

Exponential Decay – Practice Problems

1. Garden City introduced a recycling program. The goal of the program is to reduce the number of pounds of trash sent to landfills by 25% each year. In 2010, Garden City produced 100,000 tons of trash. If the recycling program were to reach its goal, how many tons of trash can Garden City expect to produce in the year 2025?

$$y = 100,000 (0.75)^x$$

$$y = 100,000 (0.75)^{15}$$

$$y = 1336.3 \text{ tons} \quad \frac{2025}{-2010} = 15 \text{ yrs}$$

How to find the Factor:

25% decrease means 75% remaining each year. Factor = 0.75

OR

$$\begin{aligned} \text{Factor} &= 1 + \text{Rate} \\ &= 1 - 0.25 \\ &= 0.75 \end{aligned}$$

-0.25 because decreasing by 25%

2. A city of 2,950,000 has a 2.5% annual decrease in population. Determine the city's population after each of the following:

$$y = 2,950,000 (0.975)^x$$

a. 1 year

$$y = 2,950,000 (0.975)^1 = 2,876,250 \text{ people}$$

b. 5 years

$$y = 2,950,000 (0.975)^5 = 2,599,232 \text{ people}$$

c. 15 years

$$y = 2,950,000 (0.975)^{15} = 2,017,861 \text{ people}$$

d. 25 years

$$y = 2,950,000 (0.975)^{25} = 1,566,525 \text{ people}$$

3. Several species of whale have been declared endangered. When the populations of a particular whale species fall dangerously low, biologists encourage governments to agree to a ban on hunting the species. Suppose that, in the year 2000, there were only 5,000 whales of a particular species and that the population was predicted to continue to decline as shown in the table.

a. Which equation below models this population pattern?

A. $W = 5,000(0.1^y)$

B. $W = 5,000(0.9^y)$

C. $W = 5,000 - 500^y$

D. $W = 5,000^y$

Year (y)	Whales (w)
0 (2000)	5,000
1	4,500
2	4,050
3	3,645
4	3,281
5	2,952
6	2,657

b. What is the decay factor for the relationship? Explain how you determined your answer.

Factor

The decay factor is 0.9. I found this by "dividing up the table," dividing one number by the previous number in the table.

c. Define all the numbers and variables in the equation in the context of the problem.

The number of whales " y " years after 2000.

$$W = 5,000(0.9^y)$$

Each year 90% of the population remains.

There were 5000 whales in year 2000.

Number of years since 2000.

d. According to this model, what will the whale population be in 2007?

$$W = 5,000(0.9)^7$$

$$W = 2391 \text{ whales}$$

e. What is the decay rate?

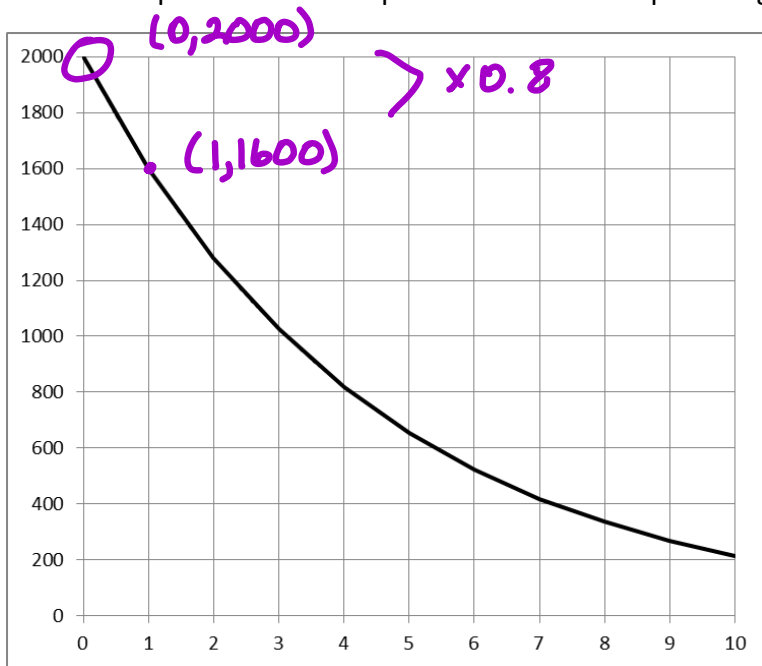
$$\text{Factor} = 1 + \text{Rate}$$

$$0.9 = 1 + \text{Rate}$$

$$\begin{array}{r} -1 \quad -1 \\ \hline -0.1 = \text{Rate} \end{array}$$

10% decrease

4. Write an equation for the exponential relationship in the graph below. Show your work.



$$y = 2000(0.8)^x$$

5. The population of a certain species of bird is decreasing by 10% each year. If the population in year 0 is 15,000, which of the following statements are true? *Select all that apply.*

- In year 7, there will be fewer than 7,000 of this species of bird.
- After 5 years, there will be approximately 8,857 of this species of bird.
- The graph of this relationship shows exponential decay.
- The decay factor is 0.10.
- After 1 year, there will be 13,500 of this species of bird.

1 yr. $y = 15,000(0.9)^1$
 $y = 13,500$

5 yrs. $y = 15,000(0.9)^5$
 $y = 8,857$

7 yrs. $y = 15,000(0.9)^7$
 $y = 7,174$

6. In some areas, home values have decreased over the past 10 years. The table shows the decrease in the home value of one house.

Year	Home Value
0	\$250,000
1	\$242,500
2	\$235,225
3	\$228,168
4	\$221,323
5	\$214,684

> x 0.97
> x 0.97
> x 0.97
> x 0.97
> x 0.97

Using the tiles, write an equation that represents the value of the home, v , after t years.

1.03 0.03 250,000

0.97 1.30 t

$$v = (250,000) \left((0.97)^t \right)$$

7. A tree farm has begun to harvest a section of trees that was planted a number of years ago. The table shows the number of trees remaining for each of 8 years of harvesting.

Year	0	1	2	3	4	5	6	7	8
Trees remaining	10,000	9502	9026	8574	8145	7737	7350	6892	6543

0.95 0.95 0.95 0.95 0.95 0.95 0.95 0.95

- a. Suppose the relationship between the year and the number of trees remaining is exponential. Approximate the decay factor for this relationship. Explain how you found this.

The decay factor is 0.95. I found this by "dividing up the table," dividing one number by the previous number in the table.

- b. Write an equation for the relationship between time and trees remaining.

$$y = 10,000 (0.95)^x$$

- c. Use your equation to find how many trees will be left after 20 years?

$$y = 10,000 (0.95)^{20} = 3584 \text{ trees remaining}$$

9. Another tree farm plans to harvest their trees so that there was a linear relationship with the initial data presented in the table below.

Year	0	1
Trees remaining	10,000	9502

+1
-498

$$\frac{\Delta y}{\Delta x} = -\frac{498}{1}$$

- b. Write an equation for the relationship between time and trees remaining.

$$y = -498x + 10,000$$

- c. Use your equation to find how many trees will be left after 20 years?

$$y = -498(20) + 10,000 = 40 \text{ trees remaining}$$