Investigation 1

Equations for Circles and Polygons

The photo below shows a "crop circle." Not all crop circles are made in crop fields, nor are they all circles. However, the term "crop circles" is often used to describe all such designs. Designs like these have appeared in fields around the world. At first, the origins of the crop circles were unknown. However, in many cases, the people who made them have come forward and taken credit for their work.

Getting Ready for Problem 1.1

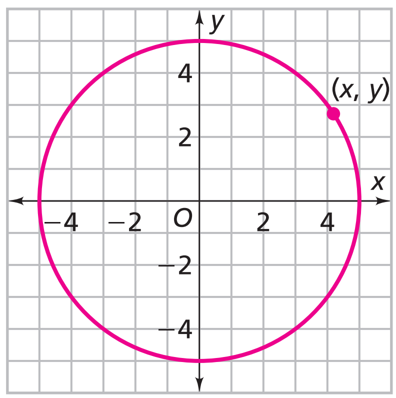
Suppose you are planning to make a crop circle design like the one above.

- How can you outline the circle accurately?
- How can you locate sides and vertices of the other shapes in the design?
- How can you use equations and coordinate graphs to help plan your design?

1.1 Equations for Circles

You can outline the outer circle of a crop circle by using a rope. Anchor one end of the rope where you want the center of the circle. Hold the other end and, with the rope pulled taut, walk around the center point.

To plan the other parts of the design, it helps to draw the circle on a coordinate grid. In this problem, you will find an equation relating the coordinates of the points on a circle.



On the circle above, are there points for which it is easy to find the coordinates?

What mathematical ideas can help you find coordinates of other points on the circle?

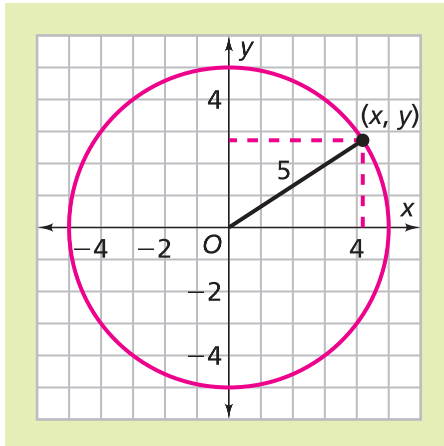
Problem 1.1 Equations for Circles

A. **1.** The circle above has a radius of 5 units and is centered at the origin. Estimate the missing coordinate for these points on the circle. If there is more than one possible point, give the missing coordinate for each possibility.

a. (0, ■)	**b.** (■, 0)	**c.** (3, ■)
d. (4, ■)	**e.** (■, −3)	**f.** (■, 4)
g. (−2, ■)	**h.** (■, 2)	**i.** (■, 5)

2. Which of your coordinates from part (1) do you think are exactly correct? How do you know?

B. Think about a point (x, y) starting at $(5, 0)$ and moving counterclockwise, tracing around the circle.



1. How does the y-coordinate of the point change as the x-coordinate approaches zero and then becomes negative?
2. The radius from the origin $(0, 0)$ to the point (x, y) has a length of 5 units. The diagram shows that you can make two right triangles with the radius as the hypotenuse. How do these triangles change as the point moves around the circle?
3. Use what you know about the relationship among the side lengths of a right triangle to write an equation relating x and y to the radius, 5.
4. Kaitlyn says that the relationship is $x + y = 5$ or $y = 5 - x$. Is she correct? Explain.
5. Does every point on the circle satisfy your equation? Explain.

C. These points are all on the circle. Check that they satisfy the equation you wrote in Question B part (3).

1. $(3, 4)$ **2.** $(-4, 3)$ **3.** $(\sqrt{13}, \sqrt{12})$ **4.** $(0, -5)$

5. Does any point *not* on the circle satisfy the equation? Explain.

D. **1.** Give the coordinates of three points in the interior of the circle. What can you say about the x- and y-coordinates of points inside the circle?

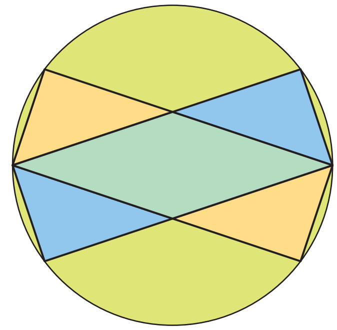
2. Use your equation from Question B to help you write an *inequality* that describes the points in the interior of the circle.

