Lesson 12: Systems of Inequalities Word Problems (Answer Key)

1. The ninth graders are hosting the next school dance. They would like to make at least a $500 profit from selling tickets. The ninth graders estimate that at most 300 students will attend the dance. They will earn $3 for each ticket purchased in advance and $4 for each ticket purchased at the door.

- Write a system of inequalities to represent this situation.
- Graph each inequality on the grid.
- Suppose only 30 people buy advance tickets. How many people would need to buy tickets at the door? (Identify one realistic solution) Justify your answer.

What do we know:
Make at least $500 At most 300 students will attend
$3 for advance & $4 for tickets at door

We must write two inequalities. We know information about the **cost of tickets** and the **number of expected attendees.**

Let x = the number of people who purchase tickets in advance
Let y = the number of people who purchase tickets at the door

Verbal model for cost of tickets:
Advance purchase + Door purchase is at least $500
3x + 4y ≥ 500
3x + 4y ≥ 500

Verbal model for number of expected attendees
At most 300 students will attend
x + y ≤ 300 (The number of students total is the number of advance purchasers + the number of door purchasers (x + y)
x + y ≤ 300

- **Our system of inequalities for this situation is:**
  3x + 4y ≥ 500 & x + y ≤ 300

The red line represents:
3x + 4y ≥ 500

The x-intercept (let y = 0)
3x + 4(0) = 500
3x = 500
x=166.67

The y-intercept (let x = 0)
3(0) +4y = 500
4y = 500
Y = 125

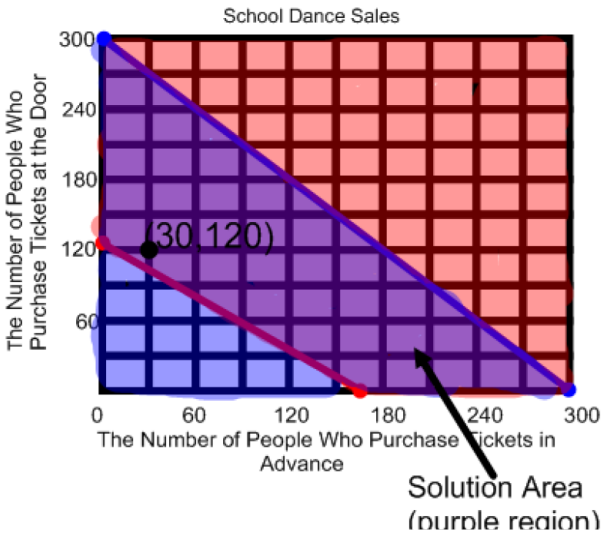
Shading: Substitute (0,0)
3x + 4y ≥ 500
3(0) +4(0) ≥500
0≥ 500 is not true

The blue line represents:
x + y ≤ 300

The x-intercept (let y = 0)
x + 0 = 300
x = 300

The y-intercept (let x = 0)
0 + y =300
y= 300

Shading: Substitute (0,0)
x+y ≤ 300
0+0 ≤ 300
0≤ 300 is true



According to the graph, if 30 people buy advance tickets, then about 120 would need to buy tickets at the door in order for the 9th graders to make their goal of at least $500.

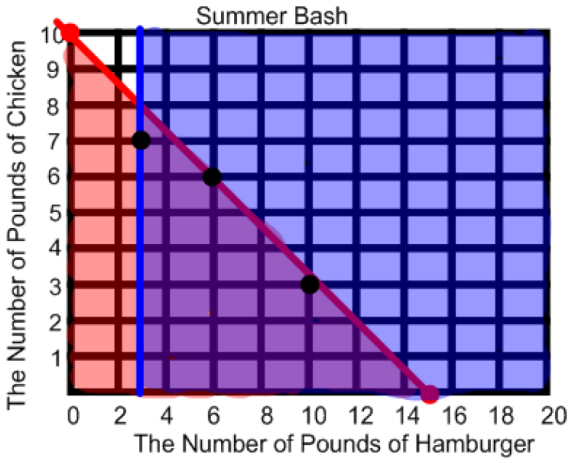
Justify:

3x+4y ≥ 500
3(30) +4(120) ≥ 500
90 + 480 ≥ 500
570 ≥ 500 ☺

x+y ≤ 300
30 + 120 ≤ 300 ☺
150≤ 300

2. In order to prepare for your summer bash, you go to the supermarket to buy hamburgers and chicken. Hamburgers cost $2 per pound and chicken costs $3 per pound. You have no more than $30 to spend. You expect to purchase at least 3 pounds of hamburgers.

