Savings Account - Compound Interest

The formula for the bank amount for any year is $A = P(1+r)^t$, where t = number of years the investment has been in the bank, P = original amount invested, and r = the interest rate expressed as a decimal.

1. a) Why does the formula use 1 + r?

1+r is the growth factor, representing the 100% of the initial amount plus the interest rate. It makes it easier to work with if you are switching between interest rates.

b) Use this formula to compute the value of the original $2,000 investment after 40 years at an interest rate of 3% if no additional monthly contributions are made.

$y = 2000(1+0.03)^{40} = \$6524.08$

↳ remember we are working with dollars which get rounded to the hundredths place.

Now consider, what happens if the bank compounds the interest four times a year, or twice a year. The formula is changed to $A = P\left(1+\frac{r}{n}\right)^{nt}$, where n is the number of times per year the amount is compounded. (Remember: These formulas are for the situation in which no additional money is being contributed by the investor.)

2. In the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$,

a) What does the fraction $\frac{r}{n}$ represent?

This represents the rate divided by the number of times during the year that interest will be applied.

b) What does the exponent nt represent?

This represents the number of times interest will be applied over the course of the investment.

3. a) Use this new formula to compute the value of the original $2,000 investment after 40 years at an interest rate of 3% if the interest is compounded four times a year and if no additional monthly contributions are made.

$y = 2000\left(1+\frac{0.03}{4}\right)^{4\cdot 40} = \$6610.57$

b) Notice the answer to Question 3.a) is greater than the amount calculated for Question 1 which invested the same $2,000 for 40 years at 3% interest compounded yearly. Why?

$6610.75 vs. $6524.08

Interest is being applied more frequently to money that already includes added interest.

People generally contribute a monthly amount to their savings. This results in the money growing much faster than just waiting for a lump sum to grow to the desired value.

4. **a)** To observe this, use the Compound Interest Simulator to determine how long it will take an investment of $1,000 at 5% to reach $2,000 without any monthly contributions.

Between 14 and 15 years.

After 15 years you will have $2078.93

b) Now, use the simulator to determine how long it will take the same investment of $1,000 at 5% to reach $2,000 when the investor also makes monthly contributions of $50.

Between 1 and 2 years.

After 2 years you will have $2332.50

To further observe the effects of compounding interest, imagine we have two people who start saving for retirement.

Person A invests $2,000 at age 30 and then makes a monthly contribution of $200 until age 65; the account has an annual interest rate of 4.5%. Person B executes the same plan, but begins at age 40. This means she only has 25 years of investing compared to person A's 35 years.

A: $204,926.58 B: $112,967.37

While ten years may not seem like much, in terms of compound interest, it is. Using the simulator, determine how much her delay of ten year will have cost person B when she retires.

It will cost Person B $91,959.21 to delay 10 years.

5. Use the simulator to explore how much money you would need to invest to have $1 million by the time you reach age 65. Try to do this with and without monthly contributions. Try different interest rates as well. Find at least three combinations that yield $1 million, and record them in the table below. Assuming you are 14, 51 years for investment.

Principal ($)	Monthly Contribution ($)	Annual Interest Rate (%)	Length of Investment	
$50,000	$130	5% (monthly)	51	$1,003,242.03
$20,000	—	8% (annually)	51	$1,013,074.83
$20,000	—	7.7% (monthly)	51	$1,002,434.47
$20,000	$20	7.7% (quarterly)	50	$1,044,188.42
	$50	7.7% (monthly)	47	$1,016,947.75

6. How long will it take an investment of $10,000 to reach $1 million if the rate of return is 10% with no additional contributions?

49 years compounded annually
48 years compounded semi-anually
47 years compounded quarterly

7. What rate of return would be necessary with an investment of $10,000 to make $1 million by the time you reach age 35? 21 year investment

24.6% compounded annually
23.2% semiannually
22.6% quarterly
22.2% monthly

8. Use the Internet to find the current interest rates for online banks and traditional banks (like the one down the street). Use this information to compare the investment results after 20 years on a $40,000 investment.

BANK NAME	INTEREST RATE	BALANCE OF $40,000 INVESTMENT AFTER 20 YEARS

What Did you find?

9. Search online or talk to a banker to find the answer to the following question: What's the difference between annual *rate* and annual *yield*?

The rate does not include the effect of compound interest.
5% rate compounded quarterly will give a yield of 5.09%
5% rate compounded monthly will give a yield of 5.12%

10. At what age should you start investing your money?

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