ACE Applications | Connections | Extensions

Applications

1. Several students were working on Question A of Problem 5.1. They wondered what would happen if they extended their table. Do you agree or disagree with each conjecture below? Explain.

Heidi's conjecture:
The 1^x column will contain only ones.

Evan's conjecture:
The bottom right corner of any table will always have the largest value.

Roger's conjecture:
So far, every number in the 2 column is even. Eventually an odd number will show up if I extend the table far enough.

Jean's conjecture:
Any odd power (an odd row) will have all odd numbers in it.

Chaska's conjecture:
To get from one row to the next in the tens column multiply the number you have by 10. For example $10^5 = 100{,}000$, so $10^6 = 100{,}000 \times 10 = 1{,}000{,}000$.

Tim's conjecture:
The row where $x = 2$ will always have square numbers in it.

2. **Multiple Choice** Which expression is equivalent to $2^9 \times 2^{10}$?

   A. 2^{90} **B.** 2^{19} **C.** 4^{19} **D.** 2^{18}

Use the properties of exponents to write each expression as a single power. Check your answers.

3. $5^6 \times 8^6$
4. $(7^5)^3$
5. $\frac{8^{15}}{8^{10}}$

For Exercises 6–11, tell whether the statement is *true* or *false*. Explain.

6. $6^3 \times 6^5 = 6^8$
7. $2^3 \times 3^2 = 6^5$
8. $3^8 = 9^4$
9. $4^3 + 5^3 = 9^3$
10. $2^3 + 2^5 = 2^3(1 + 2^2)$
11. $\frac{5^{12}}{5^4} = 5^3$

12. Multiple Choice Which number is the ones digit of $2^{10} \times 3^{10}$?

F. 2 **G.** 4 **H.** 6 **J.** 8

For Exercises 13 and 14, find the ones digit of the product.

13. $4^{15} \times 3^{15}$

14. $7^{15} \times 4^{20}$

15. Manuela came to the following conclusion about power of 2.

> It must be true that $2^{10} = 2^4 \cdot 2^6$, because I can group
> $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as
> $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$

a. Verify that Manuela is correct by evaluating both sides of the equation $2^{10} = 2^4 \cdot 2^6$.

b. Use Manuela's idea of grouping factors to write three other expressions that are equivalent to 2^{10}. Evaluate each expression you find to verify that it is equivalent to 2^{10}.

c. The standard form for 2^7 is 128, and the standard form for 2^5 is 32. Use these facts to evaluate 2^{12}. Explain your work.

d. Test Manuela's idea to see if it works for exponential expressions with other bases, such as 3^8 or $(1.5)^{11}$. Test several cases. Give an argument supporting your conclusion.

For Exercises 16–21, tell whether each expression is equivalent to 1.25^{10}. Explain your reasoning.

16. $(1.25)^5 \cdot (1.25)^5$

17. $(1.25)^3 \times (1.25)^7$

18. $(1.25) \times 10$

19. $(1.25) + 10$

20. $(1.25^5)^2$

21. $(1.25)^5 \cdot (1.25)^2$

For Exercises 22–25, tell whether each expression is equivalent to $(1.5)^7$. Explain your reasoning.

22. $1.5^5 \times 1.5^2$

23. $1.5^3 \times 1.5^4$

24. 1.5×7

25. $(1.5) + 7$

26. Some students are trying to solve problems with rational exponents. Which of these solutions is correct?

Stu's Solution

$$81^{\frac{3}{4}} = \left(81^{\frac{1}{4}}\right)^3$$
$$= (3^3)$$
$$= 27$$

Carrie's Solution

$$125^{\frac{7}{3}} = 125^{\frac{6}{3} + \frac{1}{3}}$$
$$= 125^2 \cdot 125^{\frac{1}{3}}$$
$$= 15{,}625 \cdot 5$$
$$= 78{,}125$$

For Exercises 27–30, use the properties of exponents to evaluate each expression.

27. $\left(756^{\frac{1}{7}}\right)^7$

28. $342^{\frac{5}{2}} \div 342^{\frac{3}{2}}$

29. $3^{35} \cdot 3^{-35}$

30. $\left(\frac{1}{2}\right)^{40} \cdot 2^{40}$

For Exercises 31–36, decide if each statement is *always true, always false,* or *sometimes true*. Explain.

