# Warm Up

11/19

Write an equation for a situation where you start with $1 and you get an additional $5 everyday.

Let x = # of days

Let y = amount of money

|  | x | y |  |
|---|---|---|---|
|  | 0 | 1 |  |
| +1 |  |  | +5 |
|  | 1 | 6 |  |
| +1 |  |  | +5 |
|  | 2 | 11 |  |
| +1 |  |  | +5 |
|  | 3 | 16 |  |

$$\frac{\Delta y}{\Delta x} = \frac{5}{1}$$

$$y = 5x + 1$$

What if you started with $1 and each day the amount of money you have is multiplied by 5?

How would this equation be different?

| Days | $ |
|---|---|
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |
| 3 | 125 |
| 4 | 625 |

+1 +1 +1 +1

>+4 >+20 >+100 >+500

Linear?

$\frac{\Delta y}{\Delta x} = \frac{4}{1} \neq \frac{20}{1} \neq \frac{100}{1} \neq \frac{500}{1}$

**Not Linear because there is no constant slope**

Equation

| Days | $ |
|---|---|
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |
| 3 | 125 |
| 4 | 625 |

+1 +1 +1 +1

>×5 >×5 >×5 >×5

$y = 5^x$ $\quad$ $y = 5^x + 1$

**Which one?**

Try out 2 data points.

(2, 25)

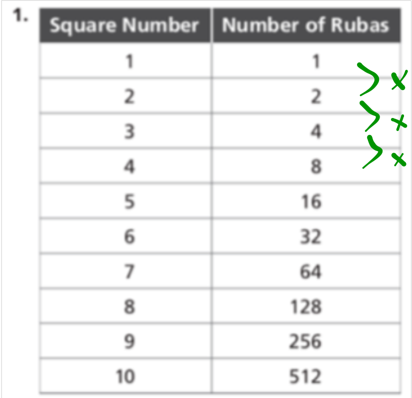
(4, 625)

$y = 5^2 = 25$ $\quad$ $y = 26$

$y = 5^4 = 625$ $\quad$ $y = 626$

$\boxed{y = 5^x}$ $\quad$ $y = 5^x + 1$

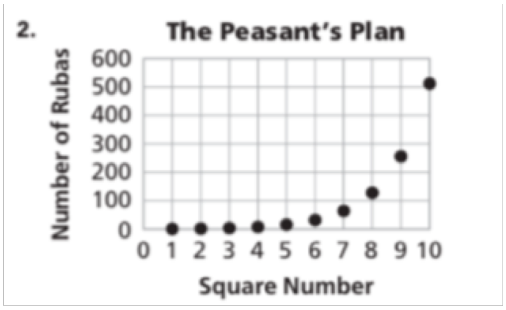
# Problem 1.2 Recap

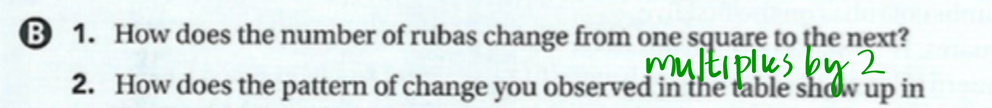


1.

| Square Number | Number of Rubas |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 4 |
| 4 | 8 |
| 5 | 16 |
| 6 | 32 |
| 7 | 64 |
| 8 | 128 |
| 9 | 256 |
| 10 | 512 |

×2
×2
×2



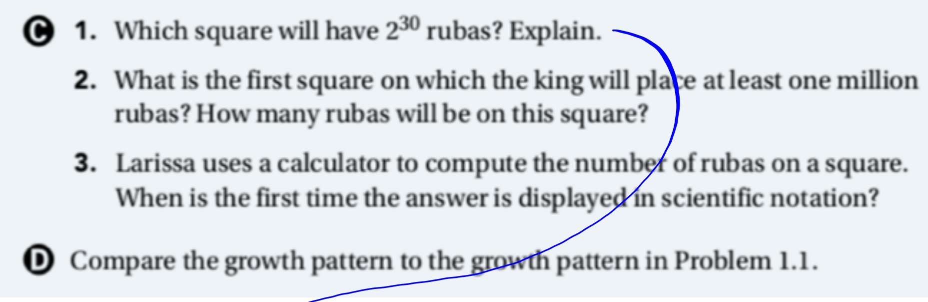


**B** 1. How does the number of rubas change from one square to the next?

multiplies by 2

2. How does the pattern of change you observed in the table show up in the graph? How does it show up in the equation?

$$y = \frac{1}{2} \cdot 2^x = \frac{2^x}{2} = 2^x \div 2$$



**C** 1. Which square will have $2^{30}$ rubas? Explain.

2. What is the first square on which the king will place at least one million rubas? How many rubas will be on this square?

3. Larissa uses a calculator to compute the number of rubas on a square. When is the first time the answer is displayed in scientific notation?

**D** Compare the growth pattern to the growth pattern in Problem 1.1.

$2^{30} = 1{,}073{,}741{,}824$

Use our equation:

30th square

$y = \frac{1}{2}(2^{30}) = 536{,}870{,}912$

only $\frac{1}{2}$ as many rubas as we want on square 30.

$y = \frac{1}{2}(2^{30}) \cdot 2$

We need to multiply by another "2"
which would put us on square 31.