- Write a system of inequalities to represent this situation.
- Graph the system of inequalities on the grid.
- Give three possible combinations for buying hamburgers and chicken for your summer bash.
- Justify your answers.



What do we know:
Hamburgers -$2 Chicken - $3 No more than $30 to spend
Purchase at least 3 pounds of hamburger

We must write two inequalities. We know information about the **cost of hamburgers and chicken** and about **how much hamburger you will purchase**.

Let x = the number of pounds of hamburger
Let y = the number of pounds of chicken

Verbal Model:
Cost of hamburger + Cost of chicken is no more than $30
2x + 3y ≤ 30
2x + 3y ≤ 30

Purchase at least 3 pounds of hamburgers
x ≥ 3 (hamburgers are greater than or equal to 3 pounds)

- **Our system of inequalities for this situation is:**
  2x + 3y ≤ 30 & x ≥ 3

Let the red line represent:
2x + 3y ≤ 30

X intercept: (let y = 0)
2x+3(0) =30
2x = 30
X = 15

Y intercept (let x = 0)
2(0) + 3y = 30
3y = 30
Y = 10

Shading; Substitute (0,0)
2(0) +3(0) ≤ 30
0 ≤ 30 is true
(shade the half plane that contains (0,0)).

Let the blue line represent:
x ≥ 3

x = 3 – this is a vertical line through the x intercept: x = 3.

Shading: Substitute (0,0)
0 ≥ 3 is not true
You expect buy at least 3 lbs, so that means 3 lbs or more. Therefore, you have to shade 3 or greater.

Three possible combinations for buying hamburgers and chicken are:

- **3 pounds of hamburger and 7 pounds of chicken.**
  Justify: 2(3) + 3(7) ≤ 30 & x≥3
  27 ≤ 30 ☺ **3 ≥ 3** ☺

- **6 pounds of hamburger and 6 pounds of chicken.**
  Justify: 2(6) + 3(6) ≤ 30 & x ≥ 3
  30 ≤ 30 ☺ **6 ≥ 3** ☺

- **10 pounds of hamburger and 3 pounds of chicken.**
  Justify: 2(10) + 3(3) ≤ 30 & x ≥ 3
  29 ≤ 30 ☺ **10≥ 3** ☺

Answers will vary. Any ordered pair within the purple shaded region is correct.

3. Jenny is making jewelry for an Arts and Crafts show. She would like to make at least $100 in sales. She estimates that she will sell at most 50 pieces of jewelry. The bracelets that she is selling cost $2 and the necklaces cost $3.

- Write a system of inequalities to represent this situation.
- Graph each inequality on the grid below.
- Give two possible combinations of bracelets and necklaces that can be sold in order for Jenny to meet her goal. Justify your answer.

What do we know:
Bracelets -$2 **Necklaces - $3** **Make at least $100**
Will sell at most 50 pieces of jewelry

We must write two inequalities. We know information about the **sales of bracelets and necklaces** and **how many pieces of jewelry she will sell.**
Let x = the number of bracelets
Let y = the number of necklaces

Verbal Model:
Bracelet Sales + Necklace sales is at least $100
2x + 3y ≥ 100
$2x + 3y \geq 100$

Sell at most 50 pieces of jewelry
Bracelets + necklaces ≤ 50
$x + y \leq 50$

- **Our system of inequalities for this situation is:**
  $2x + 3y \geq 100$ & $x + y \leq 50$