31. $2^n \cdot 2^n = 2(2^n)$

32. $2^n \cdot 2^n = (2^n)^2$

33. 2^n is less than 2^{n-1}.

34. b^n is less than b^{n-1}.

35. For the expression 3^x, when x is negative, 3^x will be smaller than 1.

36. For the expression b^x, when x is negative, b^x will be smaller than 1.

37. In 1867, the United States of America purchased the territory of Alaska from the Russian Empire. Its 586,412 square miles cost \$7.2 million. The United States paid roughly two cents per acre of land. Assume that the price of land in Alaska has increased in value by 5% a year since the purchase.

a. Write an equation that represents the price per acre in the year n.

b. What was the cost of an acre in 1900? In 2000?

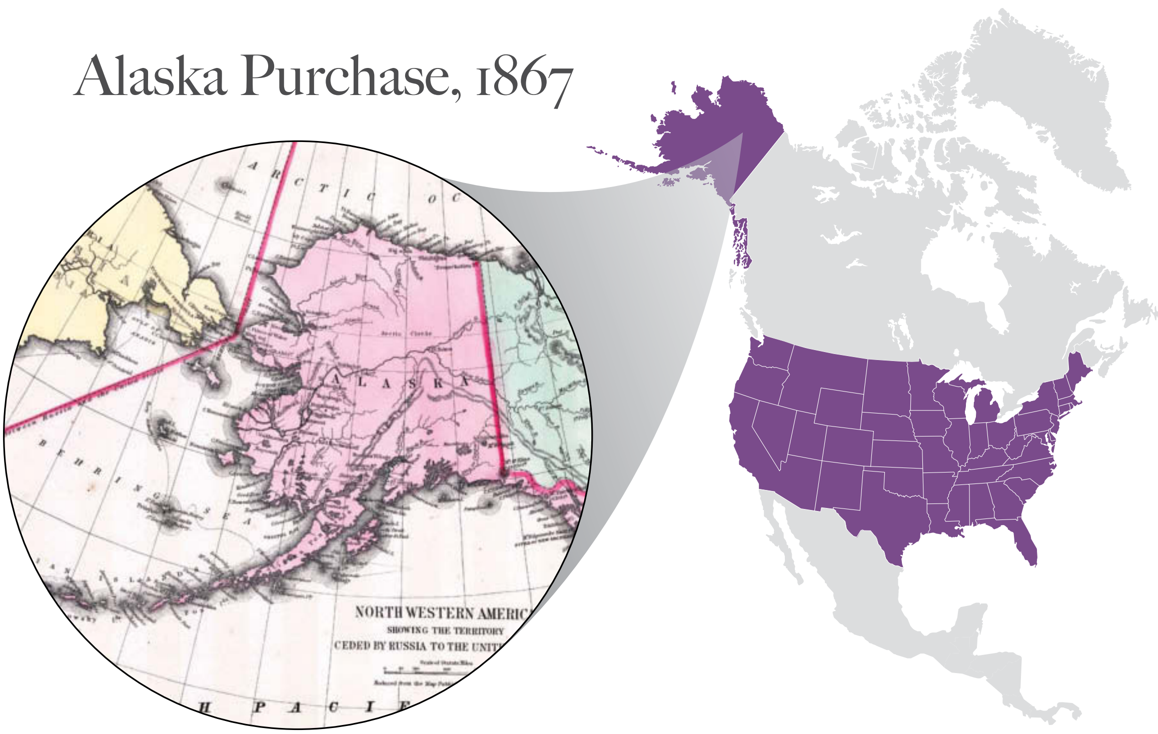
c. In what year did the cost reach approximately \$1 per acre? \$100 per acre?

d. Gia calculated the cost per acre after n years on her calculator. She got the answer 2.453774647E28. For what year was she trying to find the cost?

