Investigation 2

Linear Equations and Inequalities

Connecting geometry and algebra can help you solve problems. In the last Investigation, you used algebra to describe and reason about geometric shapes in the coordinate plane. Now, you will use coordinate geometry to help you think about algebraic equations and inequalities.

Suppose the managers of a shopping center want to upgrade their security system. Two providers bid for the job.

- Super Locks will charge $3,975 to install the equipment and then $6.00 per day to monitor the system and respond to alerts.
- Fail Safe will charge $995 to install the equipment and then $17.95 per day to monitor the system and respond to alerts.

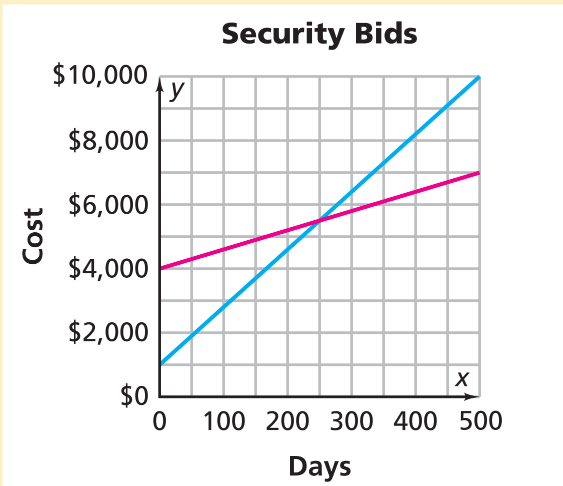
Both companies are reliable and capable, so the choice comes down to cost.

Getting Ready for Problem 2.1

- What kinds of equations will show how the costs for the two companies are a function of the number of days?
- What patterns do you expect to see in graphs of the equations?
- How can you use a graph to answer questions about which company offers the best price?

2.1 Graphs of Linear Systems

The cost of the security services from Super Locks and Fail Safe depends on the number of days the company provides service. The graph below shows the bids for both companies.



Problem 2.1 Graphs of Linear Systems

A. Use the graphs to estimate the answers to these questions. Explain your reasoning in each case.

1. For what number of days will the costs for the two companies be the same? What is that cost?
2. For what numbers of days will Super Locks cost less than Fail Safe?
3. For what numbers of days will Super Locks cost less than $6,000?
4. What is the cost of one year of service from Fail Safe?
5. How can Fail Safe adjust its per-day charge to make its cost for 500 days of service cheaper than Super Locks' cost?

B. For each company, write an equation for the cost c for d days of security services.

C. For parts (1) and (4) of Question A, write an equation you can solve to answer the question. Then use symbolic methods to find the exact answers.

ACE **Homework starts on page 30.**

2.2 Linear Inequalities

In Problem 2.1, you used graphic and symbolic methods to analyze a **system of linear equations.** The problem conditions could be expressed as two equations relating security costs and the number of days for the business contract. The coordinates of the intersection point of the graphs satisfied both equations in the system. This point is the *solution* of the system.

Getting Ready for Problem 2.2

The cost equations for the two security companies are a system of linear equations:

$$c = 3{,}975 + 6d \quad \text{(Super Locks)}$$
$$\text{and} \quad c = 995 + 17.95d \quad \text{(Fail Safe)}$$

In previous units, you learned some methods to solve this linear system to find the number of days for which the costs are the same for both companies. Here is one possible solution method:

$$3{,}975 + 6d = 995 + 17.95d \qquad (1)$$
$$2{,}980 = 11.95d \qquad (2)$$
$$249 \approx d \qquad (3)$$

- Give a reason for each step in the solution.
- What is the overall strategy that guides the solution process?
- What does the statement $d \approx 249$ tell you?
- How can the solution to this system help you answer this question: For what numbers of days will Super Locks cost less than Fail Safe?
- What does your answer to the previous question tell you about solutions to the inequality $3{,}975 + 6d < 995 + 17.95d$?

