Investigation 3

ACE Assignment Choices

Differentiated Instruction

Problem 3.1

Core 1–8, 28–35
Other Unassigned choices from previous problems

Problem 3.2

Core 15–27, 36–48
Other 49, 50, 63, and unassigned choices from previous problems

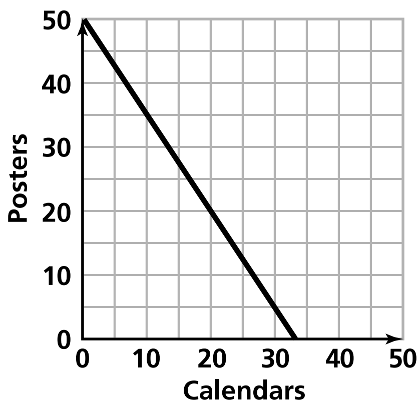
Problem 3.3

Core 10–14
Other 9, 51–62, 64, 65 and unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 2 and other ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 36–48, 50–52, 57–62: *Moving Striaght Ahead*; 50–52: *Say It With Symbols;* 53, 56: *Filling and Wrapping;* 54, 55: *Frogs, Fleas, and Painted Cubes*

Applications

1. a. $I = 3c + 2p$
   b. $3(25) + 2(18) = 111$
   c. $3(12) + 2(15) = 66$
   d. $3(20) + 2(12) = 84$
   e. Some possible pairs include (0, 50), (10, 35), (20, 20), (30, 5)
   f. The graphs may look something like the ones above at the right.



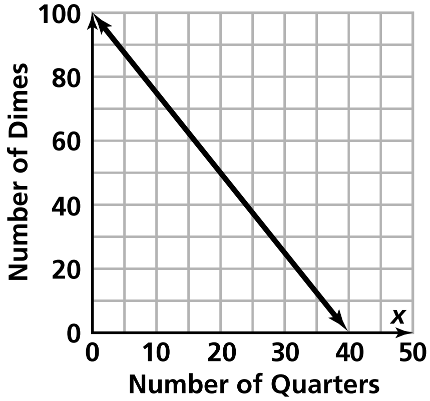
Note: The scales can be determined by finding the points where the line intercepts the horizontal and vertical axes. Since there is no dependence relationship between the posters and the calendars, it does not matter which variable is on the horizontal axis and which is on the vertical axis. The intercepts for the graph on the left are (33.3, 0) and (0, 50) and the graph the right (0, 33.3) and (50, 0).

   g. Estimates of other pairs might include (4, 44), (16, 26), (26, 11), etc. for the graph on the left and (44, 4), (26, 16), (11, 26), etc. for the graph on the right. Note to the teacher: The two graphs are mirror images of each other after a reflection in line $y = x$. So the coordinates of the graph on the left can be used to find the coordinates for the graph on the right by applying the transformation $(x, y) \rightarrow (y, x)$.

2. a. 40 quarters; 100 dimes

   b. $0.25x + 0.10y = 10$; where x is the number of quarters and y is the number of dimes.

   c.



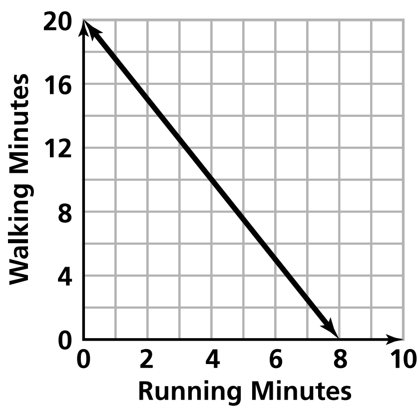
   d. Only positive whole number pairs which you can read from the graph will work. They include (8, 80), (16, 60), (24, 40), (32, 20), etc.

3. a. 160 m; $200(4) + 80(8) = 1{,}440$, so Eric could get $1{,}600 - 1{,}440 = 160$m from the goal.

   b. $200x + 80y = d$

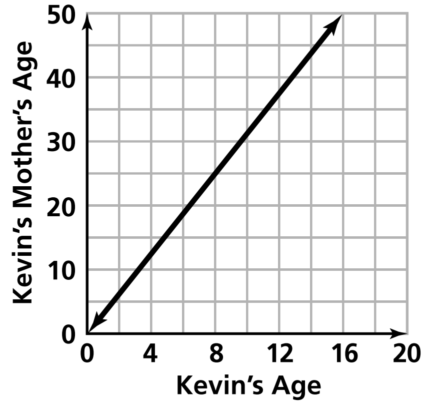
   c. $200x + 80y = 1{,}600$ in case (4, 10), (0, 20), (8, 0), etc.

   d.