38. Suppose n is the number of years after the United States purchased the territory of Alaska, in March of 1867. The equation $v = 7{,}200{,}000 \cdot (1.05)^n$ models the total value v of the territory. It is based on a 5% increase per year. Calculate the value of the territory during each month below. Explain what exponent you would use.

a. April 1867 **b.** May 1867 **c.** September 1867

d. June 1868 **e.** November 1868



39. Copy and complete this table.

Powers of Ten

Standard Form	Exponential Form
10,000	10^4
1,000	10^3
100	10^2
10	10^1
1	10^0
$\frac{1}{10} = 0.1$	10^{-1}
$\frac{1}{100} = 0.01$	10^{-2}
$\frac{1}{1{,}000} = 0.001$	■
$\frac{1}{10{,}000} = 0.0001$	■
■	10^{-5}
■	10^{-6}

40. Write each number in standard form as a decimal.

3×10^{-1} $\quad$ 1.5×10^{-2} $\quad$ 1.5×10^{-3}

41. If you use your calculator to compute $2 \div 2^{12}$, the display might show something like this:

4.8828125E-4

The display means 4.8828125×10^{-4}, which is a number in scientific notation. Scientific notation uses two parts. The first is a number greater than or equal to 1 but less than 10 (in this case, 4.8828125). The second is a power of 10 (in this case, 10^{-4}). You can convert 4.8828125×10^{-4} to standard form in this way.

$$4.8828125 \times 10^{-4} = 4.8828125 \times \frac{1}{10{,}000} = 0.00048828125$$

a. Write each number in standard notation.

1.2×10^{-1} $\quad$ 1.2×10^{-2} $\quad$ 1.2×10^{-3} $\quad$ 1.2×10^{-8}

b. Suppose you have the expression 1.2×10^{-n}, where n is any whole number greater than or equal to 1. Using what you discovered in part (a), explain how you would write the expression in standard notation.

42. Write each number in scientific notation.

a. 2,000,000 **b.** 28,000,000 **c.** 19,900,000,000

d. 0.12489 **e.** 0.0058421998 **f.** 0.0010201

43. When Tia divided 0.0000015 by 1,000,000 on her calculator, she got 1.5E−12, which means 1.5×10^{-12}.

a. Write a different division problem that will give the result 1.5E−12 on your calculator.

b. Write a multiplication problem that will give the result 1.5E−12 on your calculator.

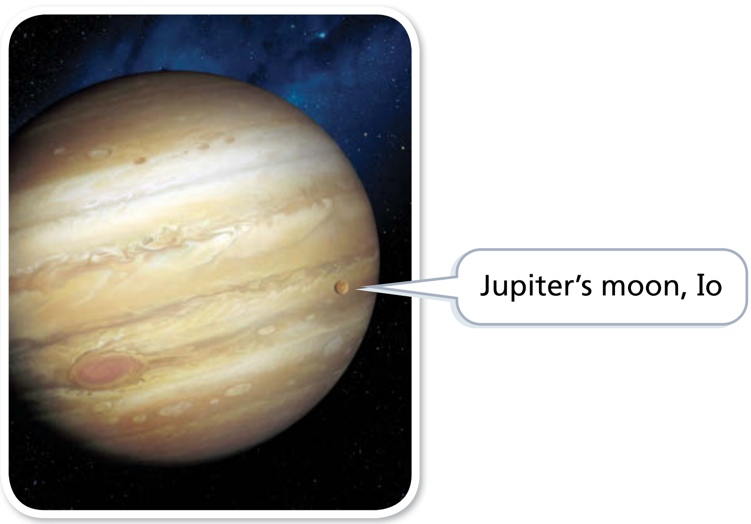
44. The radius of the moon is about 1.74×10^6 meters.

a. Express the radius of the moon in standard notation.

b. The largest circle that will fit on your textbook page has a radius of 10.795 cm. Express this radius in meters, using scientific notation.

c. Suppose a circle has the same radius as the moon. By what scale factor would you reduce the circle to fit on your textbook page?

d. Earth's moon is about the same size as Io, one of Jupiter's moons. What is the ratio of the moon's radius to the radius of Jupiter (6.99×10^7 meters)?



45. The number 2^7 is written in standard form as 128 and in scientific notation as 1.28×10^2. The number $\left(\frac{1}{2}\right)^7$, or $(0.5)^7$, is written in standard form as 0.0078125 and in scientific notation as 7.8125×10^{-3}. Write each number in scientific notation.

a. 2^8 **b.** $\left(\frac{1}{2}\right)^8$ **c.** 20^8 **d.** $\left(\frac{1}{20}\right)^8$

46. **a.** The boxes in the table below represent decreasing y-values. The decay factor for the y-values is $\frac{1}{3}$. Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
y	30	10	■	■	■	■	■	■	■

b. For $x = 12$, a calculator gives a y-value of 5.645029269E−5. What does that mean?

c. Write the y-values for $x = 8$, 9, 10, and 11 in scientific notation.