E. How can you change your equation from Question B to represent a circle with a radius of 1, 3, or 10 units?

ACE **Homework starts on page 12.**

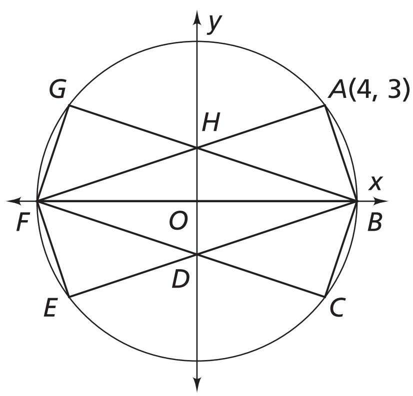
1.2 Parallels and Perpendiculars

The design at the right is made from a circle and two overlapping rectangles. One way to make a crop circle with this design is to place stakes at key points and connect the stakes with string outlining the regions. However, you first need to find the location of these points. You can use what you know about coordinate geometry to analyze the design's key points and features.



Problem 1.2 Parallels and Perpendiculars

This diagram shows some of the key points in the design. The design has reflection symmetry in both the x-axis and the y-axis. The radius is 5 units.



A. Find the coordinates of points B, C, E, F, and G.

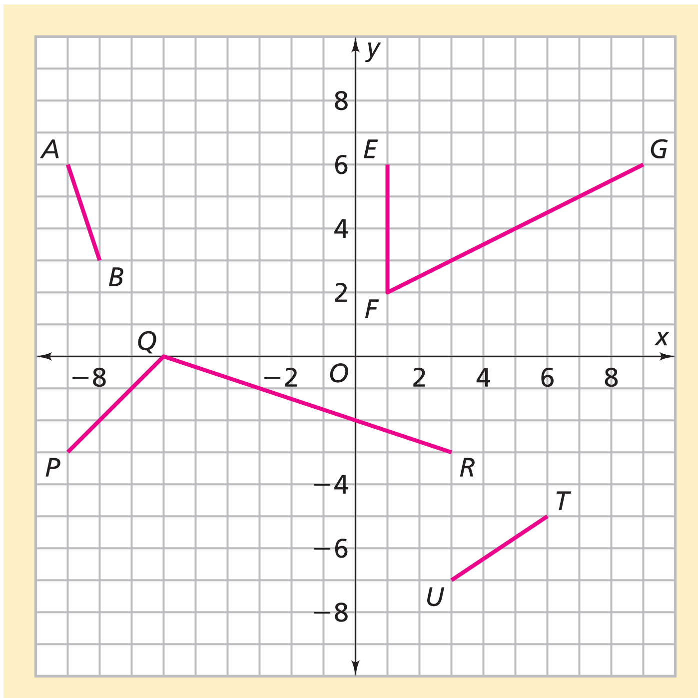
B. List all pairs of parallel lines. How do the slopes of the lines in each pair compare? Explain why this makes sense.

C. List all pairs of perpendicular lines. How do the slopes of the lines in each pair compare? Explain why this makes sense.

D. Locate a new point $K(2, y)$ on the circle. Draw a line segment from point K to the point $(5, 0)$. Can you draw a rectangle with this segment as one side and all its vertices on the circle? If so, give the coordinates of the vertices.

E. 1. Kara was sketching on grid paper to try out some design ideas. She got interrupted! On a copy of Kara's diagram below, complete the polygons specified. (There may be more than one way to draw each one.) The polygons should all fit on the grid and should not overlap.

- Rectangle *ABCD*
- Parallelogram *EFGH*
- Parallelogram *PQRS*
- Rectangle *TUVW*



2. Give the coordinates of the vertex points for each figure.

3. Compare the slopes for all pairs of parallel sides. Describe the patterns you see. Are the patterns the same as you found in Question B?

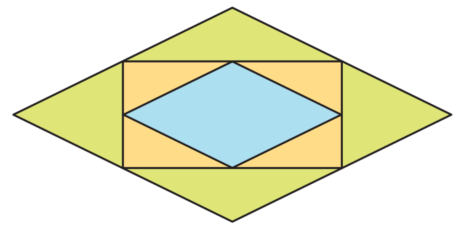
4. Compare the slopes for all pairs of perpendicular sides. Describe the patterns you see. Are the patterns the same as you found in Question C?

5. What is true about the equations for a pair of parallel lines? What is true about the equations for a pair of perpendicular lines?

ACE **Homework starts on page 12.**

1.3 Finding Midpoints

Dalton's class wants to design some interesting crop circles that are not circles. He starts with a diamond design.



To draw this diamond design, you start with the outer rhombus. You connect the midpoints of its sides to form a rectangle, and then connect the midpoints of the rectangle's sides.