4. a. $y - 3x = 1$ and $3x + 1 = y$ are correct.

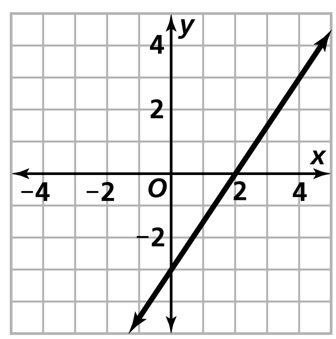
   b. Possible ordered pairs include (0,1), (5, 16), (10, 31), (15, 46), etc.



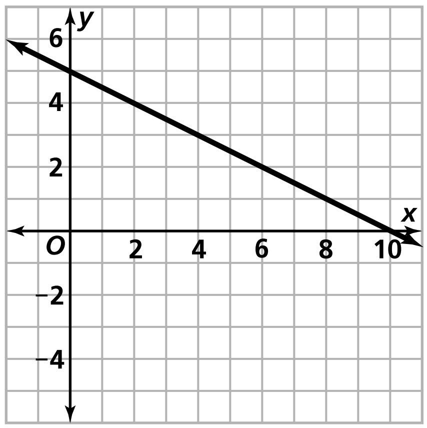
   c. Only positive whole number ordered pairs will work including (2, 7), (4, 13), (6, 19), (8, 25), etc.

   d. Reasonable solutions are (10, 31) and (15, 46).

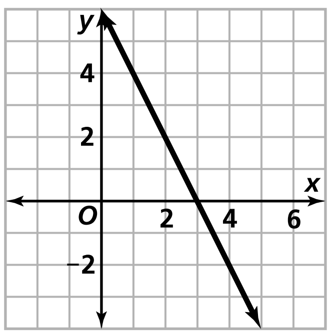
5. $6 = 3x - 2y$ has solutions $(0, -3)$, $(2, 0)$, and $(4, 3)$ among many others, including points such as $(1, -1.5)$, $(3, 1.5)$, $(5, 4.5)$, and $(-1, -4.5)$. The graph is shown below.



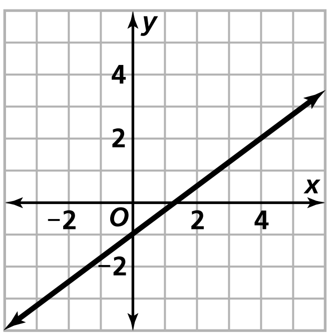
6. $10 = x + 2y$ has solutions (0, 5), (4, 3), (−2, 6), (6, 2), and many others, including points such as (2, 4), (8, 1), (1, 4.5), (3, 3.5), (5, 2.5), etc.



7. $2x + y = 6$ includes solutions (0, 6), (1, 4), (3, 0), (5, −4), and many others, including points such as (2, 2), (4, −2), (1.5, 3), etc.

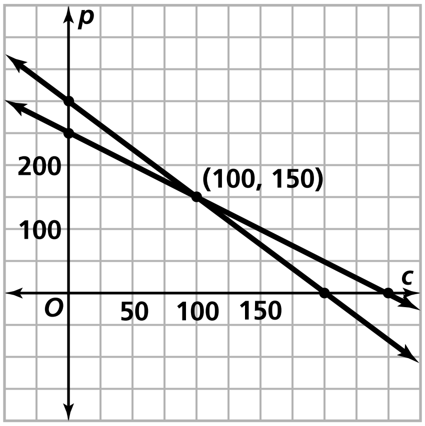


8. $-3x + 4y = -4$ has solutions including (0, −1), $(\frac{4}{3}, 0)$, (4, 2), and many others, such as points (−4, −4), (−2, −2.5), (2, 0.5), (6, 3.5), etc.

