Investigation 3

Equations With Two or More Variables

You have done a lot of work with relationships involving two related variables. However, many real-world relationships involve three or more variables. For example, consider this situation:

The eighth-graders are selling T-shirts and caps to raise money for their end-of-year party. They earn a profit of $5 per shirt and $10 per cap.

This situation involves three variables: the *number of T-shirts sold*, the *number of caps sold,* and the *profit*. The profit for the fundraiser depends on the number of caps and the number of T-shirts sold.

Getting Ready for Problem 3.1

- What equation shows how the profit p is related to the number of shirts sold s and the number of caps sold c?
- Find the profit if the students sell

  30 shirts and 50 caps
  15 shirts and 10 caps
  12 shirts and 20 caps
- What do you think it means to *solve* an equation with three variables?
- What ideas do you have for finding solutions to the equation?

3.1 Many Ways to Reach a Goal

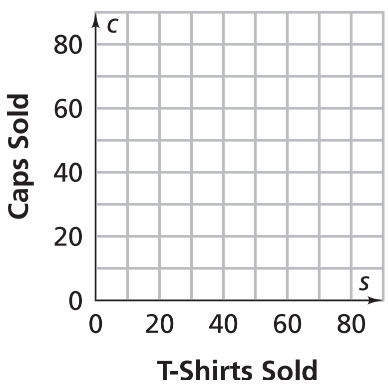
The equation relating *p, s,* and *c* represents every possible combination of T-shirts, caps, and profit values for the fundraiser. Suppose the class sets a profit goal of $P = \$600$. Finding combinations of T-shirt and cap sales that meet this goal requires solving an equation with only two variables, *s* and *c*.

Problem 3.1 Solving Equations With Two Variables

A. Find five pairs of numbers for shirt and cap sales that will allow the students to make a \$600 profit exactly.

B. **1.** Each answer for Question A can be expressed as an ordered pair (s, c). Plot these ordered pairs on a grid like the one below.

Fundraiser Sales



2. Is there a pattern in the points that suggests other solutions of the equation $600 = 5s + 10c$? Explain.

C. The equations in parts (1)–(4) are of the form $c = ax + by$ or $ax + by = c$. For each equation,

- find at least five solution pairs (x, y)
- plot the solutions
- find a pattern in the points and use the pattern to predict other solution pairs

1. $5 = x - y$

2. $10 = x + y$

3. $2x + y = 3$

4. $-3x + 2y = -4$

D. What does your work on Question C suggest about the graph of solutions for any equation of the form $ax + by = c$ or $c = ax + by$, where *a*, *b*, and *c* are fixed numbers?

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3.2 Connecting $y = mx + b$ and $ax + by = c$

There are two common forms of a linear equation.

- When the values of one variable depend on those of another, it is most natural to express the relationship as $y = mx + b$. Most of the linear equations you have seen have been in this *slope-intercept* form.
- When it is more natural to combine the values of two variables, the relationship can be expressed as $ax + by = c$. This is the *standard form* of a linear equation. The equations in Problem 3.1 were in standard form.

Getting Ready for Problem 3.2

It is easy to graph a linear equation of the form $y = mx + b$ on a calculator.

- Can you use a calculator to graph an equation of the form $ax + by = c$?
- Can you change an equation from $ax + by = c$ form to $y = mx + b$ form?
- How can rewriting the equation $600 = 5s + 10c$ (or $600 = 5x + 10y$) from Problem 3.1 in $y = mx + b$ form help you find solutions?

Problem 3.2 Connecting $y = mx + b$ and $ax + by = c$

A. Four students want to write $12x + 3y = 9$ in equivalent $y = mx + b$ form. Here are their explanations:

Jared

$12x + 3y = 9$
$3y = -12x + 9$ (1)
$y = -4x + 3$ (2)

Molly

$12x + 3y = 9$
$3y = 9 - 12x$ (1)
$y = 3 - 12x$ (2)

Ali

$12x + 3y = 9$
$4x + y = 3$ (1)
$y = -4x + 3$ (2)

Mia

$12x + 3y = 9$
$3y = 9 - 12x$ (1)
$y = 3 - 4x$ (2)
$y = 4x - 3$ (3)

1. Did each student get an equation equivalent to the original? If so, explain the reasoning for each step. If not, tell what errors the student made.
2. What does it mean for two equations to be equivalent?

B. Write each equation in $y = mx + b$ form.

1. $x - y = 4$
2. $2x + y = 9$
3. $8x + 4y = -12$
4. $12 = 3x - 6y$
5. $x + y = 2.5$
6. $600 = 5x + 10y$

C. Suppose you are given an equation in $ax + by = c$ form. How can you predict the slope, y-intercept, and x-intercept of its graph?

D. Write each equation in $ax + by = c$ form.

1. $y = 5 - 3x$
2. $y = \frac{2}{3}x + \frac{1}{4}$
3. $x = 2y - 3$
4. $2x = y + \frac{1}{2}$
5. $y - 2 = \frac{1}{4}x + 1$
6. $3y + 3 = 6x - 15$

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3.3 Intersections of Lines

At a school band concert, Christopher and Celine sell memberships for the band's booster club. An adult membership costs \$10, and a student membership costs \$5.

At the end of the evening, the students had sold 50 memberships for a total of \$400. The club president wants to know how many of the new members are adults and how many are students.

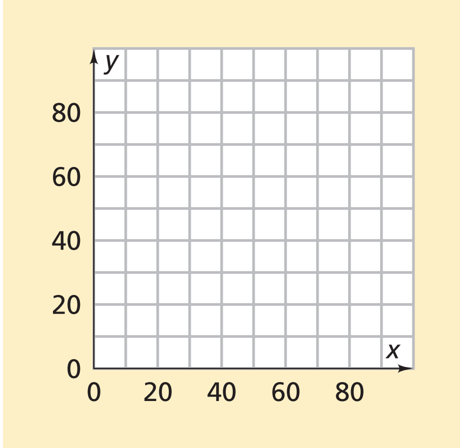
Problem 3.3 Intersections of Lines

A. Let x stand for the number of \$10 adult memberships and y for the number of \$5 student memberships.

1. What equation relates x and y to the \$400 income?
2. Give two solutions for your equation from part (1).
3. What equation relates x and y to the total of 50 new members? Are the solutions you found in part (2) also solutions of this equation?

B. 1. Graph the two equations from Question A on a single coordinate grid like the one at the right.

2. Estimate the coordinates of the point where the graphs intersect. Explain what the coordinates tell you about the numbers of adult and student memberships sold.
3. Consider the graph of the equation that relates x and y to the \$400 income. Could a point that is *not* on this graph be a solution to the equation?
4. Could there be a common solution for both of your equations that is *not* shown on your graph?



In Question A, you wrote a system of equations. One equation represents all (x, y) pairs that give a total income of \$400, and the other represents all (x, y) pairs that give a total of 50 memberships. The coordinates of the intersection point satisfy both equations, or conditions. These coordinates are the *solution to the system*.

Many real-life problems can be represented by systems of equations. In Question C, you'll practice solving such systems graphically.

C. Use graphic methods to solve each system. In each case, substitute the solution values into the equations to see if your solution is exact or an estimate.

1. $x + y = 4$ and $x - y = -2$
2. $2x + y = -1$ and $x - 2y = 7$
3. $2x + y = 3$ and $-x + 2y = 6$

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