

8-1 Study Guide and Intervention

Multiplying Monomials

Multiply Monomials A **monomial** is a number, a variable, or a product of a number and one or more variables. An expression of the form x^n is called a **power** and represents the product you obtain when x is used as a factor n times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number a and all integers m and n , $a^m \cdot a^n = a^{m+n}$.
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Example 1 Simplify $(3x^6)(5x^2)$.

$$\begin{aligned} (3x^6)(5x^2) &= (3)(5)(x^6 \cdot x^2) && \text{Associative Property} \\ &= (3 \cdot 5)(x^{6+2}) && \text{Product of Powers} \\ &= 15x^8 && \text{Simplify.} \end{aligned}$$

The product is $15x^8$.

Example 2 Simplify $(-4a^3b)(3a^2b^5)$.

$$\begin{aligned} (-4a^3b)(3a^2b^5) &= (-4)(3)(a^3 \cdot a^2)(b \cdot b^5) \\ &= -12(a^{3+2})(b^{1+5}) \\ &= -12a^5b^6 \end{aligned}$$

The product is $-12a^5b^6$.

Exercises

Simplify.

1. $y(y^5)$

$$y^6$$

2. $n^2 \cdot n^7$

$$n^9$$

3. $(-7x^2)(x^4)$

$$-7x^6$$

4. $x(x^2)(x^4)$

$$x^7$$

5. $m \cdot m^5$

$$m^6$$

6. $(-x^3)(-x^4)$

$$x^7$$

7. $(2a^2)(8a)$

$$16a^3$$

8. $(rs)(rs^3)(s^2)$

$$r^2s^6$$

9. $(x^2y)(4xy^3)$

$$4x^3y^4$$

10. $\frac{1}{3}(2a^3b)(6b^3)$

$$4a^3b^4$$

11. $(-4x^3)(-5x^7)$

$$20x^{10}$$

12. $(-3j^2k^4)(2jk^6)$

$$-6j^3k^{10}$$

13. $(5a^2bc^3)\left(\frac{1}{5}abc^4\right)$

$$a^3b^2c^7$$

14. $(-5xy)(4x^2)(y^4)$

$$-20x^3y^5$$

15. $(10x^3yz^2)(-2xy^5z)$

$$-20x^4y^6z^3$$

8-1 Study Guide and Intervention *(continued)*

Multiplying Monomials

Powers of Monomials An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor n times. To find the power of a power, multiply exponents.

Power of a Power	For any number a and all integers m and n , $(a^m)^n = a^{mn}$.
Power of a Product	For any number a and all integers m and n , $(ab)^m = a^m b^m$.

Example

Simplify $(-2ab^2)^3(a^2)^4$.

$$\begin{aligned}
 (-2ab^2)^3(a^2)^4 &= (-2ab^2)^3(a^8) && \text{Power of a Power} \\
 &= (-2)^3(a^3)(b^2)^3(a^8) && \text{Power of a Product} \\
 &= (-2)^3(a^3)(a^8)(b^2)^3 && \text{Commutative Property} \\
 &= (-2)^3(a^{11})(b^2)^3 && \text{Product of Powers} \\
 &= -8a^{11}b^6 && \text{Power of a Power}
 \end{aligned}$$

The product is $-8a^{11}b^6$.

Exercises

Simplify.

- $(y^5)^2 = y^5 \cdot y^5$
 y^{10}
- $(n^7)^4$
 n^{28}
- $(x^2)^5(x^3)$
 x^{13}
- $-3(ab^4)^3$
 $-3a^3b^{12}$
- $(-3ab^4)^3$
 $-27a^3b^{12}$
- $(4x^2b)^3$
 $64x^6b^3$
- $(4a^2)^2(b^3)$
 $16a^4b^3$
- $(4x)^2(b^3)$
 $16x^2b^3$
- $(x^2y^4)^5$
 $x^{10}y^{20}$
- $(2a^3b^2)(b^3)^2$
 $2a^3b^3$
- $(-4xy)^3(-2x^2)^3$
 $512x^9y^3$
- $(-3j^2k^3)^2(2j^2k)^3$
 $72j^{10}k^9$
- $(25a^2b)^3\left(\frac{1}{5}abc\right)^2$
 $625a^6b^5c^2$
- $(2xy)^2(-3x^2)(4y^4)$
 $-48x^4y^6$
- $(2x^3y^2z^2)^3(x^2z)^4$
 $8x^{17}y^6z^{10}$
- $(-2n^6y^5)(-6n^3y^2)(ny)^3$
 $12n^{12}y^{10}$
- $(-3a^3n^4)(-3a^3n)^4$
 $-243a^{15}n^8$
- $-3(2x)^4(4x^5y)^2$
 $-768x^{14}y^2$

