coasters?" The table shows the preferences by age group.

	Prefer Wood	Prefer Steel
Age ≤ 40 years	45	60
Age > 40 years	15	20

Does it look like younger and older riders have the same preferences in roller coaster type?

Problem 5.1

Study the roller coaster survey data by age of rider. Make a recommendation about the type of coaster that should be installed in the new park.

A Use the survey data. Is each statement *true* or *false*? Explain.

1. Younger riders are three times as likely as older riders to prefer wood-frame coasters.
2. Younger riders are three times as likely as older riders to prefer steel-frame coasters.
3. The number of riders who prefer wood-frame coasters is about three quarters of the number who prefer steel-frame coasters.
4. Younger riders are more likely than older riders to prefer steel-frame coasters.
5. Older riders are more likely than younger riders to prefer wood-frame coasters.

continued on the next page >

Problem 5.1 continued

B Suppose that a park installed one of each type of roller coaster. One day there were 210 riders over the age of 40 and 420 riders under the age of 40. Use the survey data from Question A.

1. How many riders would you expect on the wood-frame coaster and how many on the steel-frame coaster?
2. How would you expect those riders to be distributed by age and coaster type in the following table?

	Prefer Wood	Prefer Steel	Total
Age ≤ 40 years	■	■	420
Age > 40 years	■	■	210
Total	■	■	■

C If only one roller coaster type could be installed in the park, which would you recommend? Explain your choice.

ACE Homework starts on page 119.

5.2 Politics of Girls and Boys

Analyzing Data in Two-Way Tables

Every four years social studies teachers at the middle school hold a mock election. Each student registers as a Democrat, Independent, or Republican, all of which are categorical data values. Then the classes hold primary and final elections for President.

The table shows the student registrations in one class.

	Democrat	Independent	Republican
Boys	8	4	12
Girls	8	2	6

- Do you think boys and girls have different party preferences?
- What evidence could you give as support?

Problem 5.2

There are different ways to answer the question about political preferences of girls and boys in the sampled class.

A Use the table on the previous page. Do you think each statement is *true* or *false*? Justify your answers.

1. Girls and boys are equally likely to be Democrats.
2. Boys are more likely than girls to be Independents.
3. Boys are more likely than girls to be Republicans.
4. Girls are only half as likely as boys to be Republicans.

B Study the table of party choices and claims about differences between boys and girls. Notice that there are 24 boys and 16 girls in the class.

1. Copy and complete this extended table.

	Democrat	Independent	Republican	Totals
Boys	8	4	12	■
Girls	8	2	6	■
Totals	■	■	■	■

2. Do the totals of political party choices change your answers to Question A? Explain your reasoning.

C One way to compare groups with unequal numbers of members is to compute percents.

1. Copy and complete the table below to show the fractions or percents of boys and girls with each preference.

	Democrat	Independent	Republican
Boys	$\frac{8}{24}=\frac{1}{3}=33\frac{1}{3}\%$	■	■
Girls	■	■	■

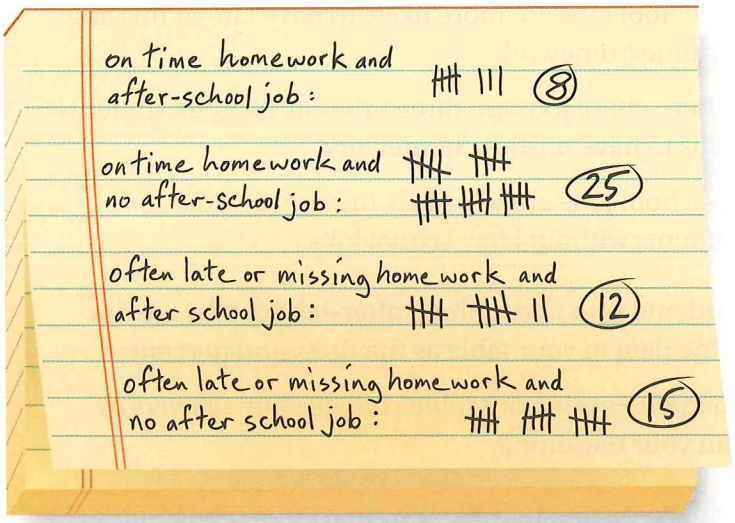
2. Do the percent calculations change your answers to Question A? Explain your reasoning.

ACE Homework starts on page 119.

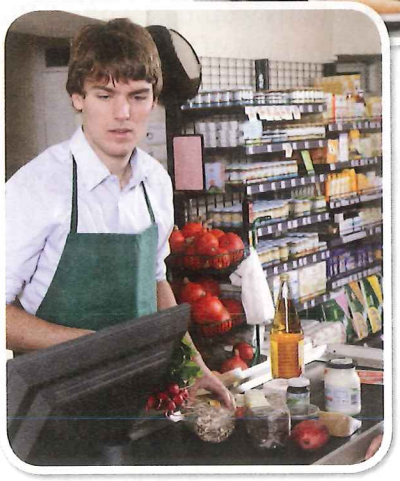
5.3 After-School Jobs and Homework

Working Backward: Setting up a Two-Way Table

The teachers at the high school did a study to see whether students who had jobs after school were more or less likely to turn in homework on time than students who did not have after-school jobs. Each student was categorized as *usually on time* or *often late* with homework and as *having a job* or *not having a job*. Here are the results.



