Applications

1. a. Plan 1: $y = x + 5$; Plan 2: $y = 1.5x + 2.5$

   b. Intersection point (5, 10) is an exact solution to the system of equations.

   c. $x + 5 = 1.5x + 2.5$ leads to $x = 5$; $(5) + 5 = 10$ or $1.5(5) + 2.5 = 10$ leads to $y = 10$.

   d. Plan 1 is a better deal for more than 5 cards purchased.

2. The change in plans leads to:

   a. Plan 1: $y = x + 4.5$; Plan 2: $y = x + 3.5$

   b. There is no intersection point since the lines are parallel with different intercepts.

   c. $x + 4.5 = x + 3.5$ leads to $1 = 0$.

   d. Plan 2 is always a better deal.

3. $(x, y) = (-3, -14)$

4. $(x, y) = (7, 28)$

5. $(x, y) = (-2, -5)$

6. No solution since $-x + 16 = -x + 8$ leads to $16 = -8$, which is a contradiction.

7. $(x, y) = (10, 164)$

8. $(x, y) = \left(\frac{29}{6}, -\frac{248}{3}\right)$

9. $y = -\frac{2}{3}x - 2$

10. $y = \frac{7}{9}x - \frac{4}{9}$

11. $y = -2x - 3$

12. $y = \frac{1}{4}x$

13. $y = x + 1$

14. $y = -5x + 3$

15. a. $p + f = 50$; $p + 6f = 115$

    b. Students might solve both equations for p, $p = 50 - f$ and $p = 115 - 6f$, then set both equations equal to each other and solve $50 - f = 115 - 6f$. $f = 13$, $p = 37$.

16. a. $m + c = 100$; $2m + 4c = 268$

    b. Students might solve both equations for m: $m = 100 - c$; $m = 134 - 2c$. $m = 66$, $c = 34$.

17. a. $x + y = 119$; $x - y = 25$. Adding the left sides of the equations and then the right sides makes the equation $2x = 144$. The first number x, is 72. Solving for y, $72 + y = 119$, so $y = 47$.

    b. $x + y = 71$; $x - y = 37$. $x = 54$, $y = 17$.

    c. $x + y = 32$; $x - y = 60$. $x = 46$, $y = -14$.

    d. $x + y = 180$; $x - y = 45$. $x = 112.5$, $y = 67.5$.

    e. Adding the left side of the equations results in $2x$, so adding the values for the sum and difference of the pair of numbers is twice the first number. To find the second number, substitute this value into either equation and solve.

18. a. $s + \ell = 29$; $0.35s + 0.50\ell = 13.45$

    b. Solve the first equation for s, $s = 29 - \ell$, and then substitute into the second equation:

    $0.35(29 - \ell) + 0.50\ell = 13.45$

    $10.15 - 0.35\ell + 0.50\ell = 13.45$

    $10.15 + 0.15\ell = 13.45$

    $\underline{-10.15 \qquad -10.15}$

    $0.15\ell = 3.30$

    c. $\ell = 22$

    d. $s = 7$

Note: For Exercises 19–22, students may represent the situations using shapes similar to Question A of Problem 2.2, or symbolically, as in Questions C and D of Problem 2.2.

19. a. $3j + 2d = 18$; $1j + 2d = 9$

    b. Students could solve both equations for $2d$: $2d = 18 - 3j$ and $2d = 9 - 1j$. Then $18 - 3j = 9 - 1j$, which is equivalent to $9 = 2j$, so $j = \$4.50$ and $d = \$2.25$.

20. a. $4s + 4c = 24$; $8s + 4c = 43$

    b. Subtracting the first equation from the second equation results in the equation $4s = 19$. $s = \$4.75$, $c = \$1.25$.

