Investigation 5

Linear Inequalities

You have studied many relationships that can be modeled by linear equations. The points that satisfy such relationships fall on a straight line. Points that do not satisfy a linear relationship (do not fall on a line) satisfy a *linear inequality*.

Graphing helps make sense of how solutions to inequalities are related to what you know about solutions to linear equations. The situations in this investigation can be modeled by linear inequalities.

5.1 Limiting Driving Miles

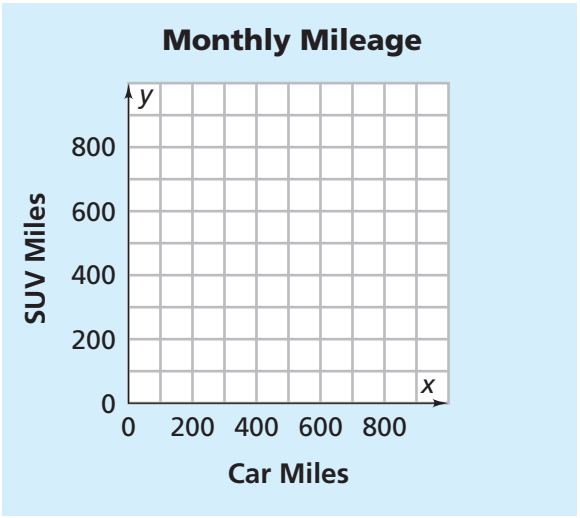
Vince reads that cars are a major source of air pollution. He decides to look at his family's driving habits. They have two vehicles, a car and an SUV. His parents estimate that the family drives about 1,200 miles each month. They decide to try to limit their driving to no more than 1,000 miles each month.

Getting Ready for Problem 5.1

- Find ten possible (*car miles, SUV miles*) pairs that give a total of no more than 1,000 miles.
- One month the family drove the car 500 miles and the SUV 500 miles. Was the total for this month "no more than" 1,000 miles?

Problem 5.1 Graphing "No More Than" Situations

A. On a copy of the grid at the right, plot the ten points you found in the Getting Ready.



B. Look at the pattern of plotted points.

1. Are there other possible (*car miles, SUV miles*) pairs that give a total of no more than 1,000 miles?
2. We refer to a part of a graph or plane as a *region*. Describe where the points are located that represent a total of no more than 1,000 miles.
3. In what region are the points that do not meet this condition located? Give some examples of such points.

C. Suppose Vince's family wants to limit their driving to at most 800 miles per month.

1. Draw a graph of (*car miles, SUV miles*) pairs that meet this condition.
2. Describe the region of the graph that includes all points that represent a total of no more than 800 miles.

D. Write inequalities to model the situations in Questions B and C.

ACE **Homework starts on page 78.**

5.2 Limiting Carbon Dioxide Emissions

Vince finds out that his family's car emits an average of 0.75 pounds of carbon dioxide (CO_2) per mile. The SUV emits an average of 1.25 pounds of CO_2 per mile.

Getting Ready for Problem 5.2

- Suppose Vince's family wants to limit CO_2 emissions from their car to at most 600 pounds per month. How many miles could they drive their car?
- Suppose Vince's family wants to limit CO_2 emissions from their SUV to at most 600 pounds per month. How many miles could they drive their SUV?
- Suppose they want to limit the total CO_2 emissions from *both* vehicles to at most 600 pounds per month. What are some (*car miles, SUV miles*) pairs that allow them to meet this condition?

Problem 5.2 Solving Linear Inequalities by Graphing

A. Suppose Vince's family wants their total CO_2 emissions to be *exactly* 600 pounds per month.

1. Give six examples of (*car miles*, *SUV miles*) that give exactly 600 pounds of CO_2 emissions per month.
2. Write an equation to model this condition.
3. Graph your equation.