For Exercises 47–49, use the properties of exponents to show that each statement is true.

47. $\frac{1}{2}(2^n) = 2^{n-1}$

48. $4^{n-1} = \frac{1}{4}(4^n)$

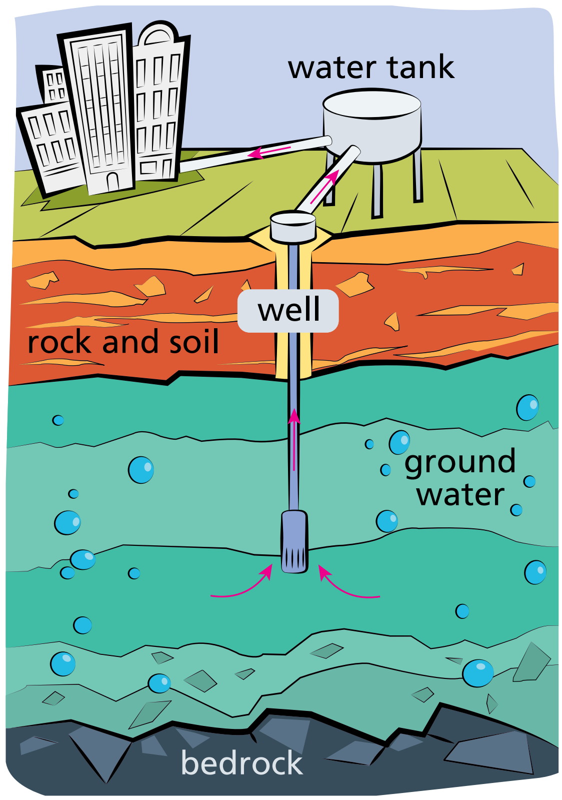
49. $25\left(5^{n-2}\right) = 5^n$

50. Use the data from Problem 5.4 to answer the following questions. Write your final answer in scientific notation.

a. How many of gallons of water are used in the United States in a year?

b. About how many times greater is the amount of water used for irrigation than the amount used for livestock?

c. Suppose 80% of water is from *surface* sources. How many gallons of freshwater are removed *from the ground* each month?



For Exercises 51–57, rewrite each expression in scientific notation.

51. $(8.2 \times 10^2) \times (2.1 \times 10^5)$

52. $(2.0 \times 10^3) \times (3.5 \times 10^6) \times (3.0 \times 10^3)$

53. $(2.0 \times 10^8) \times (1.4 \times 10^{-10})$

54. $(5.95 \times 10^8) \div (1.70 \times 10^5)$

55. $(1.28 \times 10^6) \div (5.12 \times 10^7)$

56. $(2.8 \times 10^{-4}) \div (1.4 \times 10^4)$

57. $(3.6 \times 10^2) \div (9.0 \times 10^{-3})$

For Exercises 58–62, find the missing values in each equation. Choose values such that all numbers are written in correct scientific notation.

