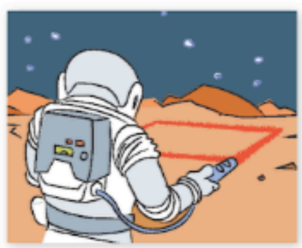
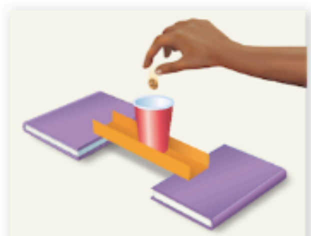
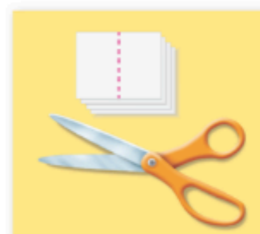
Investigation 1

The Families of Functions

In earlier Units you studied real-world situations that involved relationships between variables. Here are some examples:

- Distance traveled by riders on a bike tour related to the time riding
- Area of a rectangular piece of land with a fixed perimeter related to the width
- Strength of a bridge related to the length
- Area of ballots cut from a piece of paper related to the number of cuts

In each situation you asked these questions:

- What are the variable quantities?
- What measurement units are appropriate for the situation?

Common Core State Standards

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

F-IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-BF.B.4a Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Also N-Q.A.1, N-Q.A.2, F-IF.A.2, F-IF.B.4, F-IF.B.5, F-IF.B.6

The relationships in the examples from earlier Units are called *functions*. Some of the types of functions you studied are expressed with equations:

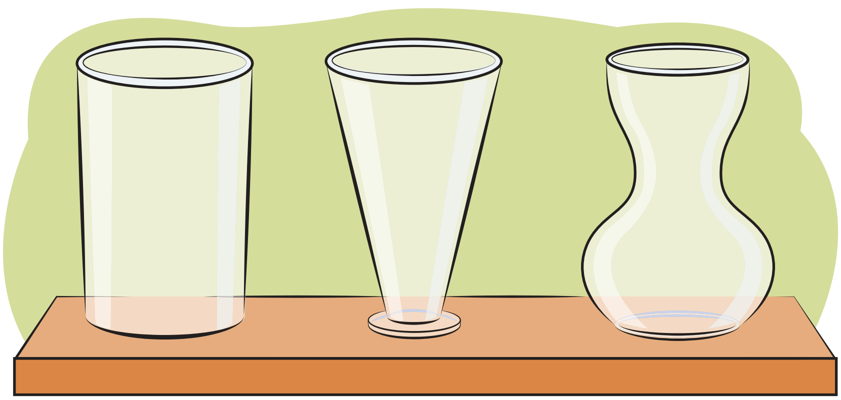
- Linear functions, with equations in the form $y = mx + b$
- Quadratic functions, with equations in the form $y = ax^2 + bx + c$
- Inverse variation functions, with equations in the form $y = \frac{k}{x}$
- Exponential functions, with equations in the form $y = a(b^x)$

In each case, every value of the independent variable x is related to exactly one value of the dependent variable y. Any relationship between variables with that property is called a **function**. Many functions have rules that are quite different from those that you have studied so far.

The Problems in this Investigation introduce the standard terminology and notation for functions. You will also study new families of special functions.

1.1 Filling Functions

Not all functions can be defined by simple algebraic rules. Suppose you fill each of these containers with water flowing in at a constant rate. Then you study the results.

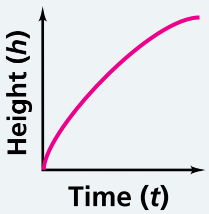


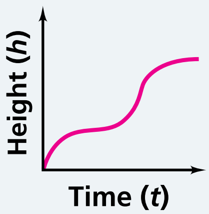
- What variables does this situation suggest?
- What are the best units to measure the quantities involved?
- What would tables, graphs, or equations of each of the functions related to filling these containers look like?

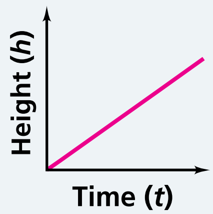
Look at each container pictured above. The height of water at any time depends on the length of time the water has been flowing into it. There is exactly one water height at each point in time. So height is a function of time.

Problem 1.1

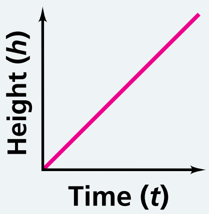
A Match each of these (*time, water height*) graphs to one of the containers on the preceding page. Explain why each graph shows the rate of filling for one of the containers. Assume the water flows at a constant rate.