It is fairly easy to find some solutions to an inequality. However, sometimes it is useful to find all the solutions by solving the inequality symbolically. The following problems will help you develop strategies for solving inequalities.

Problem 2.2 Linear Inequalities

A. For each instruction in parts (1)–(6), start with $q < r$. Tell whether performing the operation on $q < r$ will give an inequality that is still true. If so, explain why. If not, give specific examples to show why the resulting inequality is false.

1. Add 23 to both sides.
2. Subtract 35 from both sides.
3. Multiply both sides by 14.
4. Multiply both sides by -6.
5. Divide both sides by 5.
6. Divide both sides by -3.

B. What do your results from Question A suggest about how working with inequalities is similar to and different from working with equations?

C. Solve these equations and inequalities.

1. $3x + 12 = 5x - 4$
2. $3w + 12 < 5w - 4$
3. $q - 5 = 6q + 10$
4. $r - 5 > 6r + 10$

ACE **Homework starts on page 30.**

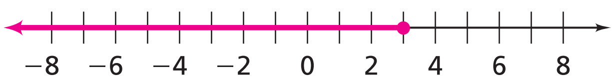
2.3 Solving Linear Inequalities

Many practical problems require solving linear inequalities. You can reason about inequalities, such as $2x - 4 < 5$ or $2x - 4 > -0.5x + 1$, using both symbolic and graphic methods. Solutions to inequalities with one variable are generally given in the form $x < a$, $x > a$, $x \le a$, or $x \ge a$.

Getting Ready for Problem 2.3

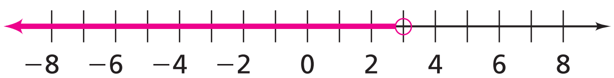
- What are some values that satisfy the inequality $3x + 4 \le 13$?
- Describe all the solutions of the inequality $3x + 4 \le 13$.

All the solutions of $3x + 4 \le 13$ can be displayed in a number-line graph. This graph represents $x \le 3$, all x-values less than or equal to 3.



- Explain why the solutions of $3x + 4 < 13$ do *not* include the value 3.

The number-line graph below represents the solutions of $3x + 4 < 13$. It shows $x < 3$, all x-values strictly less than 3. The open circle shows that 3 is not a solution.



- Make a number-line graph showing the solutions of $2x - 4 < 5$.
- Explain in words what the graph tells about the solutions.

Problem 2.3 Solving Linear Inequalities

A. Use symbolic reasoning to solve each inequality. Then make a number-line graph of the solutions. Be prepared to justify your solution steps and to explain your graphs.

1. $3x + 17 < 47$

2. $43 < 8x - 9$

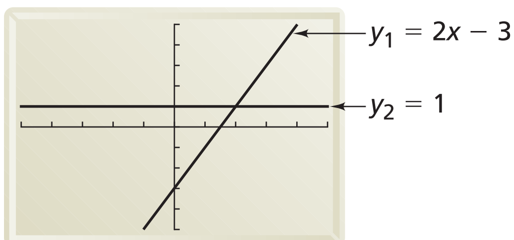
3. $-6x + 9 < 25$

4. $14x - 23 < 5x + 13$

5. $18 < -4x + 2$

6. $3{,}975 + 6d < 995 + 17.95d$

B. Luisa wants to use her graphing calculator to solve $2x - 3 \leq 1$. She graphs the linear functions $y = 2x - 3$ and $y = 1$. She uses an x- and a y-scale of 1.



1. Luisa knows that the solution for $2x - 3 = 1$ is $x = 2$. How does this relate to the graphs of the lines she drew?

2. How do the graphs show that the solution of $2x - 3 \leq 1$ is $x \leq 2$?

3. How can you use the graph to find the solution of $2x - 3 > 1$? What is the solution?

4. For one of the inequalities in Question A, sketch a graph or use your graphing calculator to find the solution. Check that your solution agrees with the one you found by using symbolic reasoning.

ACE **Homework starts on page 30.**