Answers Applications Connections Extensions

Investigation 4

ACE
Assignment Choices

Differentiated Instruction
Solutions for All Learners

Problem 4.1

Core 1–7
Other 27–37, 61, and unassigned choices from previous problems

Problem 4.2

Core 8–14
Other 38–46, 62, and unassigned choices from previous problems

Problem 4.3

Core 15–20
Other 47–50, 63, and unassigned choices from previous problems

Problem 4.4

Core 21–26
Other 51–60, 64–67, and unassigned choices from previous problems

Adapted For suggestions about adapting Exercises 2–7 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 27–43, 46: *Moving Straight Ahead*; 46: *Variables and Patterns*; 47: *Filling and Wrapping*; 48, 49, 57–60: *Say It With Symbols*; 51–56: *Frogs, Fleas, and Painted Cubes; Growing, Growing, Growing; Thinking with Mathematical Models*

Applications

1. a. Matching equations to graphs: AD is represented by $y = -x + 5{,}500$
BD is represented by $y = -0.5x + 3{,}250$
BE is represented by $y = 0.2x + 2{,}200$
CE is represented by $y = x - 1{,}000$

  b. AD intersects BE at $(2{,}750, 2{,}750)$
AD intersects CE at $(3{,}250, 2{,}250)$
BD intersects CE at $(2{,}833, 1{,}833)$

2. $(x, y) = (-3, -14)$ **3.** $(x, y) = (7, 28)$

4. $(x, y) = (-2, -5)$ **5.** $(x, y) = (-4, 20)$
6. $(x, y) = (10, 164)$ **7.** $(y, y) = \left(\frac{29}{6}, -\frac{248}{3}\right)$
8. $y = \left(-\frac{2}{3}\right)x - 2$ **9.** $y = \left(\frac{7}{9}\right)x - \frac{4}{9}$
10. $y = -2x - 3$ **11.** $y = \left(\frac{1}{4}\right)x$
12. $y = x + 1$ **13.** $y = -5x + 3$
14. $x = -1.5y - 3$

$x = \left(\frac{9}{7}\right)y + \frac{4}{7}$

$x = -0.5y - 1.5$

$x = 4y$

$x = y - 1$

$x = -0.2y + 0.6$

15. $(x, y) = (3, 0)$
16. $(x, y) = (1.5, 0.5)$
17. $(x, y) = (-15, -22)$
18. $(x, y) = (1, 1)$
19. $(x, y) = (4, 2)$
20. $(x, y) = (1, 1)$
21. $(x, y) = \left(\frac{14}{3}, 1\right)$
22. $(x, y) = \left(2, -\frac{1}{9}\right)$
23. $(x, y) = (-3, -2)$
24. $(x, y) = (-4, 2)$
25. $(x, y) = (-1.5, -3)$
26. One approach multiplies the second equation by 2 and then adds it to the first:

$$\begin{cases} 2x - 3y = 14 \\ -2x + 6y = -12 \end{cases}$$

$$3y = 2; y = \frac{2}{3}$$

$$2x - 3\left(\frac{2}{3}\right) = 14$$
$$2x = 16$$
$$x = 8$$

$$(x, y) = \left(8, \frac{2}{3}\right)$$

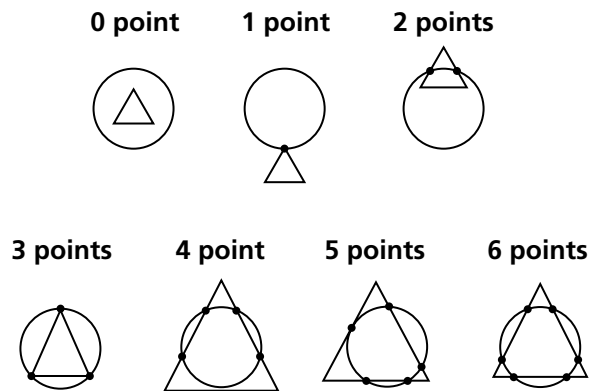(Or you could add both equations together to get $x = 8$ directly.)

Connections

27. $x = 4$ **28.** $x = -4$

29. $x = -11$ **30.** $x = 5$

31. $x = 3$ **32.** $x = -4$

33. a. $y = -13$ **b.** $y = -8$

    c. $y = -6.5$ **d.** $y = 0$

    e. $y = -7$ **f.** $y = -1.5$

34. $y = -4x + 3$ **35.** $y = (\frac{2}{3})x + 2$

36. $y = (-\frac{3}{4})x + 2$ **37.** $y = (-\frac{3}{4})x + \frac{31}{4}$

38. slope $= -1.5$; y-intercept $= 2$

39. slope $= 0.5$; y-intercept $= -1.5$

40. slope $= 1$; y-intercept $= -7$

41. slope $= 4$; y-intercept $= -8$

42. slope $= 2$; y-intercept $= 3$

43. slope $= 0$; y-intercept $= 9$

44. a. A line and a circle might intersect in 0, 1, or 2 points.