Is there evidence that students with after-school jobs are more likely to have late or missing homework than students without after-school jobs?



Problem 5.3

Use the information about the students to answer these questions.

A Make a table to display the data on students and after-school jobs.

B Use your table from Question A. Do you think each statement is *true* or *false*? Justify your answers.

1. Students without after-school jobs are more likely to have late or missing homework than students with after-school jobs.
2. Students with after-school jobs are more likely to have late or missing homework than on-time homework.
3. Students without after-school jobs are three times as likely as students with after-school jobs to have on-time homework.
4. Students with after-school jobs are less likely to have on-time homework than students without after-school jobs.

C

1. The numbers of students with and without after-school jobs are not the same. Rewrite the data in your table as fractions and percents.
2. Do the fractions and percents in your table change your answers to Question B? Explain your reasoning.

D If someone claims that the data and analysis show that after-school jobs cause students to have late or unfinished homework, what alternate explanations would you offer? What do you think could be the cause of late or unfinished homework other than after-school jobs?

Homework starts on page 119.

Applications

Classify each variable as categorical (C) or numerical (N).

1. number of text messages you send in a day
2. brands of breakfast cereal
3. heights of students in your class
4. daily maximum temperature for your city
5. breeds of dogs
6. number of hours you sleep each night
7. types of flowers available from a florist
8. number of oranges in the 5-lb bags at a supermarket
9. heights of trees that were planted one year ago
10. number of students absent from school each day for one month

List possible categories for each categorical variable.

11. types of cars
12. methods of travel to school
13. types of instruments in an orchestra
14. sports played at school

15. You can analyze data in many ways, using graphs, tables, measures of center, and measures of spread.

Graphs	Tables	Measures of Center	Measures of Spread
bar graphs	frequency table	mean	range
circle graphs	two-way table	median	interquartile range
dot plots		mode	MAD
line plots			SD
histogram			
box plot			
scatter plot			
line graph			

Make a table similar to the one below. Enter the types of graphs, measures of center, and measures of spread you can use with each data type.

What can I use?

Categorical Data	Numerical Data
Graphs:	Graphs:
Measures of center:	Measures of center:
Measures of spread:	Measures of spread:

Exercises 16 and 17 use the survival rate data of men, women, and children on the *Titanic*.

Passenger Category	Saved	Lost
Men	338	1,352
Women	316	109
Children	56	53

16. Which of these claims about survival rates on the *Titanic* are true? Explain your reasoning.

a. More men than women were saved.

b. Women were more likely than children to be lost.

c. Men were about six times as likely to be saved as children.

17. Another way to see whether men, women, and children were lost at the same rate is to find the overall survival rate for all *Titanic* passengers. Use the overall rate to find expected survival counts for each passenger category. Overall, 32% of passengers were saved and 68% lost their lives.

a. Use the total numbers of men, women, and children on board the *Titanic* and use the overall survival rates. Copy and complete the table below.

Passenger Category	Expected Saved	Expected Lost
Men	■	■
Women	■	■
Children	■	■

b. Compare the table in part (a) with the data table. Which passenger categories had greater numbers of survivors than you would expect if all categories had the same proportion? Explain.

18. Suppose you are interested in learning about the effects of parents' smoking habits on their adult children. Use the data from the table below.

	Adult children smoke	Adult children do not smoke	Total	Percent of adult children who smoke
Both parents smoke	400	1,380	■	■
One parent smokes	416	1,823	■	■
Neither parent smokes	188	1,168	■	■
Total	■	■	■	■
Percent of adults with at least one parent who smokes	■	■	■	

a. Copy and complete the table.

b. Find the percent of adult children who smoke in each situation.

i. both parents smoke

ii. one parent smokes

iii. neither parent smokes

c. Draw a bar graph to compare the three percents you found in part (b).

d. Does the table show evidence that if parents smoke, then their adult children are more likely to be smokers? Explain.

e. Does the table show evidence that if only one parent smokes, their adult children are more likely to be smokers? Explain.

f. Does the table show evidence that adult children of nonsmoking parents are smokers? Explain.

19. The table below compares a treatment for rheumatoid arthritis to a placebo. A *placebo* is a treatment that has no medicine. The outcome of the experiment reflects whether individuals showed *no improvement, some improvement,* or *marked improvement* taking either the placebo or the active medicine.

Does the table show evidence that a person given the active treatment is more likely to show at least some improvement than a person given the placebo? Explain.

Treatment	Improvement			
	None	Some	Marked	Total
Active	14 32.6%	7 16.3%	22 51.1%	43
Placebo	28 68.3%	6 14.6%	7 17.1%	41
Total	42	13	29	84

Connections

20. Fifty households on a street were asked which brand of television they owned. The table shows the results from the survey.

