Systems of Equations

Purpose: To find the common solution for two equations. This is the point where the two lines intersect; the solution is always written as a coordinate pair (x,y).

There are 4 ways to solve a system of linear equations.

- Graphing
- Equivalent Expressions
- Substitution
- Combination/Elimination

<u>Graphing</u> – If the two lines are plotted on the same graph, the common solution is the point of intersection. This answer may not always be precise.

<u>Equivalent Expressions</u> –

- Rearrange each equation so that they are **both** in slope-intercept form.
- Since we are looking for the situation where the equations have a common solution, the y-values for the two equations are equal to each other.
- Because the y-values are equal, you can set the equations equal to each other and solve for x.
- Once you have a value for x, substitute it into one of the original equations to find the value for y.
- To check you answer, substitute your solution (x,y) in to the remaining original equation to see if it works.

$y = 5x + 6$
$y = 2x + 12$

→

$5x + 6 = 2x + 12$
$3x + 6 = 12$
$3x = 6$
$x = 2$
$y = 16$

Solution: (2, 16)

<u>Substitution</u> –

- Choose an equation so that you can solve it for one of the variables. This is easiest if you have a variable with a coefficient of 1.
- Once you have solved one equation for a particular variable, substitute that value in for the variable in the second equation.
- Solve. Once you have the value for one of the variables, substitute it in to one of the original equations to find the value for the other variable.
- To check your answer, substitute your solution (x,y) in to the remaining original equation to see if it works.

$4x + 2y = 20$
$y = -3x + 13$

↓

$4x + 2(-3x + 13) = 20$
$4x – 6x + 26 = 20$
$-2x + 26 = 20$
$-2x = -6$
$x = 3$

Substitute the value of y from the second equation into the first equation and solve for x.

Solution: (3, 4)

Combination/Elimination –

- This involves combining the two equations so that you are able to eliminate one of the variables. The variable you eliminate is the one that has the same coefficient (+/-) in both equations.

$$\begin{aligned} 3x + 4y &= 19 \\ \underline{3x - 3y} &\underline{= 12} \\ 7y &= 7 \\ y &= 1 \\ x &= 5 \end{aligned}$$

Both equations contain a "3x" so we can subtract one equation from the other to eliminate "x" and solve for "y".

Solution: (5, 1)

- In some cases you do not have similar coefficients so you must change **one** equation into an equivalent form by multiplying both sides by the same number.

$$\begin{aligned} 4x + 3y &= 18 \\ 2x - 2y &= 2 \end{aligned}$$

If we multiply the second equation by 2, we will have "x's" in both equations with a coefficient of 4.

$$\downarrow$$

$$\begin{aligned} 4x + 3y &= 18 \\ \underline{4x - 4y} &\underline{= 4} \\ 7y &= 14 \\ y &= 2 \\ x &= 3 \end{aligned}$$

Subtract one equation from the other to eliminate x and solve for y.

Solution: (3, 2)

- In some cases you do not have similar coefficients so you must change **both** equations into equivalent forms by multiplying both sides of each equation by its own same number.

$$\begin{aligned} 3x + 2y &= 14 \\ 5x - 3y &= -2 \end{aligned}$$

We can multiply the top equation by 3 on both sides and the bottom equation by 2 on both sides. Then we will be able to add the equations to eliminate the y-variable and solve for x.

$$\downarrow$$

$$\begin{aligned} 9x + 6y &= 42 \\ \underline{10x - 6y} &\underline{= -4} \\ 19x &= 38 \\ x &= 2 \\ y &= 4 \end{aligned}$$

Solution: (2, 4)