B. Suppose the family wants to limit their total CO_2 emissions to *at most* 600 pounds per month.

1. Write an inequality that describes the possibilities for the miles they can drive their car if they do not drive their SUV at all.
2. Write an inequality that describes the possibilities for the miles they can drive their SUV if they do not drive their car at all.
3. Write an inequality that describes the possibilities for how many miles they can drive their car *and* their SUV.
4. Draw a graph displaying (*car miles*, *SUV miles*) pairs that satisfy the inequality you wrote in Question B, part (3).
5. Describe the region of the graph that includes all points that represent a total of no more than 600 pounds of CO_2 emissions.

C. Soo's family has a minivan and a hybrid car. The minivan emits 1.2 pounds of CO_2 per mile. The car emits 0.5 pounds of CO_2 per mile. The family wants to limit their total emissions to at most 500 pounds per month.

1. The family plans to drive both vehicles. Write an inequality to describe the possibilities for how many miles they can drive each vehicle.
2. Draw a graph displaying the (*car miles, minivan miles*) pairs that satisfy the inequality you wrote in Question C, part (1).
3. Describe the region of the graph that includes all points that satisfy the condition.

ACE **Homework starts on page 78.**

5.3 Graphs of Linear Inequalities

In the last problem, you graphed the (*car miles, SUV miles*) pairs that limited CO_2 emissions to *at most* 600 pounds. To make a correct graph, you also had to consider the fact that the numbers of miles cannot be negative. In other words, all the points are in the first quadrant.

Getting Ready for Problem 5.3

- How would the graph of the inequality from Problem 5.2 be different if Vince's family wanted their CO_2 emissions to be *at least* 600 pounds per month?
- How would the graph be different if they wanted to limit their CO_2 emissions to be *less than* 600 pounds per month?
- Is $(-100, 540)$ a possible solution pair if they want to limit their CO_2 emissions to exactly 600 pounds per month? Why or why not?

The inequalities in the next problem are not limited to the first quadrant. As you work on the problem, think about general strategies for graphing inequalities. Notice that shading is used in the graphs to indicate the region in which the points satisfying the inequality lie.

Problem 5.3 Graphs of Linear Inequalities

A. Match each inequality with its graph.

1. $y - 3x \geq 6$

2. $x - 3y \geq 6$

3. $3x + y \leq 6$

4. $x + 3y \leq 6$

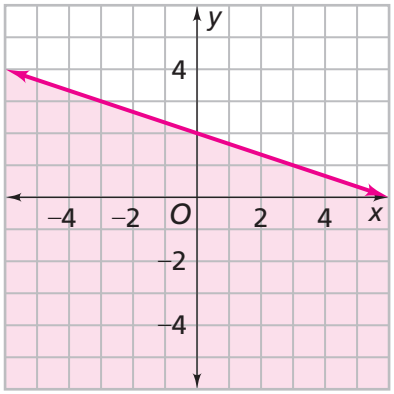
5. $y \geq -3x$

6. $y \leq -3x$

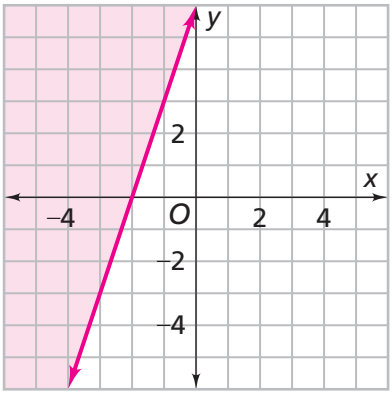
7. $x \geq -3$

8. $y \geq -3$

a.



b.



