# **A**pplications

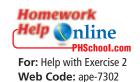
## Connections

**42** The Shapes of Algebra

### **Applications**

- **1.** For a fundraiser, students sell calendars for \$3 each and posters for \$2 each.
  - **a.** What equation shows how the income *I* for the fundraiser depends on the number calendars *c* and posters *p* that are sold?
  - **b.** What is the income if students sell 25 calendars and 18 posters?
  - c. What is the income if students sell 12 calendars and 15 posters?
  - d. What is the income if students sell 20 calendars and 12 posters?
  - **e.** Find three combinations of calendar sales and poster sales that will give an income of exactly \$100.
  - **f.** Each answer in part (e) can be written as an ordered pair (c, p). Plot the ordered pairs on a coordinate grid.
  - **g.** Use your graph to estimate three other (c, p) pairs that would meet the \$100 goal.
- **2.** Neema saves her quarters and dimes. She plans to exchange the coins for paper money when the total value equals \$10.
  - **a.** How many coins does she need to make \$10 if all the coins are quarters? If all the coins are dimes?
  - **b.** What equation relates the number of quarters *x* and dimes *y* to the goal of \$10?
  - **c.** Use the answers from part (a) to help you draw a graph showing all solutions to the equation.
  - **d.** Use the graph to find five combinations of dimes and quarters that will allow Neema to reach her goal.







Extensions

- **3.** Students in Eric's gym class must cover a distance of 1,600 meters by running or walking. Most students run part of the way and walk part of the way. Eric can run at an average speed of 200 meters per minute and walk at an average speed of 80 meters per minute.
  - **a.** Suppose Eric runs for 4 minutes and walks for 8 minutes. How close is he to the 1,600-meter goal?
  - **b.** Write an equation for the distance *d* Eric will cover if he runs for *x* minutes and walks for *y* minutes.
  - **c.** Find three combinations of running and walking times for which Eric would cover 1,600 meters.
  - **d.** Plot the ordered pairs for part (c) on a graph. Use the graph to estimate several other combinations of running and walking times for which Eric would cover 1,600 meters.
- **4.** Kevin said that if you triple his age, the result will be 1 less than his mother's age.
  - **a.** Which, if any, of these equations shows the relationship between Kevin's age *x* and his mother's age *y*? Choose all that are correct.

3x - y = 1 y - 3x = 1 3x + 1 = y 3x = 1 - y

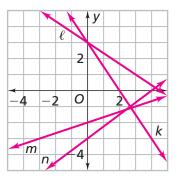
- **b.** Find three pairs of (x, y) values that satisfy the equation relating Kevin's age and his mother's age. Plot these ordered pairs, and draw the line that matches the pattern.
- **c.** Use the graph to estimate three other ordered pairs that satisfy the equation. Use the equation to check the estimates.
- **d.** Which (*x*, *y*) pairs seem to be reasonable for Kevin's age and his mother's age?

Find three pairs of (x, y) values that satisfy each equation. Plot those points and use the pattern to find two more solution pairs.

- **5.** 6 = 3x 2y (**Hint:** What is *y* if x = 0? What is *x* if y = 0?)
- **6.** 10 = x + 2y
- **7.** 2x + y = 6
- **8.** -3x + 4y = -4

**9.** Tell which line at the right is the graph of each equation in parts (a)–(d). Explain.

<b>a.</b> $2x + 3y = 9$	<b>b.</b> $2x - 3y = 9$
<b>c.</b> $x - 3y = 6$	<b>d.</b> $3x + 2y = 6$



- **10.** In Exercise 1, one equation relating the calendar and poster sales to the \$600 goal is 3c + 2p = 600. Suppose the company donating the calendars and posters said they would provide a total of 250 items.
  - **a.** What equation relates *c* and *p* to the 250 items donated?
  - **b.** Graph both equations on the same grid. Find the coordinates of the intersection point. Explain what these coordinates tell you about the fundraising situation.
- **11.** In Exercise 2, one equation relating Neema's quarters and dimes to her goal of \$10 (1,000 cents) is 25x + 10y = 1,000. Suppose Neema collects 70 coins to reach her goal.
  - **a.** What equation relates *x* and *y* to the number of coins Neema collected?
  - **b.** Graph both equations on the same grid. Find the coordinates of the intersection point. Explain what these coordinates tell you about this situation.
- **12.** In Exercise 3, one equation relating the times Eric spends running and walking to the goal of covering 1,600 meters is 200x + 80y = 1,600. Suppose Eric runs and walks for a total of 12 minutes to reach his goal.
  - **a.** What equation relates *x* and *y* to Eric's total time?
  - **b.** Graph both equations on the same grid. Find the coordinates of the intersection point. Explain what these coordinates tell you about this situation.