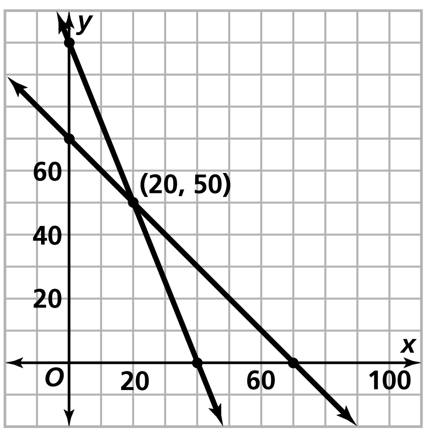


9. Students may verify their claims by comparing slopes and y-intercepts.
   a. $2x + 3y = 9$ is line l
   b. $2x - 3y = 9$ is line n
   c. $x - 3y = 6$ is line m
   d. $3x + 2y = 6$ is line k

10. a. $c + p = 250$
    b. The graphs intersect at (100, 150), meaning that 100 calendars and 150 posters will meet the limit of the donor and also the goal of earning \$600 for the fund-raiser.

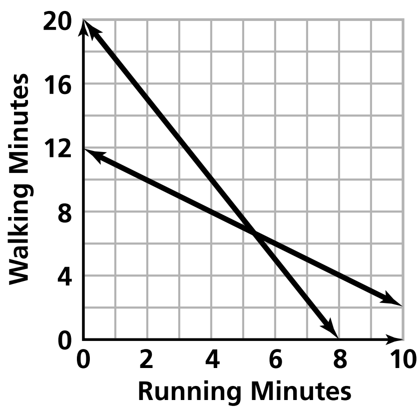


11. a. $x + y = 70$
    b. The graphs intersect at (20, 50), meaning that 20 quarters and 50 dimes will meet the condition of 70 coins and value of 1,000 cents.



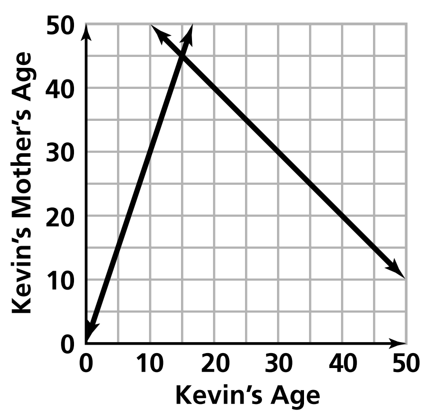
12. a. $x + y = 12$

b. The intersection point is $(5\frac{1}{3}, 6\frac{2}{3})$, though from the graph students might only estimate (5, 7) or perhaps (5.5, 6.5). This means that if one runs for a bit more than 5 minutes and walks for a bit less than 7 minutes at the predicted speeds, the goal of 1,600 meters in 12 minutes should be reached.



13. a. $x + y = 61$

b. The intersection point is (15, 46) meaning that Kevin is 15 and his mother is 46, though from the graph students might only be able to estimate (15, 45).



14. Solutions to systems by graphing will only be approximate in many cases. Always expect students to check their graphic estimates by substitution to see if they are accurate.

a. Solution is (1, 5)

b. Solution is $(-3, -3)$

c. Lines are parallel, meaning that there is no common solution to the equations.

	Equation	x-intercept	y-intercept	Slope
15.	$-4x + y = -2$	$\frac{1}{2}$	-2	4
16.	$3x + y = 5$	$\frac{5}{3}$	5	-3
17.	$-x + y = -7$	7	-7	1
18.	$-5x + y = 3$	$-\frac{3}{5}$	3	5
19.	$8x + y = -12$	$-\frac{3}{2}$	-12	-8
20.	$9x + y = 5$	$\frac{5}{9}$	5	-9
21.	$y = -2x + 5$	$\frac{5}{2}$	5	-2
22.	$y = -2x - 3$	$-\frac{3}{2}$	-3	-2
23.	$y = x - 4$	4	-4	1
24.	$y = (-\frac{3}{4})x + 3$	4	3	$-\frac{3}{4}$
25.	$y = (\frac{7}{2})x - 8$	$\frac{16}{7}$	-8	$\frac{7}{2}$
26.	$y = 0.2x - 11$	55	-11	0.2

27. a. x-intercept is always $(\frac{c}{a}, 0)$ because $ax + b(0) = c$ has that solution.

b. y-intercept is always $(0, \frac{c}{b})$ because $a(0) + by = c$ has that solution.

c. Slope is always $-\frac{a}{b}$ because $ax + by = c$ is equivalent to $by = -ax + c$ and then to $y = (-\frac{a}{b})x + (\frac{c}{b})$.

