Situations Modeled with Quadratics

Show all work in your notebook to justify your answers. Indicate which Key Feature you needed to solve for in order to answer the question.

1. The ages of three children in a family can be expressed as consecutive integers. The square of the age of the youngest child is 4 more than 8 times the age of the oldest child. Find the ages of the three children.

Let x = age of youngest
Let $x+1$ = age of middle
Let $x+2$ = age of oldest

$$x^2 = 8(x+2)+4$$
$$x^2 = 8x+16+4$$
$$x^2 = 8x+20$$
$$-x^2 \quad -x^2$$
$$-1[0 = -x^2+8x+20]$$
$$0 = x^2-8x-20$$
$$0 = (x-10)(x+2)$$

$0 = x-10$
$+10 \quad +10$
$10 = x$

$0 = x+2$
$-2 \quad -2$
$-2 = x$ — can't have a negative age

Youngest = x = 10 yrs
Middle = $x+1$ = 11 yrs
Oldest = $x+2$ = 12 yrs

Since we are solving for x we need to calculate the values of the x-intercepts (the roots). We use the second x-intercept because we can't have a negative age.

2. Justin is skeet shooting during the Olympic trials. The height of the skeet is modeled by the equation: $h = -5t^2 + 32t + 2$, where h is the height in meters t seconds after the skeet is released. The path of Justin's bullet is modeled by the equation: $h = 31.5t + 1$ with the same units. How long will it take for the bullet to hit the skeet? How high off the ground will the skeet be when it's been hit?

$h = -5t^2 + 32 + 2$

$h = 31.5t + 1$

$-5t^2 + 32 + 2 = 31.5t + 1$

$-(31.5t + 1) \quad -(31.5t + 1)$

$-5t^2 + 0.5 + 1 = 0$

$$t = \frac{-0.5 \pm \sqrt{(.5)^2 - 4(-5)(1)}}{2(-5)}$$

$$t = \frac{-0.5 \pm \sqrt{20.25}}{-10}$$

$$t = \frac{-0.5 \pm 4.5}{-10}$$

$t = \frac{4}{-10} = -0.4$

$t = \frac{-5}{-10} = 0.5$

The skeet is hit after 0.5 seconds.

$h = 31.5t + 1$

$= 31.5(0.5) + 1$

$= 16.75$ ft

$h = -5t^2 + 32t + 2$

$= -5(.5)^2 + 32(.5) + 2$

$= 16.75$ ft

The skeet will be 16.75 ft above the ground

Again, since we are solving for x we need to calculate the values of the x-intercepts (the roots). We use the second x-intercept because we can't have negative time.

3. The equation for the cost of manufacturing lawn mowers is $C = 0.008x^2 - 0.04x + 75$ where x represents the number of lawn mowers (in thousands). What number of lawnmowers should be produced to minimize costs?

$$C = 0.008x^2 - 0.04x + 75$$

$$LOS = \frac{-b}{2a} = \frac{0.04}{2(0.008)} = 2.5$$

To minimize production costs the company needs to produce 2500 lawn mowers.

The key feature that answers this question is the Line of Symmetry, which is the x-value of the vertex which in this case is a minimum.

4. In an effort to catch a criminal; a superhero is going to leap over a building and take a short cut down the ally. The function $f(x) = -16x^2 + 150x$ gives the superhero's height in feet as a function of time. The building is 425 feet high. Will the superhero make it over the building?

Show work to prove or disprove each statement.

$f(x) = -16t^2 + 150x$

$-16t^2 + 150x \stackrel{?}{=} 425$

$-425 \quad -425$

$-16t^2 + 150x - 425 = 0$

a. Yes, the superhero always makes it!

b. No, the superhero can only jump half the height of the building

c. No, the superhero will crash into the building at 351 feet

d. No, the superhero can only jump 150 feet into the air

The Discriminant will tell us if there are solutions when h = 425.

$b^2 - 4ac = 150^2 - 4(-16)(-425)$

$= -4700$ → There are no real solutions

The superhero cannot make it over the building.

The y-value of the vertex will indicate how high the superhero can jump.

$LOS = \frac{-150}{(2)(-16)} = 4.6875$

$h = -16x^2 + 150x$

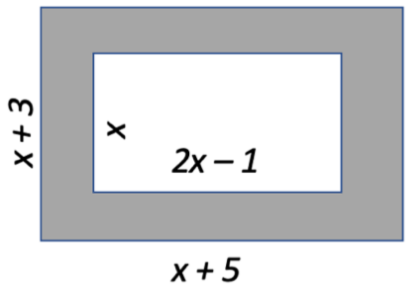
$h = -16(4.6875)^2 + 150(4.6875)$

$h = 351.5625$

The superhero can jump 351.5625 ft. high

b. False
c. True
d. False

5. If the shaded area is 23 cm^2, then what is the *smallest* possible value of the inner area?



$(x+3)(x+5) = x(2x-1)+23$

$x^2+8x+15 = 2x^2-x+23$

$-(x^2+8x+15) \quad -(x^2+8x+15)$

$0 = x^2-9x+8$

$0 = (x-8)(x-1)$

$0 = x-8$

$+8 \quad +8$

$8 = x$

$0 = x-1$

$+1 \quad +1$

$1 = x$

This will give the smallest area.