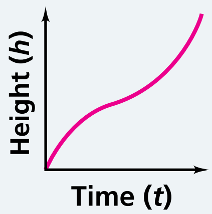


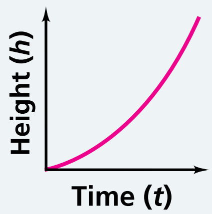




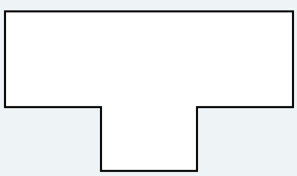
B Sketch containers that would have functions for rate of filling that would match these graphs.

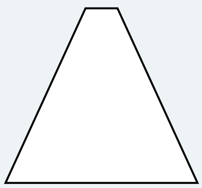


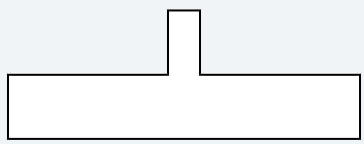




C The drawings below show a two-dimensional view of containers that have a constant dimension of 1 inch from front to back. Again, the water flows at a constant rate. Sketch graphs of the functions for rate of filling for these containers.







D Do the graphs in Questions A–C represent any functions you have seen in previous Units? Explain.

E Delsin and Christina are analyzing the graphs from Question C. Delsin says those graphs show volume as a function of time. Christina says that graphs showing how volume changes over time would be three identical graphs. They would be linear graphs for all three containers. She thinks the graphs show how height changes over time. Who is right? Explain.

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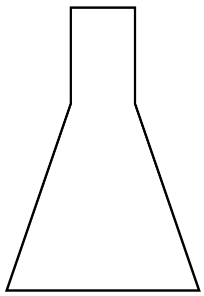
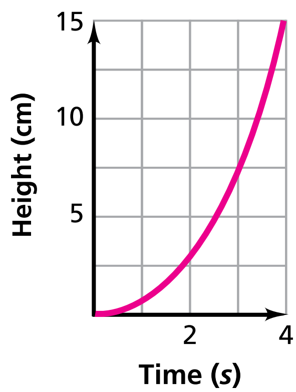
1.2 Domain, Range, and Function Notation

The drawing below shows the side view of a lab flask that you might use in chemistry class. The graph shows how the height of water in the flask changes with time as a steady flow of water is poured into it. The relation between the *height* of the water and *time* is a function.





Mathematicians use two special terms, *domain* and *range*, to describe any function.

- The **domain** of a function is the set of all possible values for the input variable. For the filling function, the domain is the set of all times for which there is a related height.
- The **range** of a function is the set of all possible values for the output variable. For the filling function, the range is the set of heights from zero to the top of the container.

A function assigns to each element of the domain exactly one element of the range. The assignment process is commonly written in symbolic form. The notation $h(t)$ is read, "the value of height h at time t." The equation $h(2) = 3$ tells you that the height of the water in the flask is 3 centimeters after 2 seconds of pouring. The notation $h(t)$ does *not* mean h times t.

- What is the domain of the height function shown in the graph?
- What range makes sense for the function shown in the graph?
- What is each equation asking and what value would you insert to make it a true statement?

$h(0) = \square$ $\qquad h(2) = \square$ $\qquad h(\square) = 1$ $\qquad h(\square) = 10$

Function notation *f(x)* is another way of expressing rules for functions. For example, the function $y = 1.5x - 4$ can be written as $f(x) = 1.5x - 4$. The notation shows the relationships between the independent and dependent variables in a short form. The sentence $f(10) = 11$ tells you that the value of y is 11 when the value of x is 10.

Problem 1.2

A For each of the following examples, describe a domain and a range that make sense for the function. Then use function notation to complete the given sentences.

1. **a.** Suppose $f(x) = 1.5x - 4$. What are the domain and range of $f(x)$?

b. $f(3) = \blacksquare$ **c.** $f(-2) = \blacksquare$ **d.** $f(\blacksquare) = 5$

e. $f(\blacksquare) = -1$ **f.** $f(n) = \blacksquare$ **g.** $f(x + 1) = \blacksquare$

2. **a.** Suppose $g(x) = \sqrt{x}$. What are the domain and range of $g(x)$?

b. $g(9) = \blacksquare$ **c.** $g(49) = \blacksquare$ **d.** $g(\blacksquare) = 5$

e. $g(t) = \blacksquare$ **f.** $g(x - 5) = \blacksquare$

3. **a.** Suppose $h(x) = 5(2^x)$. What are the domain and range of $h(x)$?

b. $h(-3) = \blacksquare$ **c.** $h(0) = \blacksquare$ **d.** $h(\blacksquare) = 40$

e. $h(m) = \blacksquare$ **f.** $h(3x) = \blacksquare$

B The graph of any function f is the set of all points with coordinates $(x, f(x))$, or the set of all points (x, y) that satisfy the equation $y = f(x)$. Sketch graphs of the three functions defined in Question A.

