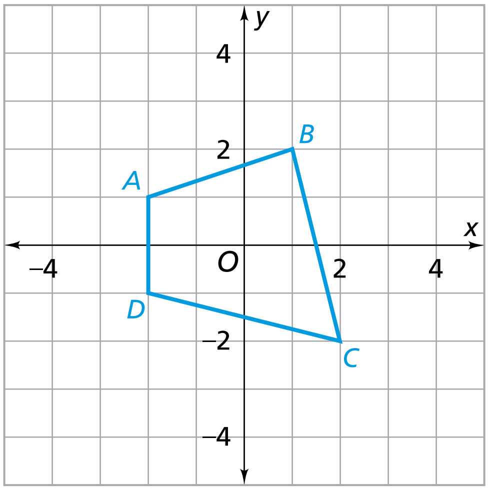
Applications

For Exercises 1–6, use the following figure.



1. Copy the figure onto grid paper. Draw the image of quadrilateral *ABCD* under a dilation with center (0, 0) and scale factor 2. Label the image $A'B'C'D'$.

2. Find the side lengths of quadrilaterals *ABCD* and $A'B'C'D'$. How are the lengths of corresponding sides related?

3. Find the perimeters of quadrilaterals *ABCD* and $A'B'C'D'$. How are the two perimeters related?

4. Find the areas of quadrilaterals *ABCD* and $A'B'C'D'$. How are the two areas related?

5. Find the slopes of the sides of quadrilaterals *ABCD* and $A'B'C'D'$.

a. How are the slopes of corresponding sides related?

b. What does your answer to part (a) suggest about the relationship between a line and its image under a dilation?

6. What dilation would transform quadrilateral $A'B'C'D'$ to quadrilateral *ABCD*?

For Exercises 7–10, suppose you dilate quadrilateral *ABCD* by a scale factor of 2. Then you flip, turn, or slide the image to quadrilateral $A''B''C''D''$.

7. Describe how the side lengths of quadrilateral $A''B''C''D''$ are related to the side lengths of each quadrilateral.

a. $ABCD$

b. $A'B'C'D'$

8. Describe how the perimeter of quadrilateral $A''B''C''D''$ is related to the perimeter of each quadrilateral.

a. $ABCD$

b. $A'B'C'D'$

9. Describe how the area of quadrilateral $A''B''C''D''$ is related to the area of each quadrilateral.

a. $ABCD$

b. $A'B'C'D'$

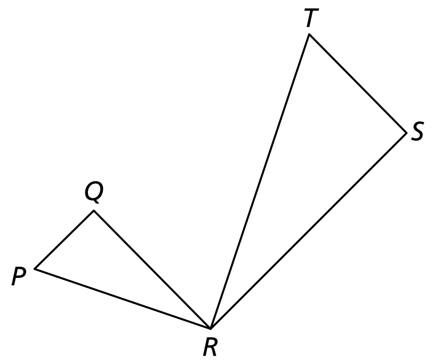
10. Describe how the slopes of the sides of quadrilateral $A''B''C''D''$ are related to the slopes of the sides of each quadrilateral.

a. $ABCD$

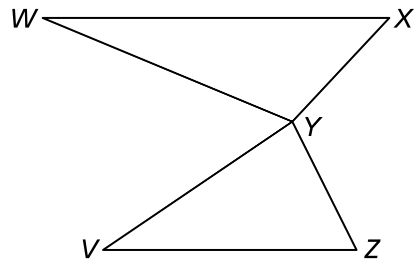
b. $A'B'C'D'$

For Exercises 11–15, tell whether the polygons in each pair are similar. For those that are similar, describe a sequence of flips, turns, slides, and/or dilations that would transform one to the other.

11.



12.



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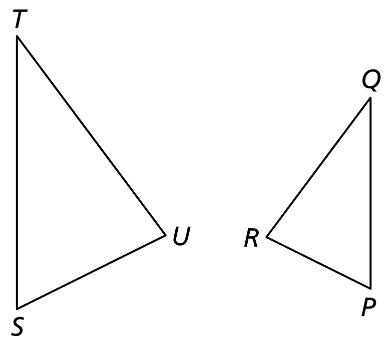
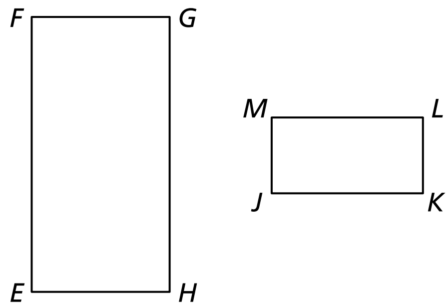
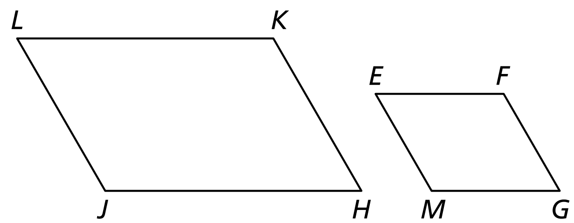


14.