0 points   **1 point**   **2 points**
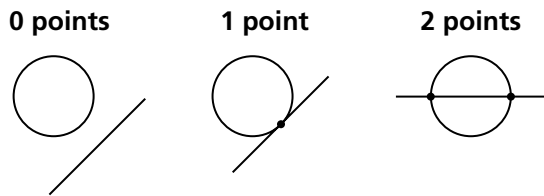
    b. Two circles might intersect in 0, 1, 2, or infinitely many points (if they are identical).

0 points   **1 point**

2 points   **Infinite**
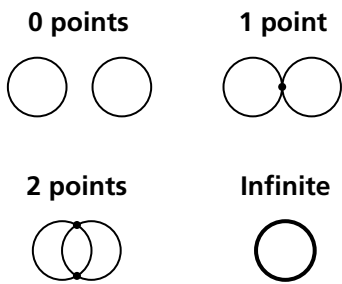
    c. A circle and a triangle might intersect in 0, 1, 2, 3, 4, 5, or 6 points.

0 point   **1 point**   **2 points**

3 points   **4 point**   **5 points**   **6 points**

    d. A circle and a rectangle might intersect in 0, 1, 2, 3, 4, 5, 6, 7, or 8 points.

0 point   **1 point**   **2 points**   **3 points**

4 points **5 points** **6 points** **7 points** **8 points**
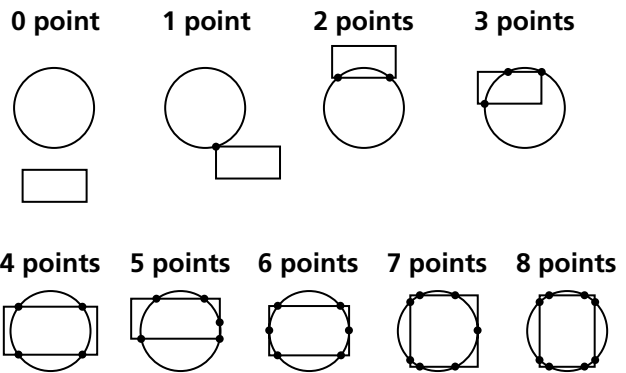
45. a. 10 (4 from each point, 5 points, but each chord counted twice)

    b. 5

    c. 16

46. C

47. a. One cover will look like this:
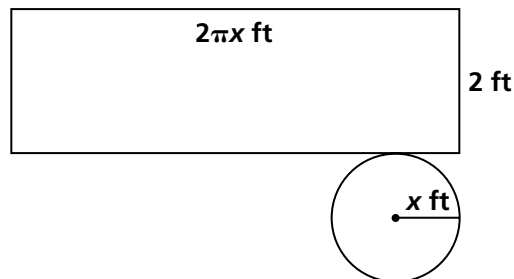
b. Area is given by $\pi x^2 + 4\pi x$ and by $\pi x(x + 4)$; volume by $2\pi x^2$.

48. J   **49.** D

50. a. Coordinates of points on the circle.

	A	B	C	D	E
X	0	3	5	0	−5
Y	5	4	0	−5	0

b. New figure is a circle with radius 10 centered at origin.

	A′	B′	C′	D′	E′
2X	0	6	10	0	−10
2Y	10	8	0	−10	0

c. New figure is a circle with radius 5 centered at $(2, 2)$:

	A	B	C	D	E
X + 2	2	5	7	2	−3
Y + 2	7	6	2	−3	2

d. Circle in (b) is similar to original with scale factor 2; circle in (c) is congruent to the original.

51. a. Line

b. Circle

c. Parabola

d. Inverse variation curve

e. Exponential curve

f. Line; the x^2 terms cancel.

g. Inverse variation curve

h. Line

52. Quadratic; the second differences are a constant 2. The equation is $y = x(x - 4)$.

53. Exponential; the equation is $y = 3^x$.

54. Linear; the equation is $y = 3x - 1$.

55. Inverse variation; the product of each pair of x and y values is 12. The equation is $xy = 12$ or, equivalently, $\frac{12}{x} = y$ or $\frac{12}{y} = x$.

56. a. Graph 2   **b.** Graph 1

c. Graph 4   **d.** Graph 3

e. Graph 6   **f.** Graph 5

57. $x = 3$

58. $\frac{-1}{2} = x$

59. $x = 3$ or $x = 4$

60. $x = -6$ or $x = 1$

Extensions

61. a. Yolanda: $y = 5.5x$; Marissa: $y = 20 + 0.5x$

b. Marissa's rate is a better deal for the customer when $x > 4$ hours.

c. They have the same charge for $x = 4$ hours.

62. a. $R = 2s - 1$

b. $T = 3s - 2$

c. $S + (2S - 1) + (3S - 2) = 21$ when $S = 4, R = 7$ and $T = 10$.