Let the red line represent:
$2x + 3y \geq 100$

X intercept: (let y = 0)
$2x+3(0) =100$
$2x = 100$
X = 50

Y intercept (let x = 0)
$2(0) + 3y = 100$
$3y = 100$
Y = 33.3

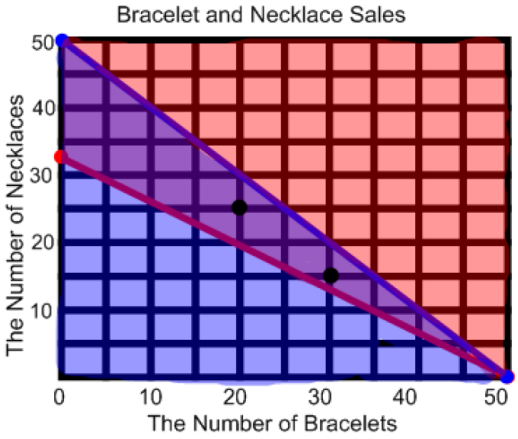
Shading; Substitute (0,0)
$2(0) +3(0)\geq 100$
$0 \geq 100$ not true
(shade the half plane that does not contain (0,0)).

Let the blue line represent:
$x + y \leq 50$

X intercept (let y = 0)
$x + 0 = 50$
x = 50

Y Intercept (let x = 0)
$0 + y = 50$
y = 50

Shading: Substitute (0,0)
$0 + 0 \leq 50$
$0 \leq 50$ is true, so shade the half plane that contains (0,0)



Two possible combinations of bracelet and necklace sales are:

- 20 bracelets and 25 necklaces.
  Justify: $2(20) + 3(25) \geq 100$ & $20+ 25 \leq 50$
  $115 \geq 100$ ☺ & $45 \leq 50$☺

- 30 bracelets and 15 necklaces.
  Justify: $2(30) + 3(15) \geq 100$ & $30+15 \leq 50$
  $105 \geq 100$ ☺ $45 \leq 50$☺

Answers will vary. Any ordered pair within the purple shaded region is correct.

4. Jason is buying wings and hot dogs for a party. One package of wings costs $7. Hot dogs cost $5 per package. He must spend no more than $40.

- Write an inequality to represent the cost of Jason's food for the party.
- Jason knows that he will be buying at least 5 packages of hot dogs. Write an inequality to represent this situation.
- Graph both inequalities. Give two options for Jason when buying wings and hot dogs.

What do we know:
Wings -$7 **Hot Dogs - $5** **Spend no more than $40**
Buy at least 5 packages of hot dogs

We must write two inequalities. We know information about the **price of wings and hot dogs** and **how many packages of hot dogs he will buy.**
Let x = the number of packages of wings
Let y = the number of packages of hot dogs

Verbal Model: (cost of Jason's Food)
Wings + Hot dogs must cost no more than $40
7x + 5y ≤ 40
7x + 5y ≤ 40

Verbal Model: (hot dog packages)
At least 5 packages of hot dogs
Y ≥ 5 (hot dogs are greater than or equal to 5)
- **Our system of inequalities for this situation is:**
  7x + 5y ≤ 40 & y ≥ 5

Let the red line represent:
7x + 5y ≤ 40

X intercept: (let y = 0)
7x+5(0) =40
7x = 40
X = 5.71

Y intercept (let x = 0)
7(0) + 5y = 40
Y = 8

Shading; Substitute (0,0)
7(0) +5(0)≤ 40
0 ≤ 40 is true
(shade the half plane that contains (0,0)).

Let the blue line represent:
y ≥ 5

y = 5 is a horizontal line
through the y-intercept of 5.

Since y is greater than or equal to 5
Shade all points that have a y coordinate greater than 5.
(0 ≥ 5 is not true, so do not shade the half plane that contains (0,0))

Jason doesn't have a whole lot of options for buying wings and hot dogs. Notice that the purple shaded region is very small.