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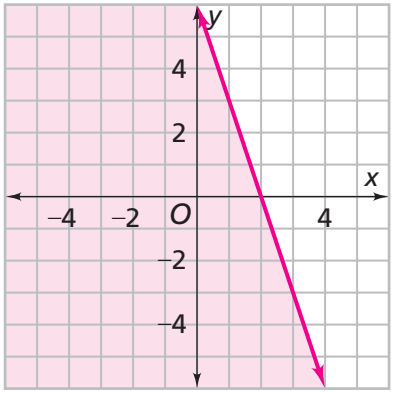

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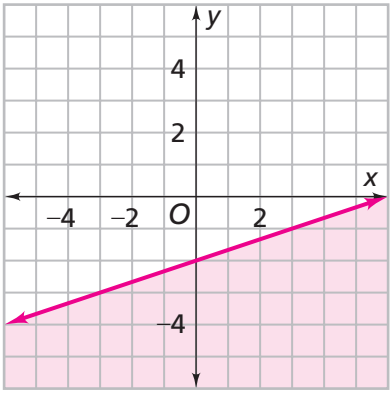

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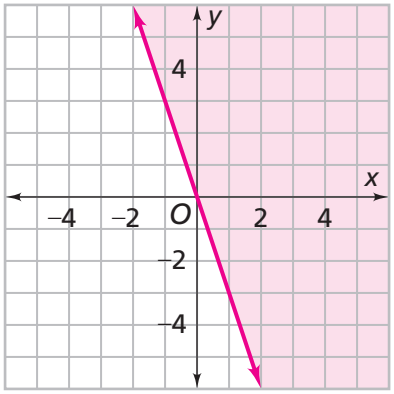

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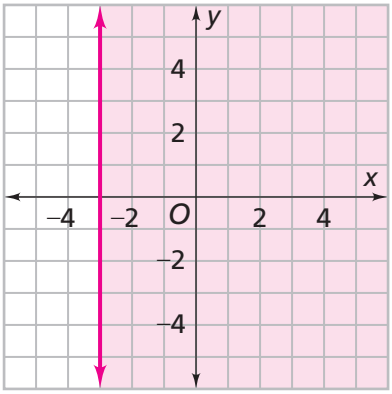

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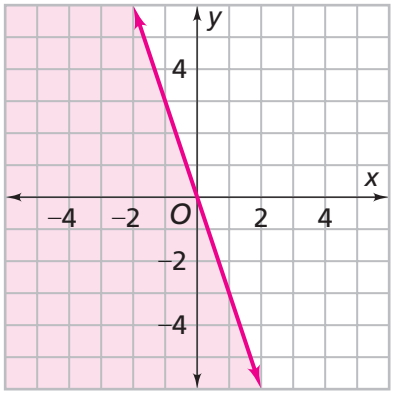
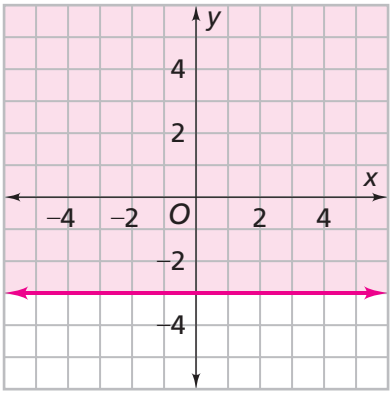


h.



B. Describe your strategies for matching the graphs and inequalities.

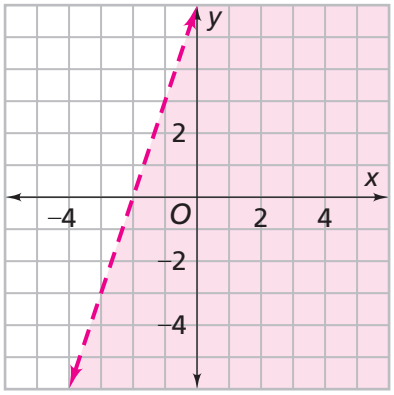
C. **1.** Rewrite the inequalities in parts (1)–(4) of Question A in either $y \leq mx + b$ or $y \geq mx + b$ form.

2. Compare this form of the inequalities with their graphs. How might this form help you determine which regions should be shaded?