58. $(2.4 \times 10^3) \times (g \times 10^h) = 6.0 \times 10^{12}$

59. $(j \times 10^2) \times (1.8 \times 10^k) = 9.0 \times 10^1$

60. $(m \times 10^7) \div (2.4 \times 10^n) = 5.0 \times 10^4$

61. $(6.48 \times 10^6) \div (p \times 10^q) = 2.16 \times 10^{-2}$

62. $(r \times 10^s) \times (r \times 10^s) = 1.6 \times 10^5$

63. Without actually graphing these equations, describe and compare their graphs. Be as specific as you can.

$y = 4^x$ $\qquad$ $y = 0.25^x$ $\qquad$ $y = 10(4^x)$ $\qquad$ $y = 10(0.25^x)$

64. Explain how each of the graphs for the equations below will differ from the graph of $y = 2^x$.

a. $y = 5(2^x)$

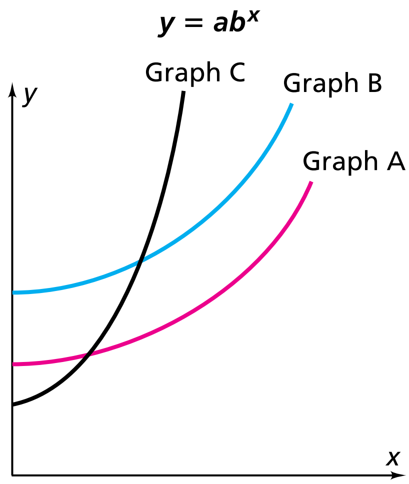
b. $y = (5 \cdot 2)^x$

c. $y = \frac{1}{2}(2^x)$

d. $y = -1(2^x)$

e. $y = \left(\frac{1}{2}\right)^x$

65. Each graph below represents an exponential equation of the form $y = a(b^x)$.



a. For which of the three functions is the value of a greatest?

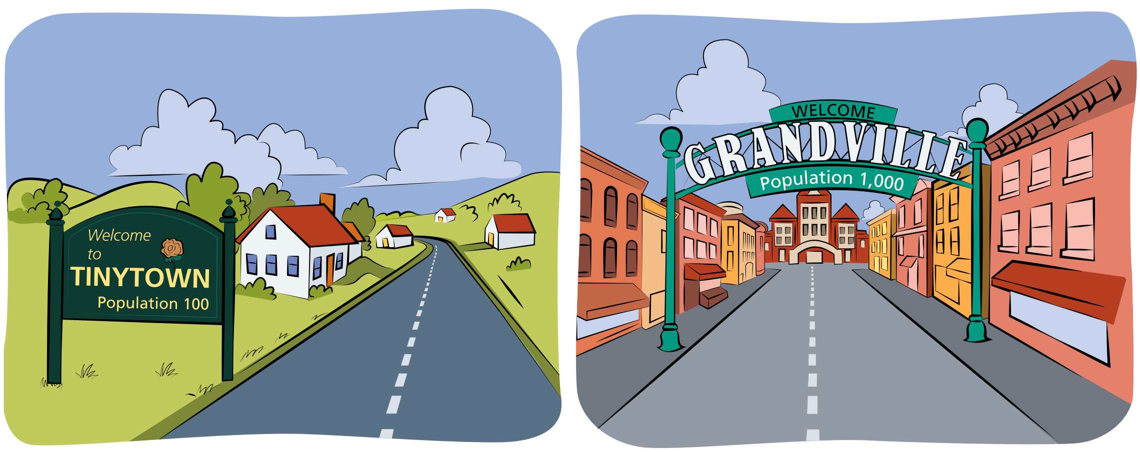
b. For which of the three functions is the value of b greatest?

66. Grandville has a population of 1,000. Its population is expected to decrease by 4% a year for the next several years. Tinytown has a population of 100. Its population is expected to increase by 4% a year for the next several years. For parts (a)–(c), explain how you found each answer.

a. What is the population of each town after 5.5 years?

b. In how many years will Tinytown have a population of approximately 1,342? Explain your method.

c. Will the populations of the two towns ever be the same? Explain.



Connections

In Exercises 67–69, tell how many zeros are in the standard form of each number.

67. 10^{10}

68. 10^{50}

69. 10^{100}

In Exercises 70 and 71, find the least integer value of x that will make each statement true.

70. $9^6 < 10^x$

71. $3^{14} < 10^x$

In Exercises 72–74, identify the greater number in each pair.

72. 6^{10} or 7^{10}

73. 8^{10} or 10^8

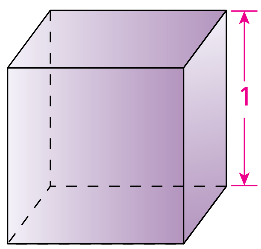
74. 6^9 or 9^6

In Exercises 75 and 76, tell whether each statement is *true* or *false*. Do not do an exact calculation. Explain your reasoning.

75. $(1.56892 \times 10^5) - (2.3456 \times 10^4) < 0$

76. $\frac{3.96395 \times 10^5}{2.888211 \times 10^7} > 1$

77. Suppose you start with a unit cube (a cube with edges of length 1 unit). In parts (a)–(c), give the volume and surface area of the cube that results from the given transformation.

