Writing Word Problems As Systems of Equations and Solving

1. The sum of two numbers is -11. Twice the first number minus the second number is 32. Find the numbers.

Let x = 1st number

Let y = 2nd number

$$\begin{cases} x+y=-11 \\ 2x-y=32 \end{cases}$$

$$+\begin{array}{l} x+y=-11 \\ 2x-y=32 \\ \hline \dfrac{3x}{3}=\dfrac{21}{3} \\ x=7 \end{array}$$

$$\begin{array}{l} x+y=-11 \\ 7+y=-11 \\ -7 \qquad -7 \\ \hline y=-18 \end{array}$$

One number is 7 and the other is -18.

2. A collection of nickels and dimes is worth $3.30. There are 42 coins in all. How many of each kind of coin are there?

Let x = # of nickels

Let y = # of dimes

$$\begin{cases} x+y=42 \\ 5x+10y=330 \end{cases}$$

$$10[x+y=42]$$
$$5x+10y=330 \Rightarrow$$

$$\begin{array}{l} 10x+10y=420 \\ -\;5x+10y=330 \\ \hline \dfrac{5x}{5}=\dfrac{90}{5} \\ x=18 \end{array}$$

$$\begin{array}{l} x+y=42 \\ 18+y=42 \\ -18 \qquad -18 \\ \hline y=24 \end{array}$$

18 nickels and 24 dimes

3. One night a theater sold 548 movie tickets. An adult's ticket cost $6.50, and a child's ticket cost $3.50. In all, $2881 was taken in. How many of each kind of ticket were sold?

Let x = # of adult tickets
Let y = # of child tickets

$$\begin{cases} x + y = 548 \\ 6.5x + 3.5y = 2881 \end{cases}$$

$x + y = 548 \Rightarrow y = 548 - x$
$6.5x + 3.5y = 2881$

$6.5x + 3.5(548 - x) = 2881$
$6.5x + 1918 - 3.5x = 2881$
$\quad -1918 \qquad\qquad -1918$
$\overline{\qquad\qquad\qquad\qquad}$
$\dfrac{3x}{3} = \dfrac{963}{3}$
$x = 321$

$x + y = 548$
$321 + y = 548$
$-321 \qquad -321$
$\overline{\qquad\qquad}$
$y = 227$

321 Adult and 227 child tickets were sold.

4. The perimeter of a rectangular field is 110 feet. The length is 7 feet more than twice the width. Find the dimensions of the field.

Let x = length of the field
Let y = width of the field

$$\begin{cases} 2x + 2y = 110 \\ x = 2y + 7 \end{cases}$$

$2x + 2y = 110$
$x = 2y + 7 \Rightarrow$

$2x + 2y = 110$
$\underline{x - 2y = 7}$
$\dfrac{3x}{3} = \dfrac{117}{3}$
$x = 39$

$x = 2y + 7$
$39 = 2y + 7$
$-7 \qquad -7$
$\overline{\qquad\qquad}$
$\dfrac{32}{2} = \dfrac{2y}{2}$
$16 = y$

The field is 39 feet long and 16 feet wide.

5. A second run movie theater charges $4 for an adult ticket and $2 for a child's ticket. One night, 380 tickets were sold for a total of $1320. How many children attended the movie that night?

Let x = # of adult tickets
Let y = # of child tickets

$$\begin{cases} x+y=380 \\ 4x+2y=1320 \end{cases}$$

$2[x+y=380]$ $\Rightarrow$ $2x+2y=760$
$4x+2y=1320$ $\quad\quad$ $-\ 4x+2y=1320$

$$\frac{2x}{2}=\frac{560}{2}$$

$$x=280$$

$x+y=380$
$280+y=380$
$-280\quad\quad -280$
$y=100$

280 Adult tickets and 100 Child tickets.

6. The state fair is a popular field trip destination. This year the senior class at High School A and the senior class at High School B both planned trips there. High School A rented and filled 8 vans and 8 buses with 240 students. High School B rented and filled 4 vans and 1 bus with 54 students. Every van had the same number of students in it as did every bus. Find the number of students in each van and each bus.

Let x = # of students/van
Let y = # of students/bus

$$\begin{cases} 8x+8y=240 \\ 4x+y=54 \end{cases}$$

$8x+8y=240$ $\rightarrow$ $8x+8y=240$ $\quad$ $4x+y=54$
$2[4x+y=54]$ $\quad\quad$ $8x+2y=108$ $\quad$ $4x+22=54$
$\quad\quad\quad\quad\quad\quad\quad\quad$ $\frac{6y}{6}=\frac{132}{6}$ $\quad$ $\frac{-22\ -22}{}$
$\quad\quad\quad\quad\quad\quad\quad\quad$ $y=22$ $\quad\quad$ $\frac{4x}{4}=\frac{32}{4}$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ $x=8$

8 students/van and 22 students/bus.

7. An orange has 20 fewer calories than a banana. If 7 bananas have the same number of calories as 9 oranges, how many calories are there in a banana?

Let x = # of calories in an orange
Let y = # of calories in a banana

$$\begin{cases} x = y - 20 \\ 7x = 9y \end{cases}$$

$$x = y - 20$$
$$7y = 9x$$

$$\Rightarrow 7y = 9(y-20)$$
$$7y = 9y - 180$$
$$\underline{-9y \quad -9y}$$
$$\frac{-2y}{-2} = \frac{-180}{-2}$$
$$y = 90$$

$$x = y - 20$$
$$x = 90 - 20$$
$$x = 70$$

There are 70 calories in an orange and 90 calories in a banana.

8. Alexis bought pizza and soda for the ski club meeting. For one meeting she bought 4 pizzas and 10 sodas for $63. The next meeting she bought 3 pizzas and 8 sodas for $48. What is the cost of one pizza?

Let x = cost of one pizza
Let y = cost of a soda

$$\begin{cases} 4x + 10y = 63 \\ 3x + 8y = 48 \end{cases}$$

$$3[4x + 10y = 63]$$
$$+4[3x + 8y = 48]$$

$$\Rightarrow \quad \begin{array}{r} 12x + 30y = 189 \\ - \ 12x + 32y = 192 \\ \hline \frac{-2y}{-2} = \frac{-3}{-2} \\ y = 1.5 \end{array}$$

$$3x + 8y = 48$$
$$3x + 8(1.5) = 48$$
$$3x + 12 = 48$$
$$\underline{-12 \quad -12}$$
$$\frac{3x}{3} = \frac{36}{3}$$
$$x = 12$$

A pizza costs $12, a drink costs $1.50.