21. **a.** $3z + 5t = 15$; $5z + 3t = 21$

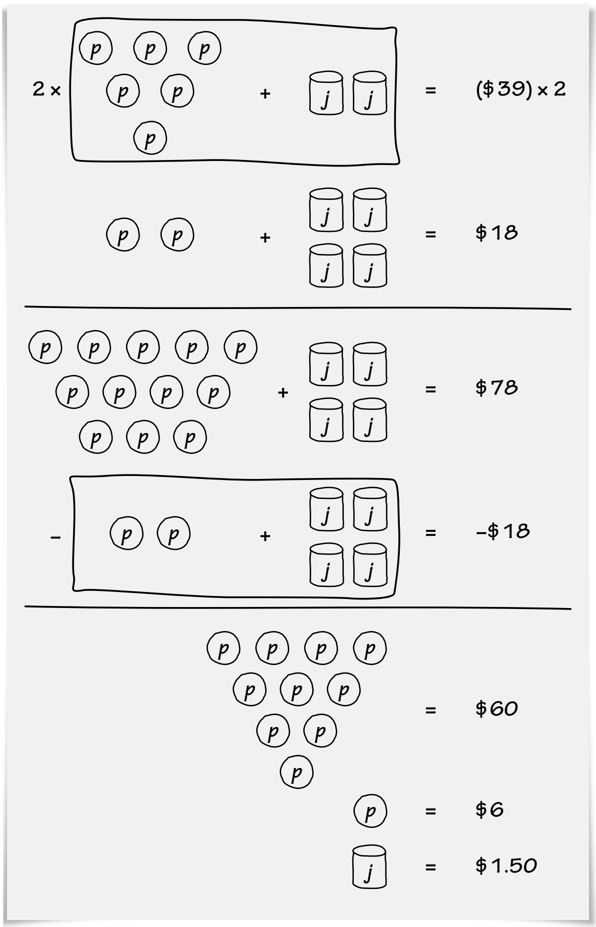
b. One method is to multiply the first equation by 5 and the second equation by 3, and then subtract.

$$\begin{array}{r} 15z + 25t = 75 \\ -15z - 9t = -63 \\ \hline 16t = 12 \end{array}$$

$t = \$.75$, $z = \$3.75$

22. **a.** $6p + 2j = 39$; $2p + 4j = 18$.

b. One strategy is to use a pictorial representation of each object.



$p = \$6.00$, $j = \$1.50$

23. $(x, y) = \left(\frac{14}{3}, 1\right)$

24. $(x, y) = \left(2, -\frac{1}{9}\right)$

25. $(x, y) = (-3, -2)$

26. $(x, y) = (-4, 2)$

27. $(x, y) = (-1.5, -3)$

28. $(x, y) = \left(8, \frac{2}{3}\right)$

29. $(x, y) = (7, -2)$

30. $(x, y) = (9, -6)$

31. $(x, y) = (3, 8)$

32. **a.** $2m + 2d = 26$, $2m - 2d = -18$; $m = 2$, $d = 11$, or February 11.

b. $2m + d = 26$, $10m - d = 10$; $m = 3$, $d = 20$, or March 20.

c. $3m + 2d = 62$, $2m - d = 4$; $m = 10$, $d = 16$, or October 16.

d. $4m + d = 42$, $m + 4d = 33$; $m = 9$, $d = 6$, or September 6.

Connections

33. $x = 4$

34. $x = -4$

35. $x = -11$

36. $x = 5$

37. $x = 3$

38. $x = -4$

39. **a.** $y = -13$

b. $y = -8$

c. $y = -6.5$

d. $y = 0$

e. $y = -7$

f. $y = -1.5$

40. $y = -4x + 3$

41. $y = \frac{2}{3}x + 2$

42. $y = -\frac{3}{1}x + 2$

43. $y = -\frac{3}{4}x + 7\frac{3}{4}$

44. slope $= -\frac{3}{2}$; y-intercept = (0, 2), or simply 2.

45. slope $= \frac{1}{2}$; y-intercept = (0, −1.5), or simply −1.5.

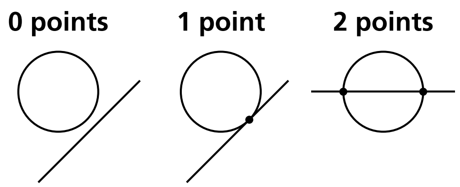
46. slope = 1; y-intercept = (0, −7), or simply −7.

47. slope = 4; y-intercept = (0, −8), or simply −8.

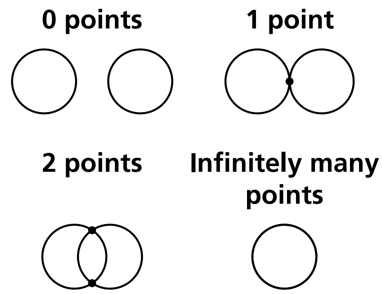
48. slope = 2; y-intercept = (0, 3), or simply 3.

49. slope = 0; y-intercept = (0, 9), or simply 9.