Connections

28–35. Check students' graphs.

28. $x < 2$ **29.** $x > 8$ **30.** $x \leq -16$ **31.** $x \geq -5$

32. $x \leq -4$ **33.** $x \leq -3$ **34.** $x < 3$ **35.** $x > 3$

36. Lines with slope 4 **37.** Lines with slope 6

38. Lines with slope -1 **39.** Lines with slope $-\frac{1}{4}$

40. Lines with slope $-\frac{3}{4}$ **41.** Lines with slope -7

42. Lines with slope $\frac{1}{4}$ **43.** Lines with slope $\frac{3}{2}$

44. Lines with slope $-\frac{1}{6}$ **45.** Lines with slope $-\frac{1}{2}$

46. Lines with slope -4 **47.** Lines with slope $\frac{3}{2}$

48. a. No: $3(-2) - 5(-4) = 14$

b. Yes **c.** No

d. Yes **e.** Yes

f. Yes

49. a. $z = 180 - (x + y)$ or $z = 180 - x - y$ or $x + y + z = 180$.

b. $40 = 180 - (x + y)$ is satisfied by (x, y) like (120, 20), (100, 40), (40, 100), and so on.

50. A **51.** J

52. a. Slopes of parallel lines will be $-\frac{a}{b}$ where $b \neq 0$.

b. Slopes of perpendicular lines will be $\frac{b}{a}$ where $a \neq 0$.

53. a. Answers will vary. Students may be quite inaccurate if they are counting grid squares to get the area.

b. The labels are rectangles. In each case, one dimension of the rectangle must wrap around the can, so this dimension must match the circumference of the circular base. The other dimension of the rectangular label must match the height of the can. If we switch the ℓ and w, then the labels are still the same size and shape. **Note:** Students investigated this idea in *Filling and Wrapping* when they used a sheet of 8.5" × 11" paper to make two different cylinders, one 8.5" high and the other 11" high.

54. C

55. a. $x < -11$ or $x > 3$

b. $-11 < x < 3$

56. a. The figure will be a right prism with triangular bases.

b. The surface area will be about 28 grid squares. (For exact: $24 + 2\sqrt{3}$ or 27.5 grid squares.)

c. The volume will be the area of the base times the height or about $2 \times 4 = 8$ cubic units. (Exact: $\sqrt{3} \times 4 \approx 6.9$ cubic units)

57. b, c, and d all have slope $-\frac{1}{2}$.

58. Parallel; Their slopes are both $-\frac{1}{2}$. The equations in $y = mx + b$ form are $y = -\frac{1}{2}x + 2$ and $y = -\frac{1}{2}x + 10$.

59. Perpendicular; their slopes are negative reciprocals since the reciprocal of 1 is itself.

60. Neither, they are the same line.

61. Perpendicular; the slope of the equation $y = -3 + 5x$ is 5 and the slope of $y = -\frac{x}{5} + 3$ is $-\frac{1}{5}$. Since 5 and $-\frac{1}{5}$ are negative reciprocals the lines are perpendicular.

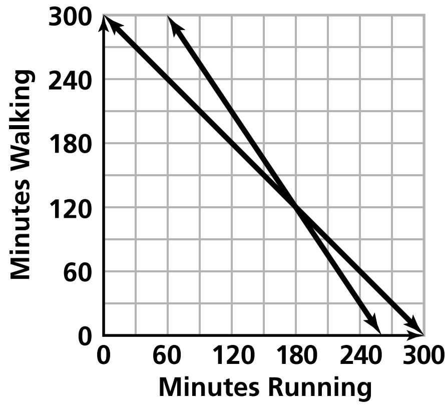
62. Neither; the slope of the line represented by the equation $10x + 5y = 20$ is -2, and the slope for the second line is 10.

Extensions

63. a. $\frac{x}{60} + \frac{y}{60} = 5$ **b.** $\frac{x}{10} + \frac{y}{15} = 26.2$

c. Solutions to the equation in part (a): (0, 300), (300, 0), (150, 150), etc.