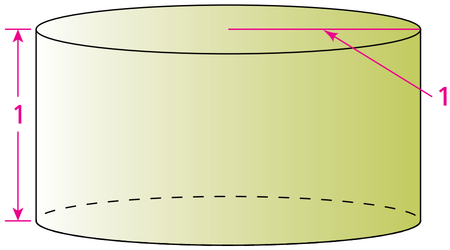


a. Each edge length is doubled.

b. Each edge length is tripled.

c. Each edge is enlarged by a scale factor of 100.

78. Suppose you start with a cylinder that has a radius of 1 unit and a height of 1 unit. In parts (a)–(c), give the volume of the cylinder that results from the given transformation.



a. The radius and height are doubled.

b. The radius and height are tripled.

c. The radius and height are enlarged by a scale factor of 100.

79. a. Tell which of the following numbers are prime. (There may be more than one.)

$2^2 - 1$ $\quad$ $2^3 - 1$ $\quad$ $2^4 - 1$ $\quad$ $2^5 - 1$ $\quad$ $2^6 - 1$

b. Find another prime number that can be written in the form $2^n - 1$.

80. In parts (a)–(d), find the sum of the proper factors for each number.

a. 2^2

b. 2^3

c. 2^4

d. 2^5

e. What do you notice about the sums in parts (a)–(d)?

81. The expression $\frac{20}{10^2}$ can be written in many equivalent forms, including $\frac{2}{10}$, $\frac{1}{5}$, 0.2, and $\frac{2(10^2)}{10^3}$. In parts (a) and (b), write two equivalent forms for each expression.

a. $\frac{3(10)^5}{10^7}$

b. $\frac{5(10)^5}{25(10)^7}$

Extensions

In Exercises 82–86, predict the ones digit for the standard form of each number.

82. 7^{100}

83. 6^{200}

84. 17^{100}

85. 31^{10}

86. 12^{100}

For Exercises 87 and 88, find the value of *a* that makes each number sentence true.

87. $a^7 = 823{,}543$

88. $a^6 = 1{,}771{,}561$

89. Explain how you can use your calculator to find the ones digit of the standard form of 3^{30}.

90. **Multiple Choice** In the powers table you completed in Problem 5.1, look for patterns in the ones digit of square numbers. Which number is *not* a square number? Explain.

A. 289 **B.** 784 **C.** 1,392 **D.** 10,000

91. **a.** Find the sum for each row in the table below.

Sums of Powers of $\frac{1}{2}$

Row 1	$\frac{1}{2}$
Row 2	$\frac{1}{2} + \left(\frac{1}{2}\right)^2$
Row 3	$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3$
Row 4	$\frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$

b. Study the pattern. Suppose the pattern continues. Write the expression that would be in row 5 and evaluate the sum.

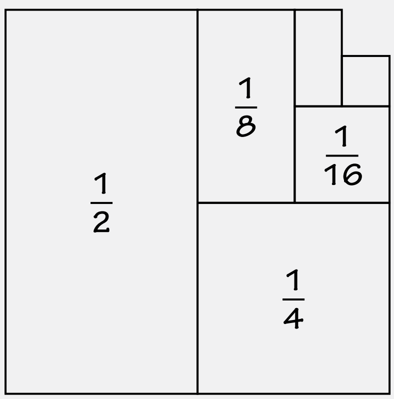
c. What would be the sum of the expression in row 10? What would you find if you evaluated the sum for row 20?

d. Describe the pattern of sums in words and with a symbolic expression.

e. For which row does the sum first exceed 0.9?

f. As the row number increases, the sum gets closer and closer to what number?

g. Celeste claims the pattern is related to the pattern of the areas of the ballots cut in Problem 4.1. She drew the picture below to explain her thinking.



$$\text{row } 6 = \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6$$

What relationship do you think she has observed?

92. Chen, from Problem 4.1, decides to make his ballots starting with a sheet of paper with an area of 1 square foot.

a. Copy and extend this table to show the area of each ballot after each of the first 8 cuts.

Areas of Ballots

Number of Cuts	Area (ft^2)
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$

b. Write an equation for the area A of a ballot after any cut n.

c. Use your equation to find the area of a ballot after 20 cuts. Write your answer in scientific notation.

93. In 1803, the United States bought the 828,000-square-mile Louisiana Purchase for \$15,000,000. Suppose one of your ancestors was given 1 acre of the Louisiana Purchase. Assuming an annual increase in value of 4%, what was the value of this acre in 2003? (640 acres = 1 square mile)