C Not all relationships between variables are functions. For example, the next table shows age in years and height in inches of students on a middle school basketball team.

Heights and Ages

Age (years)	11	11	12	12	12	13	13	13	13	14
Height (inches)	60	58	62	67	62	65	68	72	68	70

Why is *height* not a function of *age* in this case?

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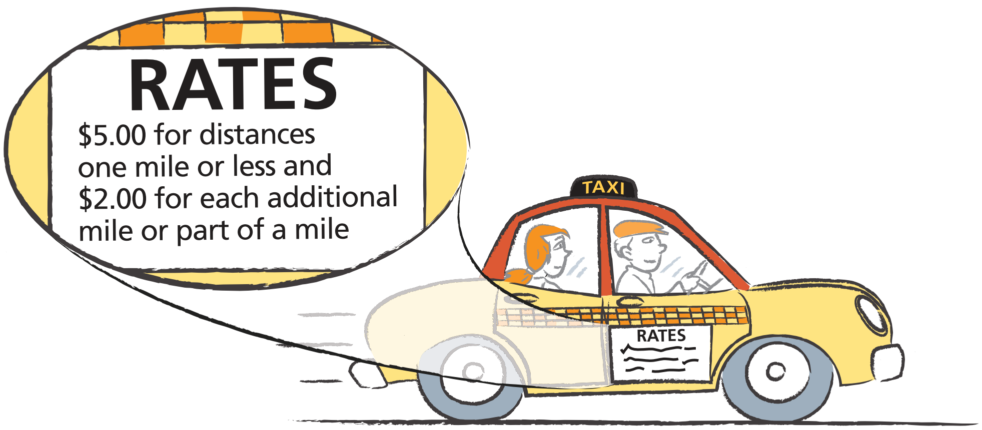
1.3 Taxi Fares, Time Payments, and Step Functions

In many important applications of mathematics a function is defined in words. However, it may not be easy to write a simple algebraic expression for that function. For example, the fare for riding in a taxicab might be calculated with a function such as the one described on the cab below:



In this taxi you would pay $5.00 for any trip 1 mile or less in length. You'd pay $7.00 for any trip longer than 1 mile but not longer than 2 miles. You'd pay $9.00 for any trip longer than 2 miles but not longer than 3 miles, and so on.

- Suppose you made a table and graph for the function relating taxicab fare to distance traveled. What patterns would you expect to see?

Analyzing relationships like the taxicab fare scheme is easier if you calculate some sample values and look for a pattern.

Problem 1.3

A **1.** Complete the table at the right to show taxicab fares for trips from 1 to 8 miles.

Taxi Fares

Distance (mi)	1	2	3	4	5	6	7	8
Fare ($)	5	7						

2. Graph the data points in the table of sample taxi fares. Complete the graph so it shows the fares for any distances between 0 and 8 miles.

3. Does the completed graph represent a function? If not, why not? If it does, what are its domain and range?

Problem 1.3 continued

B When stores want to sell things quickly, they offer deals that spread payments over many months with 0% interest. Suppose that you buy a new bicycle for $240. You get a deal that requires $10 monthly payments.

1. Complete the following table to show the unpaid balance for months 0 to 7.

Account Balance

Month	0	1	2	3	4	5	6	7
Unpaid Balance ($)	240	230	■	■	■	■	■	■

2. Graph the data points in the table of unpaid balances. Then complete the graph so it shows the unpaid balance at any time between 0 and 7 months.

3. Does the completed graph represent a function? If not, why not? If it does, what are its domain and range?

C In many quantitative problems it makes sense to round numbers to a nearby whole-number value. There are three common ways to do this rounding:

- The *standard rounding rule* takes the integer closest to x, with values exactly halfway between integers rounded up
- The *ceiling rule* takes the nearest integer greater than or equal to x
- The *floor rule* takes the nearest integer less than or equal to x

1. Complete the following table of values for the standard, ceiling, and floor rounding rules.

Comparing Standard, Ceiling, and Floor Functions

x	−3.25	−2.75	−0.5	0.6	1.7	2.21	3.5
Standard	■	■	■	■	■	■	■
Ceiling	■	■	■	■	■	■	■
Floor	■	■	■	■	■	■	■

2. Pick one of the three rules and draw a graph of its values for $x = -3$ to $x = 3$.

3. Do any of the rules define functions? If not, why not? If so, what are the domain(s) and range(s)?

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Problem 1.3 continued

D The relationships between variables that you analyzed in Questions A–C are examples of a special type of functions called **step functions.**

1. How do the graphs show why the step function name makes sense?
2. Why is it hard to give an algebraic expression for calculating values of such functions?