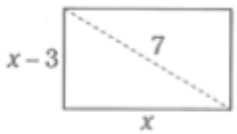
To solve for x you need to determine the x-intercepts.

Inner area $= x(2x-1)$

$= (1)(2(1)-1)$

$= 1 \text{ unit}^2$

6. A rectangular flag is designed so that the width is 3 feet less than the length. The diagonal measure of the flag is 7 feet. Find the length and the width.



$a^2+b^2=c^2$

$(x-3)^2+x^2=7^2$

$x^2-6x+9+x^2=49$

$2x^2-6x+9=49$

$-49 \quad -49$

$\frac{2x^2}{2}-\frac{6x}{2}-\frac{40}{2}=\frac{0}{2}$

$x^2-3x-20=0$

$+20 \quad +20$

$x^2-3x=20$

$+2.25 \quad +2.25$

$\sqrt{(x-1.5)^2}=\sqrt{22.25}$

$x-1.5=\pm 4.72$

$+1.5 \quad +1.5$

$x=1.5+4.72=6.22$

$x=1.5-4.72=-3.22$

can't have a negative length

To solve for x you need to determine the x-intercepts.

Length = x = 6.22 ft
Width = x − 3 = 3.22 ft

7. A small independent motion picture company determines the profit P for producing n DVD copies of a recent release is $P = -0.02n^2 + 3.40n - 16$. P is the profit in thousands of dollars and n is the number of DVDs in thousands.

a. How many DVDs should the company produce to maximize the profit?

b. What will the maximum profit be?

$P = -0.02n^2 + 3.4n - 16$

a. $LOS = \frac{-b}{2a} = \frac{-3.4}{2(-.02)} = 85$

The Line of Symmetry will tell you the number of DVDs that will result in the maximum profit.

Producing 85,000 DVD's will result in the greatest profit.

b. $P = -0.02n^2 + 3.4n - 16$

$P = -0.02(85)^2 + 3.4(85) - 16$

$P = 128.5$

The y-value of the vertex will tell you the maximum profit.

The maximum profit will be $128,500

8. A pelican flying in the air over water drops a crab from a height of 30 feet. The distance the crab is from the water as it falls can be represented by the function $h(t) = -16t^2 + 30$, where t is time, in seconds.

   To catch the crab as it falls, a gull flies along a path represented by the function $g(t) = -8t + 15$.

   Can the gull catch the crab before the crab hits the water? (If yes, how long after the crab is dropped, and how high above the water?)

$h = -16t^2 + 30$ ← crab falling

$h = -8t + 15$ ← gull flight path

$$-16t^2 + 30 = -8t + 15$$
$$+8t \quad +8t$$
$$-16t^2 + 8t + 30 = 15$$
$$-15 \quad -15$$
$$-16t^2 + 8t + 15 = 0$$

To solve for x you need to determine the x-intercepts.

$$t = \frac{-8 \pm \sqrt{8^2 - 4(-16)(15)}}{2(-16)}$$

$$t = \frac{-8 \pm \sqrt{1024}}{-32}$$

$$t = \frac{-8 + 32}{-32} = -0.75$$ can't have negative time

$$t = \frac{-8 - 32}{-32} = 1.25$$

The crab is caught 1.25 sec. after it is dropped.

$h = -16t^2 + 30$

$h = -16(1.25)^2 + 30$

$h = 5$

$h = -8t + 15$

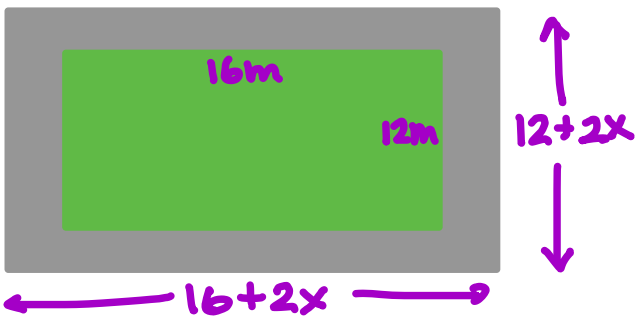
$h = -8(1.25) + 15$

$h = 5$

The gull will catch the crab 5 feet above the water.

9. A garden measuring 12 meters by 16 meters. A pedestrian pathway will be installed all around it, increasing the total area (walkway + garden) to 285 square meters. What will be the width of the pathway? (Hint: Draw a picture to help.)

x = width of walkway



Total Area = $(2x+16)(2x+12)$

$285 = 4x^2 + 56x + 192$

$-285 \qquad -285$

$0 = 4x^2 + 56x - 93$

$0 = x^2 + 14x - 23.25$

$+23.25 \qquad +23.25$

$23.25 = x^2 + 14x$

$+49 \qquad +49$

$72.25 = (x+7)^2$

$\sqrt{72.25} = \sqrt{(x+7)^2}$

$\pm 8.5 = x + 7$

$-7 \qquad -7$

$-7 \pm 8.5 = x$

$x = -15.5$ → can't have a negative length

$x = 1.5$

To solve for x you need to determine the x-intercepts.

The walkway is 1.5 meters wide.