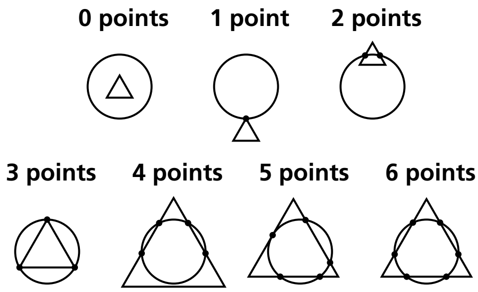
50. **a.** A line and a circle might intersect in 0, 1, or 2 points, as shown in these drawings:



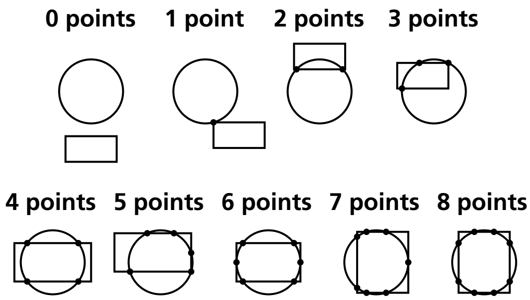
b. Two circles might intersect in 0, 1, 2, or infinitely many points (if they are identical).



c. A circle and a triangle might intersect in 0, 1, 2, 3, 4, 5, or 6 points.



d. A circle and a rectangle might intersect in 0, 1, 2, 3, 4, 5, 6, 7, or 8 points.



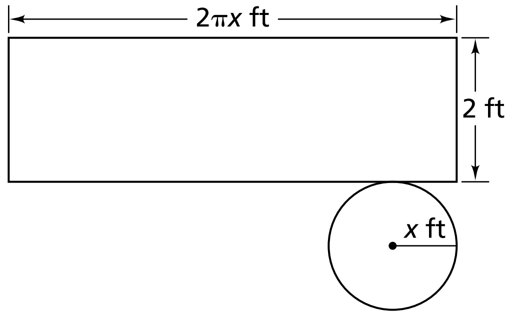
51. **a.** 10 chords

b. 5 points

c. 16 regions

52. C

53. **a.** One cover will look like this:



b. Area is given by $\pi x^2 + 4\pi x$ and by $\pi x(x + 4)$; volume by $2\pi x^2$.

54. J

55. D

56. Red Line: $y = x + 3$
Blue Line: $y = 2x + 2$
$x = 1$, $y = 4$

57. Red Line: $y = -3x - 3$
Blue Line: $y = 4x + 4$
$x = -1$, $y = 0$

58. Red Line: $y = \frac{1}{2}x + 2$
Blue Line: $y = 2x - 4$
$x = 4$, $y = 4$

59. Red Line: $y = 3$
Blue Line: $x = 5$
$x = 5$, $y = 3$

60. The solution for the system of equations is the intersection point on the two graphs.

61. Answers will vary. One way to check is to graph both equations and verify that their intersection point corresponds to the point given. Example solution: $y = x - 1$, $y = -x + 5$.

62. Any two equations of the form $y = kx$, such as $y = 2x$ and $y = -\frac{1}{2}x$.

63. Answers will vary. Sample: $2x + 2y = -12$, $x - 2y = 0$.

64. Answers will vary. Sample: $x - 2y = 0$, $x + 2y = 1$.

65. Quadratic; the second differences are a constant 2. The equation is $y = x(x - 4)$.

66. Exponential; the equation is $y = 3^x$.

67. Linear; the equation is $y = 3x - 1$.

68. Inverse variation; the product of each pair of x- and y-values is 12. The equation is $xy = 12$, or, equivalently, $y = \frac{12}{x}$ or $x = \frac{12}{y}$.

69. Students should recognize that the solution of the two linear equations is the intersection of the two graphs. Students may graph the four equations to find the intersections. They might also write the equations and solve them symbolically, Line *a*: $y = -4x - 6$, Line *b*: $y = 2x$, Line *c*: $y = -x + 3$, Line *d*: $y = \frac{1}{2}x + 3$. Another strategy is simply by inspecting the tables to see if there is a common set of points (e.g., the solutions can be found in the tables for lines *a* and *b*, and *a* and *c*)

a. $(x, y) = (-1, -2)$

b. $(x, y) = (-3, 6)$

c. $(x, y) = (-2, 2)$

d. $(x, y) = (1, 2)$

e. $(x, y) = (2, 4)$

f. $(x, y) = (0, 3)$

70. $x = 3$

71. $x = -\frac{1}{2}$

72. $x = 3$ or $x = 4$

73. $x = -6$ or $x = 1$

74. **a.** Graph 2

b. Graph 1

c. Graph 4

d. Graph 3

e. Graph 6

f. Graph 5

Extensions

75. **a.** Antonia: $y = 5.5x$; Marissa: $y = 20 + 0.5x$

b. Marissa's rate is a better deal for the customer when $x > 4$ hours.

c. They have the same charge for $x = 4$ hours.