# 1.3 Making a New Offer

## Growth Factors

constantly multiplying by a #

The patterns of change in the number of ballots in Problem 1.1 and in the number of rubas in Problem 1.2 show **exponential growth.** In each case, you can find the value for any cut or square by multiplying the previous value by a fixed number. This fixed number is called the **growth factor.** These relationships are called **exponential functions.** The number of the cut or square is the *independent variable*. The number of pieces of paper or rubas on the square is the *dependent variable*.

the # we are constantly multiplying by

- What are the growth factors for Problems 1.1 and 1.2?

2

## Exponential Growth:

Constantly multiplying by a factor

## Growth Factor:

| x | y |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

+1 +1 +1

×2 ×2 ×2

Growth Factor

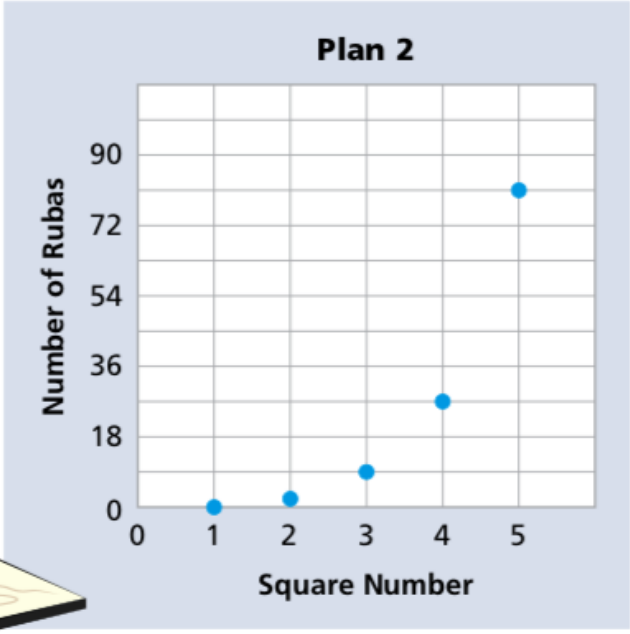
## Exponential Function

Constant factor

The king told the queen about the reward he promised the peasant. The queen said, “You have promised her more money than the entire royal treasury! You must convince her to accept a different reward.”

## Plan 2—The King’s New Plan

After much thought, the king came up with Plan 2. He would make a new board with only 16 squares. Then he would place 1 ruba on the first square and 3 rubas on the second. He drew a graph to show the number of rubas on the first five squares. He would continue this pattern until all 16 squares were filled.



## Plan 3—The Queen's Plan

The queen was unconvinced about the king's new plan. She devised Plan 3. Using a board with 12 squares, she would place 1 ruba on the first square. She would use the equation $r = 4^{n-1}$ to figure out how many rubas to put on each square. In the equation, r ~~is~~ 4 the number of rubas on square n.

$$r = \frac{4^n}{4}$$ ← growth factor

**Plan 1**:

64 squares on the board
1 ruba on first square
2 rubas on second square
4 rubas on third square
8 rubas on fourth square

$y = \frac{2^n}{2}$

**Plan 2**:

16 squares on the board
1 ruba on first square
3 rubas on second square
(see graph)

mult by 3

$y = \frac{3^n}{3}$

**Plan 3**:

12 squares on the board
1 ruba on first square
Would use the equation:

$$r = \frac{4^n}{4}$$

mult by 4

## Problem 1.3

Making a new offer

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**A** **1.** In the table below, Plan 1 is the reward the peasant requested. Plan 2 is the king's new plan. Plan 3 is the queen's plan. Copy and extend the table to show the number of rubas on squares 1 to ~~10~~ 8 for each plan.

**Reward Plans**

| Square Number | Number of Rubas | | |
|---|---|---|---|
| | Plan 1 | Plan 2 | Plan 3 |
| 1 | 1 | 1 | 1 |
| 2 | 2 | 3 | 4 |
| 3 | 4 | ■ | ■ |
| 4 | ■ | ■ | ■ |

## Problem 1.3 continued

**2.** **a.** What are the independent and dependent variables in each plan?

**b.** How are the patterns of change in the number of rubas under Plans 2 and 3 similar to Plan 1? How are they different from Plan 1?

**3.** Do the growth patterns for Plans 2 and 3 represent exponential functions? If so, what is the growth factor for each? Explain.

**Ⓑ** **1.** Write an equation for the relationship between the number of the square $n$ and the number of rubas $r$ for Plan 2.

**2.** Make a graph of Plan 3 for $n = 1$ to ~~10~~ 8. How does your graph compare to the graphs for Plans 1 and 2?

**3.** How is the growth factor represented in the equations and graphs for Plans 2 and 3?

**Ⓒ** The king's financial advisor said that either Plan 2 or Plan 3 would devastate the royal treasury. She proposed a fourth plan.

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### Plan 4—The Financial Advisor's Plan

The advisor proposed Plan 4. The king would put 20 rubas on the first square, 25 on the second, 30 on the third, and so on. He would increase the number of rubas by 5 for each square. He would continue this pattern until all 64 squares are covered.

1. Compare the growth pattern of Plan 4 to Plans 1, 2, and 3. Is the pattern in Plan 4 an exponential function? Explain.
2. Write an equation that represents the relationship in Plan 4.

Ⓓ For each plan, how many rubas are on the final square? List them from least to greatest.

# Homework

Finish classwork