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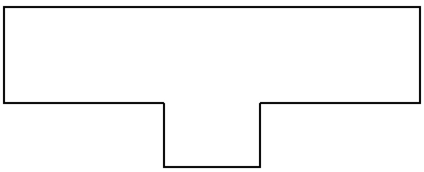
1.4 Piecewise-Defined Functions

Suppose the container below has constant width from front to back. Water pours into it at a constant rate.



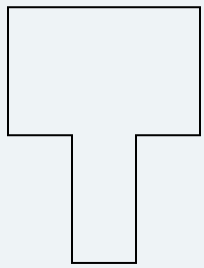
- How rapidly will the water level rise?
- What sort of graph would you expect for the function relating water height to time?
- What kind of algebraic rule would express the height function?

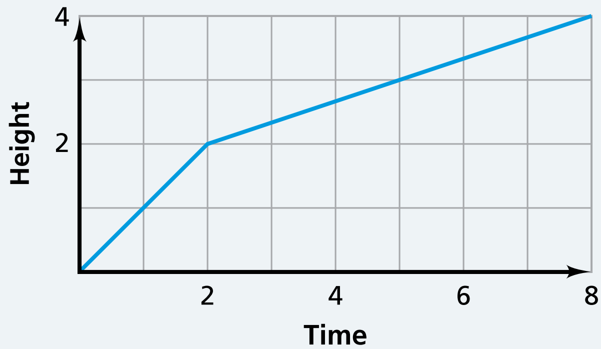
There is a function that describes the rate at which water fills the container pictured above. You can write rules that relate the dependent and independent variables for that function. However, the rules or expressions for that function and others like it are different on different pieces of the domain. Such relationships are called **piecewise-defined functions.**

The following Problem will enhance your understanding of piecewise-defined functions. You will work with some function rules that are stated in words and others given as algebraic expressions.

Problem 1.4

A The graph below models the rate at which the water level would rise in the container pictured to its left.





1. Why does it make sense that the graph should consist of two different linear pieces?
2. What algebraic expressions would you use to model the pieces of the rate of filling function?

B The absolute value function has the following piecewise definition:

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

1. Copy and complete this table of values for the absolute value function $v(x) = |x|$.

x	−3	−2.5	−2	−1.5	−1	0	1	1.5	2	2.5	3
$v(x) = \|x\|$	3	■	■	■	■	■	■	■	■	■	■

2. Use the data for the points $(x, |x|)$ to draw a graph of $v(x)$.

C Use results from Question B to draw graphs of these variations on the absolute value function.

1. $a(x) = |x| + 1$
2. $b(x) = |x| - 2$
3. $c(x) = v(x) - 1$
4. Explain how these graphs are related to the graph of $v(x) = |x|$.

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Problem 1.4 continued

D Competitors in iron-man triathlon races swim, bicycle, and run a total of 140.6 miles. Suppose that one racer's pace in each part of the race was as shown below.

1. What were the racer's average speeds for swimming, biking, and running?
2. Sketch a graph showing the racer's progress over the whole race. Assume that the pace on each segment of the race is constant .
3. What are the domain and range of the function graphed in part (2)?
4. Based on your graph, estimate the racer's average rate for the entire triathlon.

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1.5 Inverse Functions

If an airplane averages 500 miles per hour in flight, you know that the time and distance traveled are related. Look at the two tables below. Numbers in the first table show distance as a function of time. Numbers in the second table show time as a function of distance.

The equations $d = 500t$ and $f(t) = 500t$ both show how distance traveled is related to time in flight.

Using units of measure helps to keep track of the domain and range. Here is an example.

2,000 miles $=$ 500 mph $\times$ 4 hours

Time and Distance

Time (hours)	Distance (miles)
0	0
1	500
2	1,000
3	1,500
4	2,000
■	■
t	$500t$

The equations $t = d \div 500$ and $g(d) = d \div 500$ both show how time in flight is related to distance traveled.

- How do units of measure help make sense of this sentence?