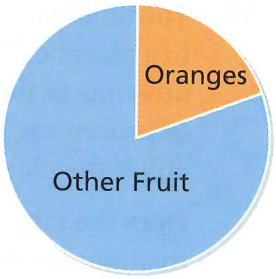
TV Brands	A	B	C	D	E	F
TV Owners	9	4	12	8	7	10

a. Draw a horizontal bar graph of the data.

b. Suppose you wanted to use the data to determine buying patterns of people living in the city. Is it possible that this sample is *biased* for particular TV brands? Explain your reasoning.

c. Which measures—mode, median, or mean—would you use to describe the typical television brand owned? Explain.

21. The circle graph shows the results of a survey in which people were asked, "What is your favorite fruit?" The angle of 68° represents 277 people who said their favorite fruit is oranges. Find the sample size used to the nearest 10 people.



Find the measure of the angle of a circle graph for each frequency.

22. 23 in a sample of 180

23. 128 in a sample of 720

24. 238 in a sample of 1,250

25. A gymnast received scores from five judges in the state competition.

Parallel Bars: 7.6, 8.2, 8.5, 8.2, and 8.9

a. What happens to the mean of scores when you multiply each data point by 2? By $\frac{2}{3}$? By 0.2?

b. Why do you think the mean changes like that? Explain.

26. Multiple Choice A store owner keeps a tally of the sizes of shoes bought at her store. Which measure of central tendency best describes the average shoe size sold?

A. mean **B.** mode **C.** median **D.** range

27. Multiple Choice Suppose all the students who took a math test yesterday scored over 75. Three students missed the test. Their scores are listed as 0 until they take the test. Which measure best represents the data?

F. mean **G.** range **H.** median **J.** standard deviation

28. Multiple Choice A bag contains red and black chips. The probability of selecting a red chip from the bag is $\frac{1}{4}$. What is the probability of drawing a black chip?

A. $\frac{1}{4}$ **B.** $\frac{1}{2}$ **C.** $\frac{3}{4}$ **D.** None of these

29. A student scored 40 out of 100 points on this week's test. Her teacher announced that this week's test will be averaged with next week's test. Can the student still get a C if she scores a 100 on next week's test? The lowest C is 70 points. She reasons, "My average will be 70, a C, because half of 40 is 20 and half of 100 is 50 and 20 plus 50 is 70." Does her method always work? Explain.

30. Multiple Choice Which situation can be represented using a scatter plot?

F. Jennifer keeps a list of the amount of time she spends on her social studies homework each day.

G. Mr. Jones wants to see if his students' shoe sizes are directly related to their heights.

H. Mr. DiSanti records his customers' best video game scores during the summer.

J. Sam keeps track of his algebra grades for the quarter.

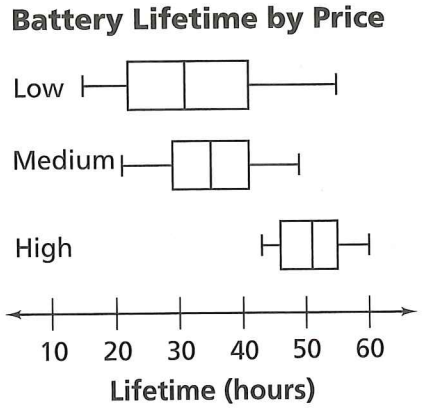
31. a. Make a scatter plot of the data in the table below.

Hours at Mall	10	8	9	3	1	2	5	6	7	8	2	3
Dollars Spent	42	14	25	21	9	32	50	60	16	22	100	45

b. What type of correlation (positive, negative, or zero) exists between the number of hours spent in the mall and the number of dollars spent?

Extensions

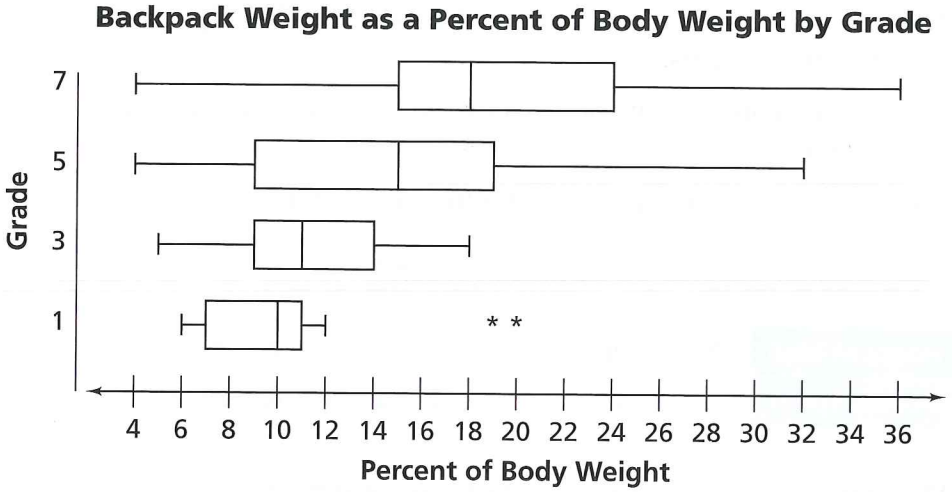
32. The triple box plot below shows the distribution of the lifetime (in hours) of three different batteries (low, medium, and high price).



a. The two variables in the box plot are *battery lifetime* and *battery price*. Which variable is numerical and which is categorical?

b. Does the graph support the claim that battery life depends on price? Explain your thinking.