- **13.** In Exercise 4, one equation relating the ages of Kevin and his mother is y 3x = 1. The sum of Kevin's age and his mother's age is 61 years.
  - **a.** What equation relates Kevin's and his mother's ages to their total age?
  - **b.** Graph both equations on the same grid. Find the coordinates of the intersection point. Explain what these coordinates tell you about the ages of Kevin and his mother.
- 14. Use graphing methods to solve each system of equations. [Hint: If you are using a graphing calculator, you can determine a good graphing window by first finding the x- and y-intercepts of each graph. For instance, find the x-intercept of 3x + 4y = 8 by substituting 0 for y, and then find the y-intercept by substituting 0 for x.

If 
$$y = 0$$
, then  $3x + 4(0) = 8$ , so  $x = \frac{8}{3}$ . The *x*-intercept is  $(\frac{8}{3}, 0)$ .  
If  $x = 0$ , then  $3(0) + 4y = 8$ , so  $y = 2$ . The *y*-intercept is  $(0, 2)$ .]

- **a.** x y = -4 and x + y = 6
- **b.** -2x + y = 3 and x + 2y = -9
- **c.** -2x + y = 1 and 4x 2y = 6

Write the equation in ax + by = c form. Identify the *x*-intercept, *y*-intercept, and slope.

<b>15.</b> $y = 4x - 2$	<b>16.</b> $y = -3x + 5$	<b>17.</b> $y = x - 7$
<b>18.</b> $y = 5x + 3$	<b>19.</b> $y = -8x - 12$	<b>20.</b> $y = -9x + 5$

For Exercises 21–26, write the equation in y = mx + b form. Identify the *x*-intercept, *y*-intercept, and slope.

<b>21.</b> $-2x - y = -5$	<b>22.</b> $6x + 3y = -9$	<b>23.</b> $x - y = 4$
<b>24.</b> $3x + 4y = 12$	<b>25.</b> $-7x + 2y = -16$	<b>26.</b> $x - 5y = 55$

- **27.** Look back over your work for Exercises 15–26. Look for patterns relating the standard form of the equation, ax + by = c, to the *x*-intercept, *y*-intercept, and slope.
  - **a.** Write a general formula for calculating the *x*-intercept from the values of *a*, *b*, and *c*.
  - **b.** Write a general formula for calculating the *y*-intercept from the values of *a*, *b*, and *c*.
  - **c.** Write a general formula for calculating the slope from the values of *a*, *b*, and *c*.

#### Connections

Solve the inequality and graph the solution on a number line.

<b>28.</b> <i>x</i> + 3 < 5	<b>29.</b> $x - 12 > -4$
<b>30.</b> $14 + x \le -2$	<b>31.</b> $2x + 7 \ge -3$
<b>32.</b> $7x + 3 \le -17 + 2x$	<b>33.</b> $-3 - 4x \ge 5x + 24$
<b>34.</b> $2x - 4 + 7x < -6x + 41$	<b>35.</b> $12x - 3 + 5 - 4x > 24 - 2x + 8$

Write an equation of a line parallel to the given line.

<b>36.</b> $y = 4x + 6$	<b>37.</b> $-6x + y = 3$	<b>38.</b> $x + y = 9$
<b>39.</b> $x + 4y = -20$	<b>40.</b> $y = -\frac{3}{4}x - 2$	<b>41.</b> $7x + y = -12$

For Exercises 42–47, write an equation of a line perpendicular to the given line.

<b>42.</b> $y = -4x + 2$	<b>43.</b> $y = -\frac{2}{3}x - 7$	<b>44.</b> $y = 6x + 12$
<b>45.</b> $-2x + y = -1$	<b>46.</b> $x - 4y = 20$	<b>47.</b> $2x + 3y = 8$

**48.** Tell whether each ordered pair is a solution of 3x - 5y = 15. Show how you know.

<b>a.</b> (-2, -4)	<b>b.</b> (0, -3)	<b>c.</b> (-10,9)
<b>d.</b> (-5, -6)	<b>e.</b> (-10, -9)	<b>f.</b> (-4, -5.4)

-Go 🕔

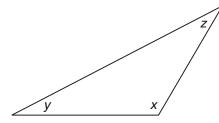
nline

For: Multiple-Choice Skills Practice Web Code: apa-7354

**PHSchool.com** 

**49.** The angle measures of the triangle below are x, y, and z.

- **a.** What equation shows how *z* depends on *x* and *y*?
- **b.** Find five combinations of values for *x* and *y* for which the value of *z* is 40.