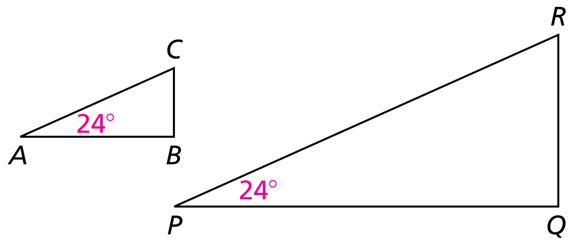


15.

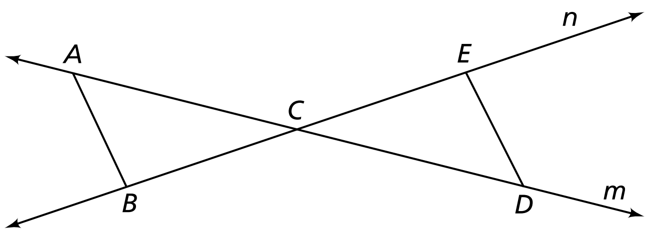


For Exercises 16–20, determine whether each statement is *true* or *false*. Justify your answer.

16. If $\angle P \cong \angle A$, $PQ = 2.5 \cdot AB$, and $PR = 2.5 \cdot AC$, you can conclude that the triangles are similar without measuring any more angles or sides.



17. If $\overline{AB}$ is parallel to $\overline{DE}$, then triangles ABC and DEC are similar.

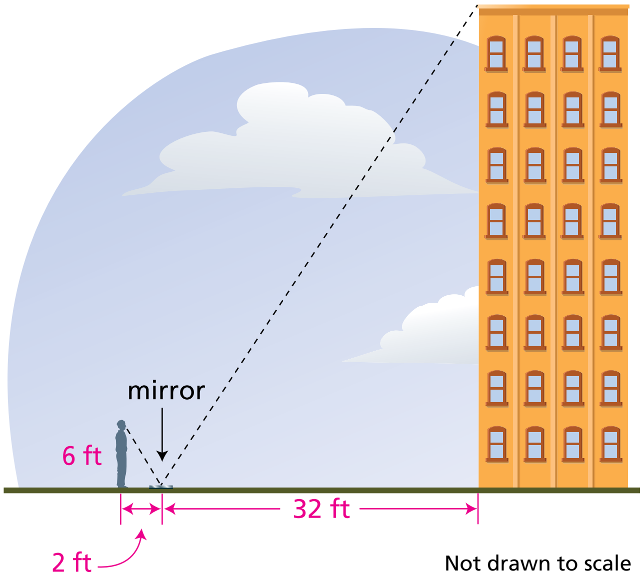


18. Any two equilateral triangles are similar to each other.

19. If corresponding angles of two polygons are congruent, then the polygons are similar.

20. Any two isosceles triangles are similar to each other.

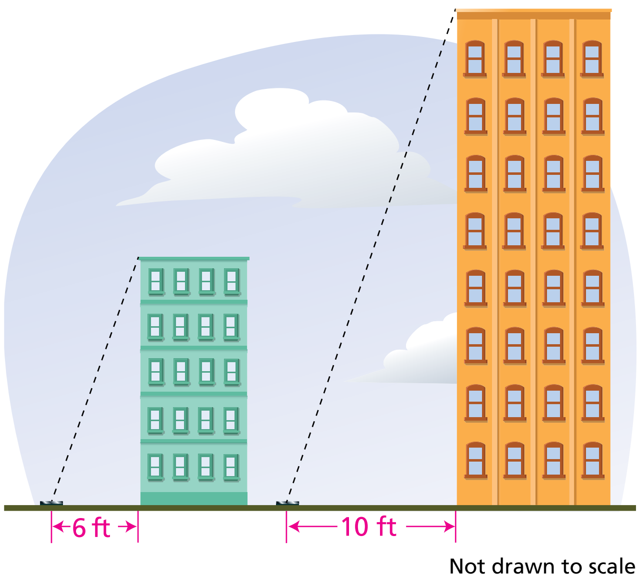
21. Stan uses the mirror method to estimate the height of a building. His measurements are shown in the diagram below.



a. How tall is the building?

b. How do you know that your calculation is correct?

22. One afternoon, the building in Exercise 21 casts a shadow that is 10 feet long, while a nearby building casts a shadow that is 6 feet long.



a. How tall is the shorter building?

b. How do you know that your calculation is correct?

Connections

23. A sphere has a radius of 5 centimeters.

a. What are the volume and surface area of the sphere?

b. The sphere is dilated by a scale factor of 2.

i. What is the surface area of the image?

ii. What is the volume of the image?

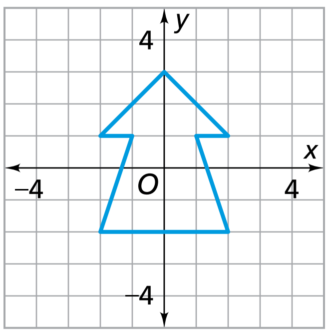
c. How can you find the answers to part (b) quickly by using your results from part (a)?