33. Students collected two sets of data, the weight of a student and the weight of the student's backpack, from Grades 1, 3, 5, and 7. Then they computed the ratio of backpack weight to student weight as a percent. The graph below shows the data.



a. Which box plot has the greatest interquartile range? What does this tell you about the middle 50% of the backpack weights for that grade compared to the other grades?

b. What is the median of the data for Grade 1? What does this tell you about the data for these students?

c. Suppose that some health officials claim that backpacks should be only 15% of a student's weight. From the graph, are there any grades for which this is *not* the case? If so, which grades? Explain.

d. Do the box plots support the claim that students at higher grades tend to carry heavier backpacks? Explain.

34. A survey of people's favorite colors reported the results below.

Red: 12% **Orange:** 14% **Purple:** 28% **Blue:** 30% **Green:** 16%

a. Make a circle graph and a bar graph to show the results.

b. How do you use percents to make the circle graph? The bar graph?

c. What is the least number of people who could have taken the survey? Explain.

Mathematical Reflections 5

In this Investigation you analyzed data in two-way tables to find similarities and differences in groups. You learned how to draw accurate conclusions from these data arrays. The following questions will help you summarize what you have learned.

Think about these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. **What** are categorical variables and what do they measure?
2. **Suppose** a survey asked teenagers and adults whether or not they use text messaging.
   a. **How** could you arrange the data to compare the groups?
   b. **How** would you decide that the two groups—teenagers and adults—were different in their use of text messaging?
   c. **Suppose** that one analysis compared only the numbers in each group—teenage text messager, teenage non-text messager, adult text messager, and adult non-text messager. How might the analysis result in misleading conclusions?

Common Core Mathematical Practices

As you worked on the Problems in this Investigation, you used prior knowledge to make sense of them. You also applied Mathematical Practices to solve the Problems. Think back over your work, the ways you thought about the Problems, and how you used Mathematical Practices.

Tori described her thoughts in the following way:

In Problem 5.1, Question A, we were confused about how to make sense of the data presented in the table showing customers' preferences about which kind of roller coaster they like to ride. The data are grouped in two age categories.

Jen wondered how many customers were surveyed. So we decided to add up columns first. We saw that 60 customers prefer wood coasters and 80 customers prefer steel coasters. Then we added up rows. We saw that there were 105 customers surveyed who were 40 or younger; there were 35 customers who were older than that.

So when we find the total number of preferences separated by kind of coaster, does it equal the total number of customers separated by age? It does! We think 140 customers were surveyed.

Common Core Standards for Mathematical Practice

MP4 Model with mathematics.

- What other Mathematical Practices can you identify in Tori's reasoning?
- Describe a Mathematical Practice that you and your classmates used to solve a different Problem in this Investigation.

Looking Back

While working on the problems in this Unit, you extended your skills in writing equations to express linear relationships. You also learned about a type of nonlinear relationship called an inverse variation. You used inverse and linear relationships to solve problems and make predictions. You learned new ways to measure the spread of a data distribution and the strength of an association between two variables.

Use Your Understanding

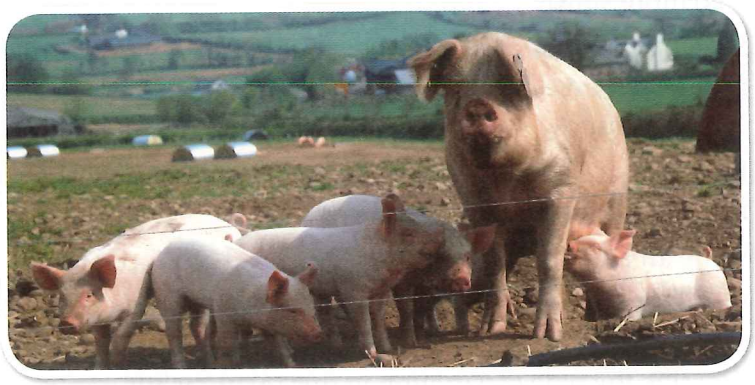
Solve the following problems to test your understanding of linear relationships, inverse variation, the correlation coefficient, and standard deviation. These problems are about a park with a small farm, a train, and a snack bar.

1. This table shows the growth of one pig raised on the farm.

Average Growth of a Properly Fed Pig

Age (mo)	0	1	2	3	4	5	6
Weight (lb)	3	48	92	137	182	228	273

SOURCE: Your 4-H Market Hog Project, Iowa State University.



a. Make a graph of the (age, weight) data. Draw a line that fits the data.

b. Find a linear equation in the form $y = mx + b$ for the line in part (a).

c. What do the values of m and b in your equation tell you about the pig's growth?

d. Use your equation to estimate the pig's weight at 3.5 months and at 7 months.

2. A group of students suspect that farm animals eat less when the weather is warm. They asked the farm staff to record the food an adult goat ate on days with different average temperatures.

Food Consumption of a Goat

Average Daily Temperature (°F)	30	40	45	55	60	75	85	90
Food Eaten (kg)	3.9	3.6	3.4	3.0	2.7	2.5	2.2	1.9

   a. Make a graph of the (temperature, food eaten) data. Draw a line that fits the data.

   b. Find a linear equation in the form $y = mx + b$ for your line in part (a).

   c. What do the values of m and b in your equation tell you about the relationship between average daily temperature and the goat's food consumption?

   d. Use your equation to predict how much the goat would eat on a day with an average temperature of 50°F and on a day with an average temperature of 70°F.