Two options for buying wings and hot dogs are:

(1,5) 1 package of wings and 5 packages of hot dogs.
7(1) + 5(5) ≤ 40 y≥ 5
32 ≤ 40 ☺ 5 ≥ 5 ☺

(2,5) 2 packages of wings and 5 packages of hot dogs.
7(2) +5(5) ≤ 40 y≥ 5
39 ≤ 40 ☺ 5 ≥5 ☺

Check Your Understanding

A Dinner Theatre actress is paid $250 per day to rehearse the play and $500 per day to perform in front of an audience. In one season, an actress earned between $2000 and $5000.

- Write a system of inequalities that represents this situation. (2 points)

Let x = the number of days the actress rehearses Let y = the number of days the actress performs

We know that she earned between 2000 and 5000. This means greater than 2000 and less than 5000. So, we will write two equations based on this information.

$250x+500y > 2000$

$250x + 500y < 5000$

This is our system of equations.

- Graph the system of inequalities on the grid. (2 points)

In order to graph, we will first find the x-intercepts of both equations because we must determine the scale that will be used on the x and y axis.

$250x + 500y > 2000$ (Red Line)

x-int: (8,0) Y int: (0, 4)

$250x +500y < 5000$ (Blue Line)

x-int: (20,0) Y int: (0,10)

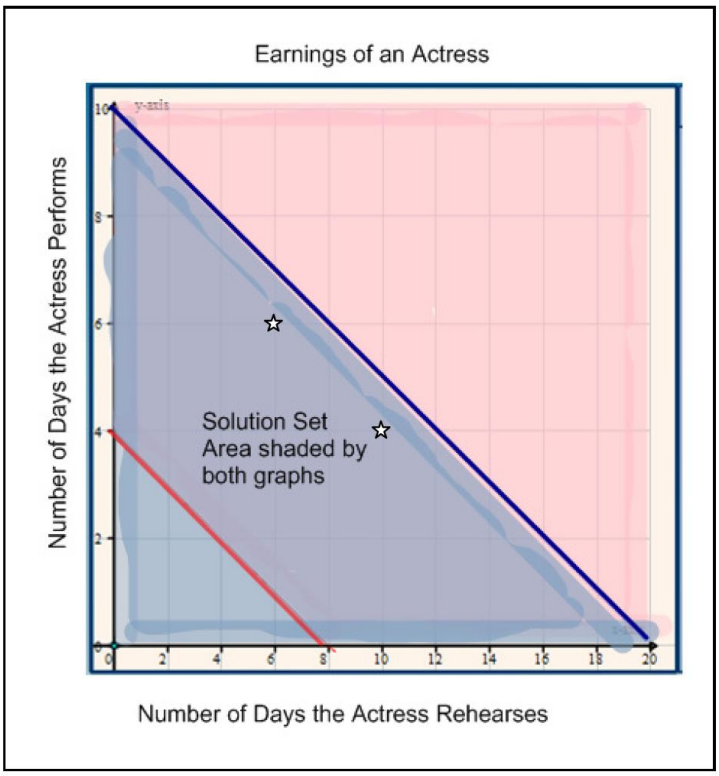
Largest x coordinate is 20, so I will use a scale of 2 on the x axis.

Largest y coordinate is 10, so I will use a scale of 1 on the y axis.

The red line's solution set does not contain (0,0) because 0 >2000 is not a true statement.

The blue line's solution set contains (0,0) because 0<5000 is a true statement.

The solution set for the system is between the red and blue lines as this is the part that was shaded by both graphs.



- Identify two different ways the actress may have earned her salary. Justify your answers. (2 points)

The actress may have earned her salary by rehearsing for 6 days and performing for 6 days. (6,6)

Justification:

$250x + 500y > 2000$ $250x + 500y < 5000$

$250(6) + 500(6) > 2000$ $250(6)+500(6) < 5000$

$4500 > 2000$ $4500 < 5000$

The actress may have rehearsed for 10 days and performed for 4 days. (10,4)

$250x + 500y > 2000$ $250x + 500y < 5000$

$250(4) + 500(4) > 2000$ $250(4)+500(4) < 5000$

$3000 > 2000$ $3000 < 5000$