D. Think about the inequality $y < 3x + 6$.

1. Does the pair $(2, 12)$ satisfy the inequality? Explain.

2. Below is the graph of $y < 3x + 6$. How is this graph different from the graphs in Question A? What is the reason for this difference?



ACE **Homework starts on page 78.**

5.4 Systems of Linear Inequalities

Vince's family determines that, on average, they drive their SUV more than twice as many miles as they drive their car. Vince writes

$$s > 2c$$

where s represents the number of miles they drive the SUV, and c represents the number of miles they drive the car.

Why does this inequality represent the situation?

The family agrees to limit the total CO_2 emissions to less than 600 pounds per month. Recalling that the car emits 0.75 pounds of CO_2 per mile and the SUV emits 1.25 pounds of CO_2 per mile, Vince writes

$$0.75c + 1.25s < 600$$

Together, the two inequalities form a **system of linear inequalities.**

$$\begin{cases} s > 2c \\ 0.75c + 1.25s < 600 \end{cases}$$

Why does this system of linear inequalities describe the situation?

How would the system change if Vince's family agrees to total emissions that are at most *600 pounds rather than* less than *600 pounds?*

Problem 5.4 Systems of Linear Inequalities

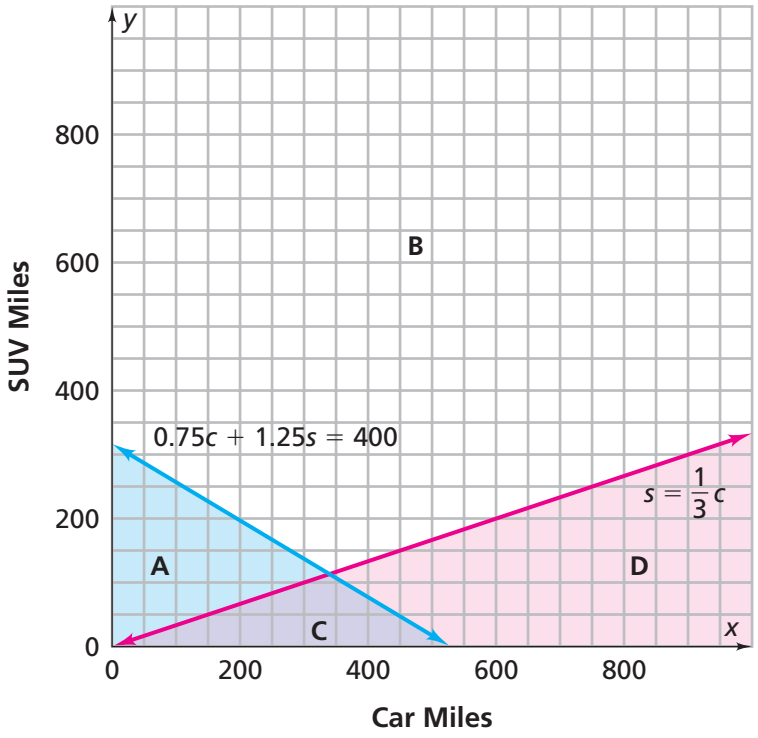
A. 1. Graph the inequality $0.75c + 1.25s < 600$. This graph shows the possible (*car miles, SUV miles*) pairs for which the total CO_2 emissions are less than 600 pounds per month.

2. On the same axis, graph the inequality $s > 2c$. This graph shows the possible (*car miles, SUV miles*) pairs for which the number of SUV miles is more than twice the number of car miles.

3. Where on the grid are the points that satisfy both conditions?

B. Nancy has a car and an SUV with the same emissions as Vince's family's vehicles. She will drive her car at least three times as much as her SUV. She wants to limit the total CO_2 emissions to at most 400 pounds per month. She draws the following graph.

Limiting CO_2 Emissions



1. Describe what information the points in each region represent in terms of the situation.

2. In which region(s) are the points that satisfy both conditions?

ACE **Homework starts on page 78.**