24. Consider the image of a figure after a dilation with center (0, 0) and scale factor 1.5, followed by a dilation with center (0, 0) and scale factor 4.

a. What is the simplest coordinate rule that relates each point of the figure to its image after the two dilations?

b. Suppose the order of the two dilations was reversed. Would the rule be different? Explain.

25. The figure below has reflectional symmetry in the *y*-axis.



a. Suppose the figure is dilated with scale factor 1.5 and center (0, 0). Would the image have the same symmetry?

b. Would the result be different with a different scale factor?

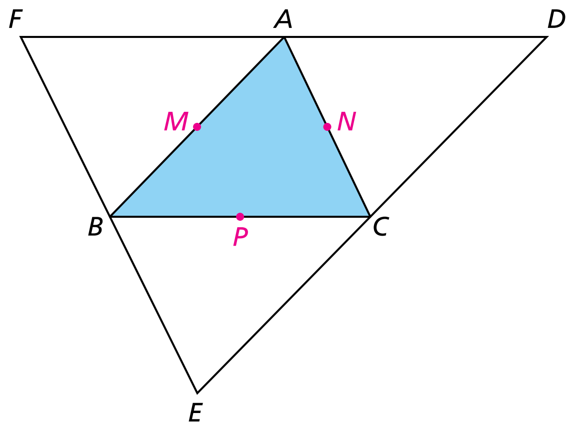
c. Would the result be different if the center of dilation was outside the figure?

For Exercises 26 and 27, determine whether the given statement is *true* or *false*. Justify your answer.

26. Any two squares are similar to each other.

27. Any two rhombuses are similar to each other.

28. The diagram below shows triangle *ABC* and its three images under a rotation of 180° about midpoints *M, N,* and *P*.

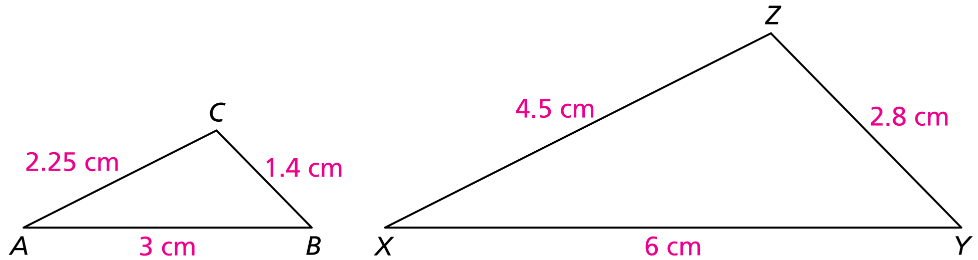


a. Are any of the triangles in the diagram similar? If so, what is the scale factor? Explain.

b. List all the parallelograms in the diagram. For each one, state why you think it is a parallelogram.

Extensions

29. Two students claim that you can determine whether two triangles are similar without measuring any angles. They say, "For example, if $XY = 2 \cdot AB$, $YZ = 2 \cdot BC$, and $ZX = 2 \cdot CA$, then triangles ABC and XYZ are similar."



a. Do you think the students are correct? Explain.

b. Do you agree with their reasoning in the following proof? Explain why or why not.

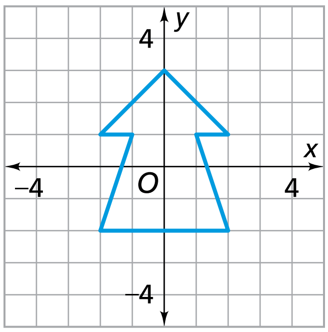
> If triangles ABC and XYZ are similar, then the lengths of corresponding sides are proportional. That is, $\frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA} = 2$.
> If you dilate triangle ABC by a factor of 2, corresponding angles will be congruent. The image $A'B'C'$ will have side lengths that are congruent to those of triangle XYZ.
>
> So, triangle $A'B'C'$ will be congruent to triangle XYZ and have congruent corresponding angles. This means that the angles of triangle ABC must be congruent to those of triangle XYZ, and the two triangles must be similar.

30. Suppose that a figure is transformed by two dilations, one after the other. The scale factors and centers of the two dilations are different. Will the final result be a dilation of the original figure with a different third center and scale factor?

Explore some simple cases to help you answer the question. For example, you might start with transformations of a simple triangle. Use centers of dilation (0, 0) and (1, 0) and scale factors 2 and 3.

31. Consider a one-directional dilation with rule $(x, y) \rightarrow (2x, y)$ instead of the basic dilation centered at the origin that "moves" $(x, y) \rightarrow (2x, 2y)$.

Experiment with a figure like the one below to see how the one-directional dilation works.



a. Does the transformation produce an image that is similar to the original figure?

b. How does the transformation affect the side lengths, angle measures, perimeter, and area of the figure?

c. How does the transformation affect the slopes of the sides of the figure?