**50.** Multiple Choice Suppose k, m, and n are numbers and k = m + n. Which of the following statements must be true?

<b>A.</b> $k - m = n$	<b>B.</b> $m - k = n$
<b>C.</b> $2k = 2m + n$	<b>D.</b> $-n = k + m$

**51.** Multiple Choice Which equation is equivalent to 3x + 5y = 15?

<b>F.</b> $3x = 5y + 15$	<b>G.</b> $x = -5y + 5$
<b>H.</b> $y = 0.6x + 3$	<b>J.</b> $y = -0.6x + 3$

- **52.** Suppose you are given the linear equation ax + by = c.
  - **a.** What is the slope of every line parallel to this line?
  - **b.** What is the slope of every line perpendicular to this line?
- **53.** You will need two sheets of grid paper and two different cans with paper labels (for example, tuna and soup cans). On grid paper, trace the top and bottom of each can. Cut these out. Now carefully remove the labels and trace these on grid paper.
  - **a.** Estimate and compare the surface areas of the cans (label + top + bottom or  $A = \ell w + 2\pi r^2$ ).



**b.** After Joel removes his two labels, he notices that the labels are the exact same size and shape. Explain how this can happen.

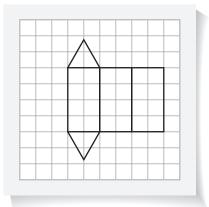
**54.** Multiple Choice Which values are solutions of the quadratic equation  $x^2 + 8x - 33 = 0$ ?

**A.** x = -11 and x = -3**B.** x = 11 and x = -3**C.** x = -11 and x = 3**D.** x = 11 and x = 3

**55.** Use the graph of  $y = x^2 + 8x - 33$  to find the solution of each inequality.

**a.**  $x^2 + 8x - 33 > 0$  **b.**  $x^2 + 8x - 33 < 0$ 

56. a. What shape will this net make if it is cut out and folded?



- **b.** Find the surface area of the shape.
- **c.** Find the volume of the shape.

**57.** Tell whether each line has a slope of 
$$-\frac{1}{2}$$
.  
**a.**  $y = \frac{-1}{-2}x + 3$   
**b.**  $y = \frac{-1}{2}x + 3$   
**c.**  $y = \frac{1}{-2}x + 3$   
**d.**  $y = -\frac{1}{2}x + 3$ 

Without graphing, decide whether the lines are parallel, perpendicular, or neither.

**58.** 3x + 6y = 12 and  $y = 10 + \frac{-1}{2}x$  **59.** y = -x + 5 and y = x + 5 **60.** y = 2 - 5x and y = -5x + 2 **61.** y = -3 + 5x and  $y = \frac{-x}{5} + 3$ **62.** 10x + 5y = 20 and y = 10x + 20

#### **Extensions**

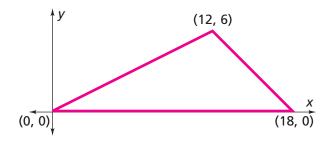
**63.** Jasmine wants to run a marathon. She knows she will have to walk some of the 26.2 miles, but she wants to finish in 5 hours. She plans to run 10-minute miles and walk 15-minute miles.

Let *x* stand for the number of minutes Jasmine runs. Let *y* stand for the number of minutes she walks.

- **a.** What equation relates *x* and *y* to the goal of completing the race in 5 hours?
- **b.** What equation relates *x* and *y* to the goal of covering 26.2 miles?
- **c.** For each equation, find several ordered-pair solutions (x, y). Then, plot the points with those coordinates and use the pattern to draw a graph of the equation. Graph both equations on the same axes.
- **d.** Use the graphs to estimate the combination of running and walking times that will allow Jasmine to complete the marathon in exactly 5 hours.
- **e.** Suppose Jasmine decides she wants to finish the marathon in less than 5 hours. Find five combinations of running and walking times that give a total time less than 5 hours.
- **f.** Express the condition that the total running and walking times must be less than 5 hours as an inequality.
- g. Make a graph of all the solutions of the inequality.
- **h.** Graph the linear equation from part (b) on the same axes as the inequality. Explain how the result shows Jasmine's options for running and walking times if she wants to finish the marathon in 5 hours or less.



64. a. Find coordinates of the midpoints of the sides of this triangle.



- **b.** A segment from one vertex of a triangle to the midpoint of the opposite side is called a *median*. Find equations of the three medians of this triangle.
- **c.** Use algebraic methods to find the coordinates of the point(s) where the median from the vertex (12, 6) intersects the other medians.
- **d.** The medians of any triangle intersect at a single point called the *centroid*. The centroid divides each median into two pieces that are related. Study the coordinates of the vertices, midpoints, and centroid of the triangle above. What is the special way in which the centroid splits each median?
- e. Use the coordinates of the centroid and the vertices to calculate the lengths of the two segments that make up each median. Explain how the results confirm your answer to part (d) or how they suggest a revision of your original idea.
- **65.** Your exploration in Exercise 64 focused on a single triangle. To test the patterns you observed, repeat the analysis from those problems with these other triangles.

**a.** A triangle with vertices (0,0), (12,0) and (-6,6)

**b.** A triangle with vertices (0, 0), (12, 0) and (4, 12)