Solutions to the equation in part (b): (0, 393), (262, 0), (100, 243), (162, 150).



d. The graph suggests a combination of running and walking times in the vicinity of 180 minutes running and 120 minutes walking. The exact solution is (186, 114), so the graph gives a good estimate. $\frac{186}{10} + \frac{114}{15} = 26.2$

e. Combinations of times less than 5 hours: (100, 199), (199, 100), (100, 150), etc.

f. $x + y < 300$ or $\frac{x}{60} + \frac{y}{60} < 5$

g. The graph of the inequality in (b) will be a region to the left and below the graph of the equation $x + y = 300$.

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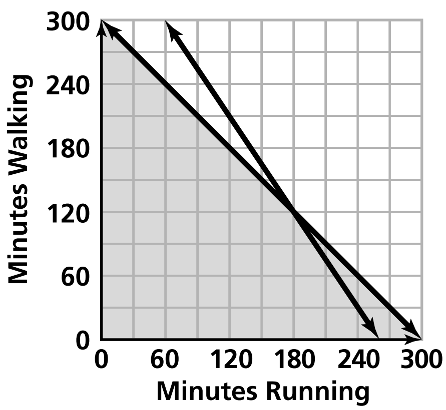
h. When the distance equation is graphed on the region showing times less than 5 hours, it indicates a segment of the graph where coordinates will represent distance of 26.2 miles and time less than or equal to 5 hours. Note that this solution is only the intersection of the line and the shaded region. We have circled the indicated part on the graph.



64. **a.** Midpoints of sides of the triangle are $(6, 3)$, $(15, 3)$, and $(9, 0)$.

b. Equations for the medians are: $y = 0.2x$; $y = 2x - 18$; $y = -(\frac{1}{4})x + 4.5$.

c. The medians intersect at $(10, 2)$.

d. The centroid splits each median into two segments that are in the ratio 2 to 1.

e. Segment lengths for the median from $(0, 0)$ to $(15, 3)$ are about 10.2 and 5.1. Segment lengths for the median from $(12, 6)$ to $(9, 0)$ are about 4.48 and 2.24. Segment lengths for the median from $(18, 0)$ to $(6, 3)$ are about 8.24 and 4.12. These results confirm the answers from part (d).

65. **a**. Midpoints of sides of the triangle are $(3, 3)$, $(-3, 3)$, and $(6, 0)$.
Equations for the medians are: $y = x$; $y = -0.5x + 3$; $y = -0.2x + 2.4$.
The medians intersect at $(2, 2)$.
Segment lengths for the median from $(0, 0)$ to $(3, 3)$ are $2\sqrt{2}$ and $\sqrt{2}$.
Segment lengths for the median from $(-6, 6)$ to $(6, 0)$ are $4\sqrt{5}$ and $2\sqrt{5}$.
Segment lengths for the median from $(12, 0)$ to $(-3, 3)$ are $2\sqrt{26}$ and $\sqrt{26}$.

b. Midpoints of sides of the triangle are $(8, 6)$, $(2, 6)$, and $(6, 0)$.
Equations for the medians are: $y = 0.75x$; $y = -6x + 36$; $y = -0.6x + 7.2$.
The medians intersect at $(\frac{16}{3}, 4)$.
Segment lengths for the median from $(0, 0)$ to $(8, 6)$ are $\frac{20}{3}$ and $\frac{10}{3}$.
Segment lengths for the median from $(12, 0)$ to $(2, 6)$ are $\frac{4\sqrt{34}}{3}$ and $\frac{2\sqrt{34}}{3}$.
Segment lengths for the median from $(4, 12)$ to $(6, 0)$ are $\frac{4\sqrt{10}}{3}$ and $\frac{2\sqrt{10}}{3}$.

Possible Answers to Mathematical Reflections

1. You will graph a straight line.
2. If you graph the lines associated with each equation, they may intersect at a point. The coordinates of the point of intersection will satisfy each equation in the system. You need to rewrite the equations in slope-intercept form if you want to use your calculator to make the graphs.
3. To get from $ax + by = c$ to $y = mx + b$ form, you need to solve for y. You could write: $by = c - ax$, and then $y = \frac{c}{b} - (\frac{a}{b})x$, and then: $y = (-\frac{a}{b})x + \frac{c}{b}$. This equation has the form $y = mx + k$, where $m = -\frac{a}{b}$ and $k = \frac{c}{b}$. To get from $y = mx + b$ to $ax + by = c$ is even easier. You just need to write: $-mx + y = b$.

   Reminder: There is a strong potential for notational confusion because the letter b stands for different quantities in the two equation forms.