76. Let R = Raj's age, s = Sarah's age, and T = Toni's age.

a. $R = 2s - 1$

b. $T = 3s - 2$

c. $s + (2s - 1) + (3s - 2) = 21$ when $s = 4$, $R = 7$, and $T = 10$

77. Let m = the number of candy bars Melissa sells and t = the number of candy bars Trevor sells.

a. $\begin{cases} m + t = 49 \\ 5t + 1 = t \end{cases}$

b. Trevor sold 41 and Melissa sold 8.

78. **a.** This system has no solution because following any of the developed solution strategies leads one to an impossible result, (0 = 3).

b. This system has infinitely many solutions because the two equations are equivalent.

c. This system has no solution. (This is the same as part (a).)

d. This system has infinitely many solutions. (This is the same as part (b).)

79. The system has two solutions $(-3, -4)$ and $(3, 4)$. This solution is suggested by the graph. A more algebraic approach could involve substituting $\frac{4}{3}x$ for y in the second equation and solving it for x.

80. **a.** 2; for example, $y = x^2$ and $y = x$ have the intersections (0, 0) and (1, 1).

b. 2; for example, $y = x^2 + 2$ and $y = -x^2 + 4$ have intersections (1, 3) and $(-1, 3)$.

c. 3; for example, $y = x(x + 1)(x - 1) = x^3 - x$ has three x-intercepts $x = 0, -1, 1$; which means that it intersects the line $y = 0$ (i.e. the x-axis) three different places.

d. 2; for example, the inverse variation function $y = \frac{1}{x}$ intersects the line $y = -x + 2.5$ at the points: $\left(\frac{1}{2}, 2\right)$ and $\left(2, \frac{1}{2}\right)$.

e. All might not have an intersection except part (c). A cubic function and a linear function defined over all real numbers will eventually intersect. Examples of nonintersecting pairs:

In part (a), quadratic $y = x^2$, linear $y = x - 2$.

In part (b), quadratic $y = x^2$, quadratic $y = -x^2 - 1$.

In part (d), inverse variation $y = \frac{1}{x}$, linear $y = 0$.

81. Answers will vary. We suggest some strategies that can be used to generate the answers.

a. Start with $x = 3$ and $y = 7$. Then get something like $x + y = 10$ and $x - y = -4$, for example. You could also use $2x + 3y = 27$, etc.

b. Same strategies will work as in part (a).

c. Write one equation $ax + by = c$ and then another $ax + by = c + 1$, for example. Just make sure slopes of lines are the same but intercepts are different.

82. **a.** No; the top two equations are exactly the same while the bottom two equations, when put into $y = mx + b$ form, are the same. Hence, the four equations represent only two distinct lines.

b–c. It does not represent either equation in the system, but it passes through the point of intersection and hence provides the y-coordinate of the solution pair.

d. The resulting equation will be linear since $ax + by = c$ and $dx + ey = f$ together imply that $(a + d)x + (b + e)y = c + f$.
The resulting equation will pass through the intersection of $ax + by = c$ and $dx + ey = f$ if such an intersection exists. If they are parallel, then the new equation will also be parallel.

83. **a.** Adding these two equations eliminates the z variable from the equation.

b. Yes, if he adds the middle equation to the bottom equation, the y and z variables will be eliminated from the equation.

c. The two resulting equations are $3x - 3y = -1$ and $-3x = -1$. Then $x = \frac{1}{3}$, $y = \frac{2}{3}$, $z = -2$.

84. **a.** 6 wafers in one serving of regular Sweetie Pies; 4 wafers in one serving of double-stuffed Sweetie Pies.

b. **i.** $6w + 3f = 160$

ii. $4w + 4f = 140$

c. Each wafer has $18\frac{1}{3}$ Calories and each layer of stuffing has $16\frac{2}{3}$ Calories.