$$11) \frac{2n^2}{n} = 2n$$

$$12) \frac{8x^3}{10x^5} = \frac{4}{5x^2}$$

$$13) \frac{12x^3}{9y^8} = \frac{4x^3}{3y^8}$$

$$14) \frac{14x^4y^7}{6x^5y^4} = \frac{7y^3}{3x}$$

$$15) \frac{11u^4}{17u^7v^9} = \frac{11}{17u^3v^9}$$

$$16) \frac{4y^4}{14yx^8} = \frac{2y^3}{7x^8}$$

$$17) \frac{12yx^4}{10yx^8} = \frac{6}{5x^4}$$

$$18) \frac{18x^8y^8}{10x^3} = \frac{9x^5y^8}{5}$$

$$19) \frac{5n^8}{20n^8} = \frac{1}{4}$$

$$20) \frac{16yx^4}{9x^8y^2} = \frac{16}{9x^4y}$$

8-2 Study Guide and Intervention

Dividing Monomials

Quotients of Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers m and n and any nonzero number a , $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer m and any real numbers a and b , $b \neq 0$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$.

Example 1 Simplify $\frac{a^4b^7}{ab^2}$. Assume

neither a nor b is equal to zero.

$$\begin{aligned} \frac{a^4b^7}{ab^2} &= \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right) && \text{Group powers with the same base.} \\ &= (a^{4-1})(b^{7-2}) && \text{Quotient of Powers} \\ &= a^3b^5 && \text{Simplify.} \end{aligned}$$

The quotient is a^3b^5 .

Example 2 Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$.

Assume that b is not equal to zero.

$$\begin{aligned} \left(\frac{2a^3b^5}{3b^2}\right)^3 &= \frac{(2a^3b^5)^3}{(3b^2)^3} && \text{Power of a Quotient} \\ &= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3} && \text{Power of a Product} \\ &= \frac{8a^9b^{15}}{27b^6} && \text{Power of a Power} \\ &= \frac{8a^9b^9}{27} && \text{Quotient of Powers} \end{aligned}$$

The quotient is $\frac{8a^9b^9}{27}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