3. A small train gives visitors rides around the park on a 5,000-meter track. The time the trip takes varies. When many people are waiting in line, the drivers go quickly. When there are fewer people waiting, they go more slowly.

   a. Sketch a graph showing how average speed (in meters per minute) changes as the trip time (in minutes) increases from 1 to 10 minutes.

   b. For what parts of your graph are the predicted speeds realistic? Explain.

   c. Write an equation relating average speed s to trip time t.

   d. Write several sentences explaining as accurately as possible how average speed changes as trip time changes. In particular, describe the type of variation in this relationship.

4. The table shows the number of train trips over 10 summer days. Find the standard deviation of the data.

Day	1	2	3	4	5	6	7	8	9	10
Trains	5	8	6	12	15	6	20	12	14	12

5. The next table shows the relationship between the number of train trips and profit at the train station snack bar over 10 summer days.

Day	1	2	3	4	5	6	7	8	9	10
Trains	5	8	6	12	15	6	20	12	14	12
Snack Bar Profit	$40	$85	$55	$110	$165	$65	$190	$125	$130	$125

   a. Make a scatter plot of the (number of trains, snack bar profit) data.

   b. Estimate the correlation coefficient for the scatter plot. Is the correlation coefficient closest to -1, -0.5, 0, 0.5, or 1? Explain how you chose your estimate.

6. Park operators asked visitors which parts of the park needed improvement. They recorded the suggestions in the table.

	Farm	Playground	Picnic Area
Kids	48	24	8
Adults	30	15	5

   a. Do kids and adults have different opinions about what areas of the park need improvement? Use the data to support your answer.

   b. Suppose someone claimed that kids value the farm more than adults do. What would you say about that claim?

   c. Suppose someone claimed that kids value the playground more than adults do. What would you say about that claim?

English / Spanish Glossary

A **additive inverses** Two numbers, a and b, that satisfy the equation $a + b = 0$. For example, 3 and -3 are additive inverses, and $\frac{1}{2}$ and $-\frac{1}{2}$ are additive inverses.

inversos de suma Dos números, a y b, que cumplen con la ecuación $a + b = 0$. Por ejemplo, 3 y -3 son inversos de suma, y $\frac{1}{2}$ y $-\frac{1}{2}$ son inversos de suma.

C **categorical variables** Variables that measure characteristics using words that represent possible responses within a given category. Frequency counts can be made of the values for a given category. The table below shows examples of categories and their possible values.

Category	Possible Values
Month people are born	January, February, March
Favorite color to wear	magenta, blue, yellow
Kinds of pets people have	cats, dogs, fish, horses

variables por categorías Variables que miden características usando "palabras" que representan respuestas posibles en una categoría dada. Se pueden contar las frecuencias de los valores para una categoría dada. La siguiente tabla muestra ejemplos de categorías y sus posibles valores.

Categoría	Valores posibles
Mes de nacimiento de las personas	enero, febrero, marzo
Color preferido para vestir	magenta, azul, amarillo
Tipos de mascotas que tienen las personas	gatos, perros, peces, caballos

correlation coefficient A measure of the strength of a linear relationship between two variables using a decimal number between -1 and 1.

coeficiente de correlación Medida del grado de la relación lineal entre dos variables, usando un número decimal que esté entre -1 y 1.

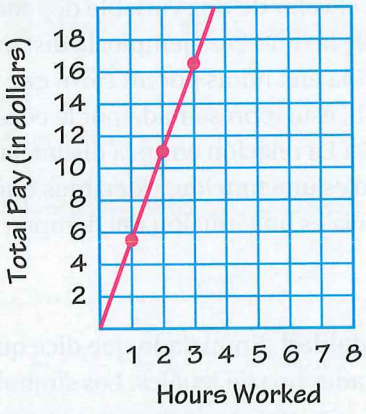
D **describe** Academic Vocabulary To explain or tell in detail. A written description can contain facts and other information needed to communicate your answer. A diagram or a graph may also be included when you describe something.

related terms *express, explain*

Hours Worked	1	2	3
Total Pay	$5.50	$11.00	$16.50

sample Describe the relationship between hours worked and pay.

The relationship is linear. Total pay varies directly with the number of hours worked. That is, as the number of hours worked increases by one, the pay increases by $ 5.50. This means that the employee earns $ 5.50 for each hour worked. I can also draw a graph that shows this relationship.



I plotted each point on the graph and I drew one line through all of the points. I can also write an equation, $P = 5.5t$, where P is the amount of money earned and t is the number of hours worked, to represent this relationship.