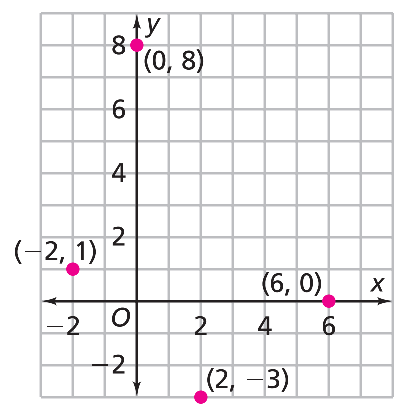
What would you need to check to know that the yellow shape is a rectangle?

How could you create this pattern by measuring and drawing or by folding paper and tracing creases?

Getting Ready for Problem 1.3

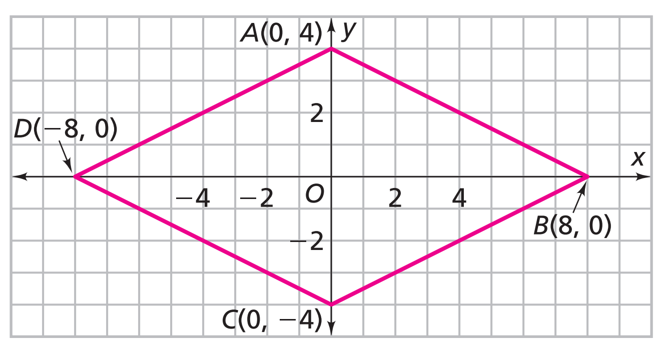
To make symmetric designs on a coordinate grid, it is helpful to have strategies for finding the coordinates of midpoints of line segments.

- How can you find the length of the line segment from (0, 8) to (6, 0) and from (−2, 1) to (2, −3)?
- How can you estimate the coordinates of the midpoint of each segment?



Problem 1.3 Finding Midpoints

A. The figure at the right is a rhombus.

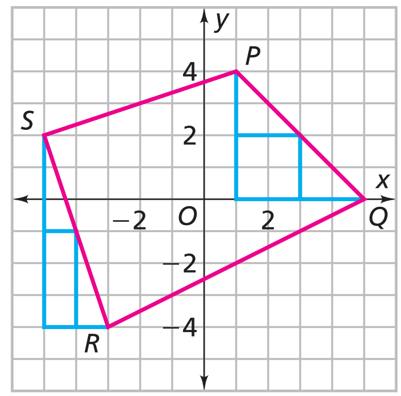


1. Estimate coordinates of P, the midpoint of side AB. Estimate the coordinates of S, the midpoint of side AD.

2. Explain why $\triangle APS$ is similar to $\triangle ABD$. What is the scale factor? How can you use these facts to check the coordinates of P and S? How can you use these facts to confirm that P and S are the midpoints?

3. Find the midpoints of sides BC and CD.
4. Check the midpoint coordinates by calculating the distance from each midpoint to the endpoints of the segments on which it is located. (Hint: Use symmetry to limit the calculations you do.)

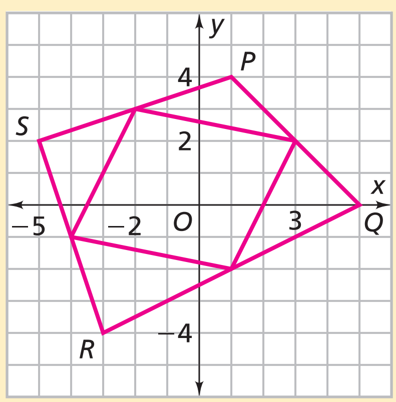
B. Dalton tries a quadrilateral that is not symmetric.



1. Dalton draws some lines on the quadrilateral to help him locate the midpoints of PQ and SR. Does this seem like a plan that would work no matter where P and Q or S and R are located? Explain.
2. Find coordinates of the midpoint of each side.
3. **a.** For each side, compare the coordinates of the endpoints to the coordinates of the midpoint. See if you can find a strategy for finding the coordinates of the midpoint of any line segment.

   b. Use your findings to complete this statement:
   The midpoint of the segment with endpoints (a, b) and (c, d) has coordinates . . .

C. Dalton connected the midpoints of the sides of $PQRS$ to form a quadrilateral.



1. The quadrilateral appears to be a parallelogram. Verify this by finding the slopes of its sides.
2. Draw several quadrilaterals of your own. For each quadrilateral, find the midpoints of the sides (by measuring or paper folding), and connect those midpoints in order.
3. Describe the pattern in your results by completing this sentence: *When the midpoints of the sides of a quadrilateral are connected in order, the resulting figure is . . .*

ACE **Homework starts on page 12.**