1. $\frac{5^5}{5^2}$ **5^3 or 125**

2. $\frac{m^6}{m^4}$ **m^2**

3. $\frac{p^5n^4}{p^2n}$ **p^3n^3**

4. $\frac{a^2}{a}$ **a**

5. $\frac{x^5y^3}{x^5y^2}$ **y**

6. $\frac{-2y^7}{14y^5}$ **$-\frac{1}{7}y^2$**

7. $\frac{xy^6}{y^4x}$ **y^2**

8. $\left(\frac{2a^2b}{a}\right)^3$ **$8a^3b^3$**

9. $\left(\frac{4p^4q^4}{3p^2q^2}\right)^3$ **$\frac{64}{27}p^6q^6$**

10. $\left(\frac{2v^5w^3}{v^4w^3}\right)^4$ **$16v^4$**

11. $\left(\frac{3r^6s^3}{2r^5s}\right)^4$ **$\frac{81}{16}r^4s^8$**

12. $\frac{r^7s^7t^2}{s^3r^3t^2}$ **r^4s^4**

8-2

Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

1. $\frac{6^5}{6^4}$ 6^1 or 6

2. $\frac{9^{12}}{9^8}$ 9^4 or 6561

3. $\frac{x^4}{x^2}$ x^2

4. $\frac{r^3s^2}{r^3s^4}$ $\frac{1}{s^2}$

5. $\frac{m}{m^3}$ $\frac{1}{m^2}$

6. $\frac{9d^7}{3d^6}$ $3d$

7. $\frac{12n^5}{36n}$ $\frac{n^4}{3}$

8. $\frac{w^4u^3}{w^4u}$ u^2

9. $\frac{a^3b^5}{ab^2}$ a^2b^3

10. $\frac{m^7n^2}{m^3n^2}$ m^4

11. $\frac{-21w^5u^2}{7w^4u^5}$ $-\frac{3w}{u^3}$

12. $\frac{32x^3y^2z^5}{-8xyz^2}$ $-4x^2yz^3$

8-2 Study Guide and Intervention *(continued)*

Dividing Monomials

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents.

Zero Exponent	For any nonzero number a , $a^0 = 1$.
Negative Exponent Property	For any nonzero number a and any integer n , $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^n} = a^{-n}$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that the denominator is not equal to zero.

$$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right) \left(\frac{a^{-3}}{a^2}\right) \left(\frac{b^6}{b^6}\right) \left(\frac{1}{c^{-5}}\right)$$

Group powers with the same base.

$$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$$

Quotient of Powers and Negative Exponent Properties

$$= \frac{1}{4}a^{-5}b^0c^5$$

Simplify.

$$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$$

Negative Exponent and Zero Exponent Properties

$$= \frac{c^5}{4a^5}$$

Simplify.

The solution is $\frac{c^5}{4a^5}$.

Exercises

Simplify. Assume that no denominator is equal to zero.

- $\frac{2^2}{2^{-3}}$ **25 or 32**
- $\frac{m}{m^{-1}}$ **m^5**
- $\frac{p^{-8}}{p^3}$ **$\frac{1}{p^{11}}$**
- $\frac{b^{-1}}{b^{-5}}$ **b**
- $\frac{(-x^{-1}y)^0}{4w^{-1}y^2}$ **$\frac{w}{4y^2}$**
- $\frac{(a^2b)^2}{(ab)^{-2}}$ **a^6b^8**
- $\frac{x^4y^0}{x^{-2}}$ **x^6**
- $\frac{(6a^{-1}b)^2}{(b^2)^4}$ **$\frac{36}{a^2b^6}$**
- $\frac{s^{-3}t^{-5}}{(2t^3)^{-1}}$ **st^2**
- $\frac{(4m^2n^2)^0}{8m^{-1}t}$ **$\frac{1}{8m^{-1}t}$**
- $\frac{(-2m^2)^{-3}}{4m^{-6}n^4}$ **$-\frac{m^3}{32n^{10}}$**

8-2

Skills Practice

Dividing Monomials

Simplify. Assume that no denominator is equal to zero.

- $\frac{6^5}{6^3}$ **6¹ or 6**
- $\frac{9^{12}}{9^8}$ **9⁴ or 6561**
- $\frac{x^4}{x^2}$ **x^2**
- $\frac{r^3s^2}{r^2s^4}$ **$\frac{1}{s^2}$**
- $\frac{m}{m^3}$ **$\frac{1}{m^2}$**
- $\frac{9d^7}{3d^6}$ **3d**
- $\frac{w^4u^3}{w^4u}$ **u^2**
- $\frac{m^7n^2}{3m^2}$ **m^4**
- $\frac{32x^3y^2z^5}{-8xyz^2}$ **$-4x^2yz^3$**
- $4 \cdot 4^{-4}$ **$\frac{1}{256}$**
- 8^{-2} **$\frac{1}{64}$**
- $\left(\frac{9}{11}\right)^{-1}$ **$\frac{11}{9}$**
- $k^0(k^4)(k^{-6})$ **$\frac{1}{k^2}$**
- $\frac{r^{-7}}{r^4}$ **$\frac{1}{r^{11}}$**
- $\frac{r^{-5}g^4}{h^{-2}}$ **$\frac{g^4h^2}{f^5}$**
- $\frac{15x^6y^{-9}}{5xy^{-11}}$ **$3x^5y^2$**
- $\frac{48x^6y^7z^5}{-6xy^5z^6}$ **$-\frac{8x^5y^2}{z}$**