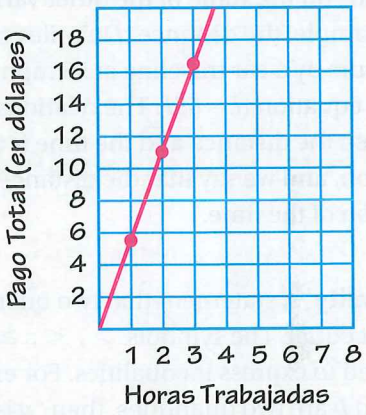
describir Vocabulario académico
Explicar o decir con detalle. Una descripción escrita puede contener datos y otro tipo de información necesaria para comunicar tu respuesta. También puedes incluir un diagrama o una gráfica cuando describes algo.

términos relacionados *expresar, explicar*

Horas trabajadas	1	2	3
Pago total	$5.50	$11.00	$16.50

ejemplo Describe la relación entre las horas trabajadas y el pago recibido.

La relación es lineal. El pago total varía directamente con el número de horas trabajadas. Es decir, a medida que el número de horas trabajadas aumenta en uno, el pago aumenta en $ 5.50 dólares. Esto significa que el empleado gana $ 5.50 por cada hora trabajada. También puedo dibujar una gráfica que muestre esta relación.



Marqué cada punto en la gráfica y tracé una línea a través de todos los puntos. Para representar estar elación, también puedo escribir una ecuación, $P = 5.5t$, donde P es la cantidad de dinero ganado y tes el número de horas trabajadas.

E **explain** Academic Vocabulary To give facts and details that make an idea easier to understand. Explaining can involve a written summary supported by a diagram, chart, table, or a combination of these.

related terms *analyze, clarify, describe, justify, tell*

sample The equation $c = 75d + 15$ gives the charge c in dollars for renting a car for d days. Explain what the numbers and variables in the equation represent.

The variable c represents the total amount the customer is charged. The variable d is the number of days the car is rented. 75 is the cost per day of renting the car. 15 is an additional one-time fee for the customer.

explicar Vocabulario académico Dar datos y detalles que hacen que una idea sea más fácil de comprender. Explicar puede implicar hacer un resumen escrito apoyado por un diagrama, una gráfica, una tabla o una combinación de éstos.

términos relacionados *analizar, aclarar, describir, justificar, decir*

ejemplo La ecuación $c = 75d + 15$ da el cargo c en dólares para alquilar un carro por d días. Explica qué representan los números y las variables en la ecuación.

La variable c representa la cantidad total que se le cobra al cliente. La variable d es el número de días que se alquiló el carro. 75 es el costo de alquilar el carro por día. 15 es un monto adicional que se le cobra al cliente sólo una vez.

F **function** A relationship between two variables in which the value of one variable depends on the value of the other variable. For example, the distance d in miles covered in t hours by a car traveling at 55 mph is given by the equation $d = 55t$. The relationship between the distance and the time is a function, and we say that the distance is a function of the time.

función Una relación entre dos variables en la que el valor de una variable depende del valor de la otra. Por ejemplo, la distancia d recorrida en t horas por un carro que viaja a 55 mph, está representada por la ecuación $d = 55t$. La relación entre la distancia y el tiempo es una función, y decimos que la distancia es una función del tiempo.

I **inequality** A statement that two quantities are not equal. The symbols $>$, $<$, $\geq$, and $\leq$ are used to express inequalities. For example, if a and b are two quantities, then "a is greater than b" is written as $a > b$, and "a is less than b" is written as $a < b$. The statement $a \geq b$ means "a is greater than or equal to b." The statement $a \leq b$ means "a is less than or equal to b."

desigualdad Enunciado que dice que dos cantidades no son iguales. Los símbolos $>$, $<$, $\geq$, y $\leq$ se usan para expresar desigualdades. Por ejemplo, si a y b son dos cantidades, entonces "a es mayor que b" se escribe $a > b$, y "a es menor que b" se escribe $a < b$. El enunciado $a \geq b$ quiere decir "a es mayor que o igual a b". El enunciado $a \leq b$ quiere decir "a es menor que o igual a b."

inverse variation A nonlinear relationship in which the product of two variables is constant. An inverse variation can be represented by an equation of the form $y = \frac{k}{x}$, or $xy = k$, where k is a constant (for $k > 0$). In an inverse variation, the values of one variable decrease as the values of the other variable increase.

variación inversa Una relación no lineal en la que el producto de dos variables es constante. Una variación inversa se puede representar por una ecuación de la forma $y = \frac{k}{x}$, ó $xy = k$, donde k es una constante (por $k > 0$). En una variación inversa, los valores de una variable disminuyen a medida que los valores de la otra variable aumentan.

linear relationship A relationship in which there is a constant rate of change between two variables. A linear relationship can be represented by a straight-line graph and by an equation of the form $y = mx + b$. In the equation, m is the slope of the line, and b is the y-intercept.

relación lineal Una relación en la que hay una tasa de cambio constante entre dos variables. Una relación lineal puede estar representada por una gráfica de línea recta y por una ecuación en la forma $y = mx + b$. En la ecuación, m es la pendiente de la recta y b es el intercepto en y.