63. a. $\begin{cases} m + t = 49 \\ 5t + 1 = m \end{cases}$

b. Trevor sold 41 and Melissa sold 8.

64. a. No solutions because the lines are disjoint and parallel; an algebraic solution: the system

$$\begin{cases} x - 2y = 3 \\ -3x + 6y = -6 \end{cases}$$

is equivalent to

$$\begin{cases} 3x - 6y = 9 \\ -3x + 6y = -6 \end{cases}$$

If $3x - 6y = 9$ and $-3x + 6y = -6$, then using the combination method: $(3x - 6y) + (-3x + 6y) = 9 + (-6)$, which leads to $0 = 3$. Since $0 = 3$ is a false statement there is no solution to this system.

b. Infinite solutions because the lines are identical; the algebraic solution for the system

$$\begin{cases} x - y = 4 \\ -x + y = -4 \end{cases}$$

If $x - y = 4$ and $-x + y = -4$, then using the combination method $(x - y) + (-x + y) = 4 + (-4)$ which leads to $0 = 0$. The solution of $0 = 0$ is a true statement and means that for all values (x, y) the equations in this system are equivalent.

c. No solutions because the lines are disjoint and parallel; An algebraic solution: the system is

$$\begin{cases} 2x - 3y = 4 \\ 4x - 6y = 7 \end{cases}$$

which is equivalent to

$$\begin{cases} 4x - 6y = 8 \\ 4x - 6y = 7 \end{cases}$$

If $4x - 6y = 8$ and $4x - 6y = 7$ then using the combination method: $4x - 6y - (4x - 6y) = 8 - 7$ which leads to $0 = 1$ which is a false statement so there is no solution to this system.

d. Infinite solutions because the lines are identical; an algebraic solution: the system

$$\begin{cases} 4x - 6y = 4 \\ -6x + 9y = -6 \end{cases}$$

is equivalent to

$$\begin{cases} 36x - 54y = 36 \\ -36x + 54y = -36 \end{cases}$$

(multiplying the first equation by 9 and the second equation by 6). If $36x - 54y = 36$ and $-36x + 54y = -36$, then using the combination method: $(36x - 54y) + (-36x + 54y) = 36 + (-36)$ which leads to $0 = 0$ which is a true statement and means that for all values (x, y) the equations in this system are equivalent.

65. Substituting $y = \frac{4}{3}x$ in the equation for

$$x^2 + (\tfrac{4}{3}x)^2 = 25$$
$$\tfrac{25}{9}x^2 = 25$$
$$x^2 = 9$$
$$x = \pm 3$$
$$x = 3 \text{ when } y = 4$$
$$x = -3 \text{ when } y = -4$$

66. Answers will vary. We suggest some strategies that can be used to generate the answers.

a. Start with $x = 3$ and $y = 7$, then get something like $x + y = 10$ and $x - y = -4$, for example. You could also use $2x + 3y = 27$, etc.

b. Same strategies as in (a) will work.

c. Write one equation $ax + by = c$ and then another $ax + by = c + 1$, for example. Just make sure slopes of lines are the same, but intercepts are different.

67. a. No, the top two equations are exactly the same while the bottom two equations, when put into $y = mx + b$ form, are the same. Hence, the four equations represent only two distinct lines.

b–c. It does not represent either equation in the system, but it passes through the point of intersection and hence provides the y-coordinate of the solution pair.

d. The resulting equation will be linear since: $ax + by = c$ and $dx + ey = f$ together imply that $(a + d)x + (b + e)y = c + f$.

The resulting equation will pass through the intersection of $ax + by = c$ and $dx + ey = f$ if such an intersection exists. If they are parallel, then the new equation will also be parallel.

Possible Answers to Mathematical Reflections

1. The goal in solving systems of linear equations like those shown is to find values of the unknowns x and y that satisfy both equations simultaneously. A solution to a system with two unknowns is either an ordered pair of numbers, no solution, or an infinite number of solutions.

2. In the given problems the most efficient strategy is likely to be:

a. Setting $4x - 5 = 1.5x + 8$ and proceeding as in solving linear equations with one variable. When the value of x is known, substitute in either equation to find the corresponding value of x.

b. Subtract one equation from the other to eliminate x. Then solve the resulting equation for y and substitute that value in either original equation to find the corresponding value of x. You could also solve both equations for x in terms of y, set the resulting expressions in y equal, and then solve for y.

c. This system could be solved by solving the second equation for x in terms of y and then substituting that expression for x in the first equation.

d. This system seems likely to be solved most easily by multiplying both sides of the second equation by 2 and then subtracting it from the first equation. This will eliminate x, providing an easy solution for y.

3. A proposed solution for a system of linear equations in two variables can be checked by substituting the numerical values for each unknown and determining whether the resulting statements are in fact equalities.