M **mathematical model** An equation or a graph that describes, at least approximately, the relationship between two variables. To make a mathematical model, acquire data, plot the data points, and, when the points show a pattern, find the equation of a line or curve that fits the trend in the data. A mathematical model allows you to make reasonable guesses for values between and beyond the data points.

modelo matemático Una ecuación o gráfica que describe, al menos aproximadamente, la relación entre dos variables. Para hacer un modelo matemático es necesario reunir datos, marcar los puntos asociados con los datos y, cuando los puntos muestren un patrón, hallar la ecuación de la línea o curva que se corresponde con la tendencia de los datos. Un modelo matemático te permite hacer predicciones razonables para valores dentro y fuera de los datos obtenidos.

multiplicative inverses Two numbers, a and b, that satisfy the equation $ab = 1$. For example, 3 and $\frac{1}{3}$ are multiplicative inverses, and $-\frac{1}{2}$ and -2 are multiplicative inverses.

inversos multiplicativos Dos números, a y b, que cumplen con la ecuación $ab = 1$. Por ejemplo, 3 y $\frac{1}{3}$ son inversos multiplicativos, y $-\frac{1}{2}$ y -2 son inversos multiplicativos.

O **outlier** A value that lies far from the "center" of a distribution. *Outlier* is a relative term, but it indicates a data point that is much higher or much lower than the values that could be normally expected for the distribution.

valor extremo Valor que se sitúa lejos del "centro" de una distribución. *El valor extremo* es un término relativo, pero indica un dato que es mucho más alto o mucho más bajo que los valores que se podrían esperar normalmente de la distribución.

R **residual** The error calculated by finding the difference between an actual data point and the value that a model for the data predicts.

residuo El error que se calcula hallando la diferencia entre un punto real y el valor que predice el modelo de datos.

S **scatter plot** A graph used to explore the relationship between two variables. The graph below is a scatter plot of (height, arm span) for several people. Each point represents the height and arm span for each person.

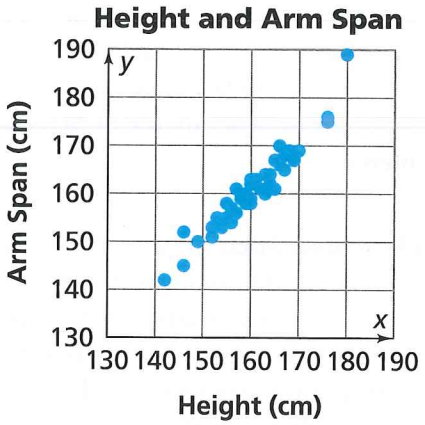
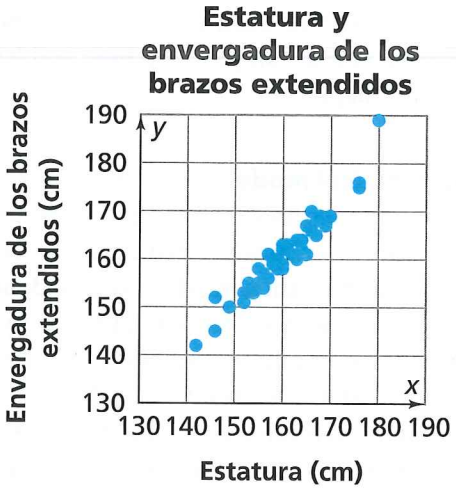
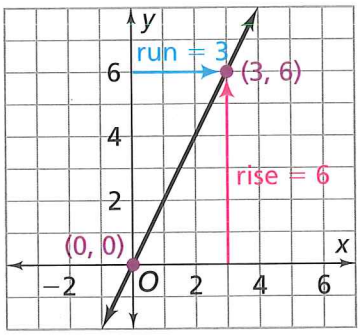


diagrama de dispersión Una gráfica que se usa para explorar la relación entre dos variables. La siguiente gráfica muestra un diagrama de dispersión (estatura, envergadura de los brazos extendidos) de varias personas. Cada punto representa la estatura y el espacio entre los brazos extendidos de cada persona.



slope The number that expresses the steepness of a line. The slope is the ratio of the vertical change to the horizontal change between any two points on the line. Sometimes this ratio is referred to as *the rise over the run*. The slope of a horizontal line is 0. Slopes are positive if the y-values increase from left to right on a coordinate grid and negative if the y-values decrease from left to right. The slope of a vertical line is undefined. The slope of a line is the same as the constant rate of change between the two variables. For example, the points (0, 0) and (3, 6) lie on the graph of $y = 2x$. Between these points, the vertical change is 6 and the horizontal change is 3, so the slope is $\frac{6}{3} = 2$, which is the coefficient of x in the equation.

pendiente El número que expresa la inclinación de una recta. La pendiente es la razón entre la variación vertical y la horizontal entre dos puntos cualesquiera de la recta. A veces a esta razón se le denomina *distancia vertical sobre distancia horizontal.* La pendiente de una recta horizontal es 0. Las pendientes son positivas si los valores de y aumentan de izquierda a derecha en una gráfica de coordenadas, y negativas si los valores de y disminuyen de izquierda a derecha. La pendiente de una recta vertical es indefinida. La pendiente de una recta es igual a la tasa de cambio constante entre dos variables. Por ejemplo, los puntos (0, 0) y (3, 6) están representados en la gráfica de $y = 2x$. Entre estos puntos, el cambio vertical es 6 y el cambio horizontal es 3, de manera que la pendiente es $\frac{6}{3} = 2$, que es el coeficiente de x en la ecuación.



solve Academic Vocabulary To determine the value or values that make a given statement true. Several methods and strategies can be used to solve a problem, including estimating, isolating the variable, drawing a graph, or using a table of values.

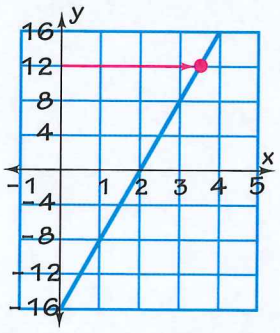
related terms *calculate, find*

sample Solve the equation $8x - 16 = 12$ for x.

I can solve the equation by isolating x on the left side of the equation.

$$8x - 16 = 12$$
$$8x = 28$$
$$x = \frac{28}{8} = \frac{7}{2} = 3.5$$

I can also sketch a graph of $y = 8x - 16$. When $y = 12$, x is between 3 and 4, so I know my solution is reasonable.



resolver Vocabulario académico Determinar el valor o los valores que hacen cierto un enunciado. Se pueden usar varios métodos y estrategias para resolver un problema incluyendo hacer una estimación, aislar la variable, dibujar una gráfica o usar una tabla de valores.

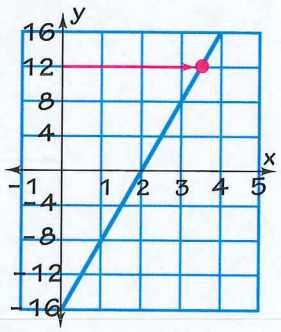
términos relacionados *calcular, hallar*

ejemplo Resuelve la ecuación $8x - 16 = 12$ para hallar el valor de x.

Puedo resolver la ecuación aislando x en el lado izquierdo de la ecuación.

$$8x - 16 = 12$$
$$8x = 28$$
$$x = \frac{28}{8} = \frac{7}{2} = 3.5$$

También puedo hacer el bosquejo de una gráfica de $y = 8x - 16$. Cuando $y = 12$, x está entre 3 y 4, por lo tanto sé que mi solución es razonable.



standard deviation Standard deviation measures the spread of a data set. The greater the standard deviation, the greater the spread of the data. To calculate the standard deviation, find the differences between the actual values and the mean. These differences are squared and averaged by dividing by $(n - 1)$. This average is the variance. Take the square root of the variance to get the standard deviation.

desviación estándar La desviación estándar mide la dispersión de un conjunto de datos. Mientras mayor sea la desviación estándar, mayor será la dispersión de los datos. Para calcular la desviación estándar, halla las diferencias entre los valores reales y la media. Estas diferencias se elevan al cuadrado y se promedian dividiendo por $(n - 1)$. Ese promedio es la varianza. Usa la raíz cuadrada de una varianza para obtener la desviación estándar.

U

unit rate A unit rate is a rate in which the second number (usually written as the denominator) is 1, or 1 of a quantity. For example, 1.9 children per family, 32 miles per gallon, and $\frac{\text{3 flavors of ice cream}}{\text{1 banana split}}$ are unit rates. Unit rates are often found by scaling other rates.

tasa por unidad Tasa en la que el segundo número (normalmente escrito como el denominador) es 1, ó 1 de una cantidad. Por ejemplo, 1.9 niños por familia, 32 millas por galón, y $\frac{\text{3 sabores de helado}}{\text{1 banana split}}$ son tasas por unidad. Las tasas por unidad se calculan a menudo poniendo a escala otras tasas.

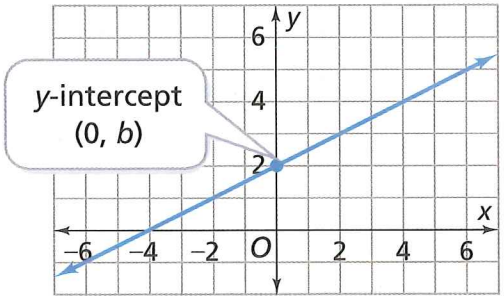
V

variance Variance is calculated from the differences between the actual value and the mean. These differences are squared and averaged by dividing by $(n - 1)$.

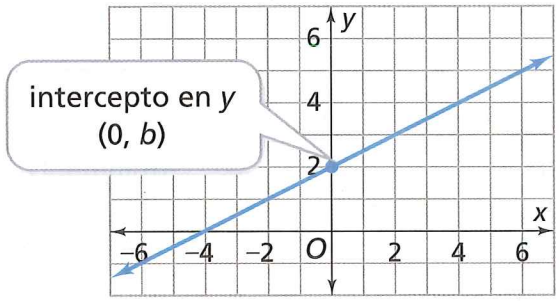
varianza La varianza se calcula a partir de las diferencias entre el valor real y la media. Estas diferencias se elevan al cuadrado y se promedian dividiendo por $(n - 1)$.

Y

y-intercept The point where the graph crosses the y-axis. In a linear equation of the form $y = mx + b$, the y-intercept is the constant, b. In the graph, the y-intercept is (0, 2) or 2.



intercepto en y El punto en el que la gráfica cruza el eje de las y. En una ecuación lineal de la forma $y = mx + b$, el intercepto en y es la constante, b. En la gráfica, el intercepto en y es (0, 2) ó 2.



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