  4 hours $=$ 2,000 miles $\div$ 500 mph

Distance and Time

Distance (miles)	Time (hours)
0	0
500	1
1,000	2
1,500	3
2,000	4
■	■
d	■

The functions $f(x) = 500x$ and $g(x) = x \div 500$ are related in a very special and useful way. The function $g(x)$ is the **inverse function** of $f(x)$, and $f(x)$ is the inverse function of $g(x)$.

- Why does the term *inverse* make sense in describing how functions $f(x)$ and $g(x)$ are related?

Working on this Problem will develop your ability to find and use inverses of familiar functions.

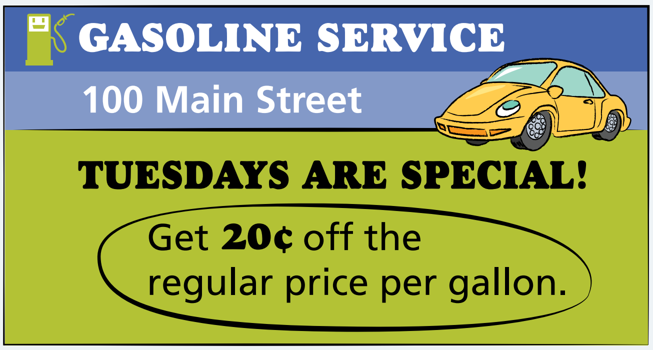
Problem 1.5

For the situations described in Questions A–E, do the following:

- Write an equation that shows how the two variables are related.
- Use function notation to write an equation that shows the same relationship.
- Write the equation and function that show the inverse relationship of the two variables.
- Explain what the function and its inverse tell about the related variables.

A A bus averages 50 miles per hour on the highway. How is the distance covered d, in miles, related to the driving time t, in hours?

B A gas station offers the price shown in the advertisement below. How is the price per gallon on Tuesday T related to the price D on other days of the week?



C The typical customer at the Spartan Deli buys food that costs $7.50. How is the Deli's daily income I related to the number of customers n?

D
1. The Spartan Deli has operating expenses of $850 per day. How is the Deli's daily profit P related to the number of customers n?
2. Amy thinks that the answer for the inverse is $n = P \div 7.50 + 850$. Becky says that the units needed for the expression on the right side of this equation would not give a number of people. Who do you agree with? Explain.

Problem 1.5 continued

E How is the area A of any square related to the length s of its sides?

F For any function $f(x)$, the inverse function is shown by the function notation $f^{-1}(x)$. The notation $f^{-1}(x)$ is read, "f inverse of x."

Suppose a function $f(x)$ tells the range value corresponding to each domain value. Then the inverse of that function reverses this relationship. Values that were in the domain are now in the range, and vice versa.

Find inverses for the functions below and explain how you know that your answers are correct.

Hint: You might find it helpful to write each function as an equation using y to name the dependent variable. For example, $f(x) = 3x$ could be written as $y = 3x$. Then the task is to find an expression that shows how to calculate x when given y.

1. $f(x) = 3x$
2. $g(x) = x + 7$
3. $h(x) = 3x + 7$
4. $j(x) = x - 7$
5. $k(x) = x^2$
6. $m(x) = \frac{1}{x}$
7. $n(x) = x^3$

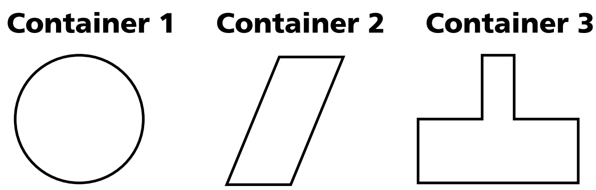
G For each function in Question F, do the following:

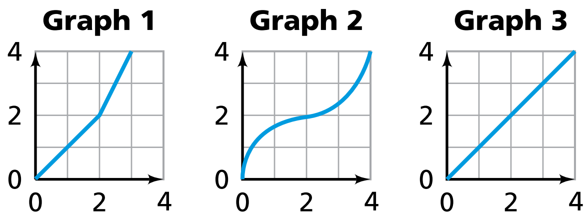
- Describe the domain and range of the function.
- Describe the domain and range of the inverse function.
- Explain any ways that the domain of the original function must be limited if it is to have the proposed inverse.
- Sketch a graph of the function and a graph of its inverse. Use separate coordinate axes for each pair of graphs.

ACE Homework starts on page 20.

Applications

1. Suppose each container below is filled at a constant rate with water. Match each container with the graph that represents the relationship between the height of the water in the container and time.

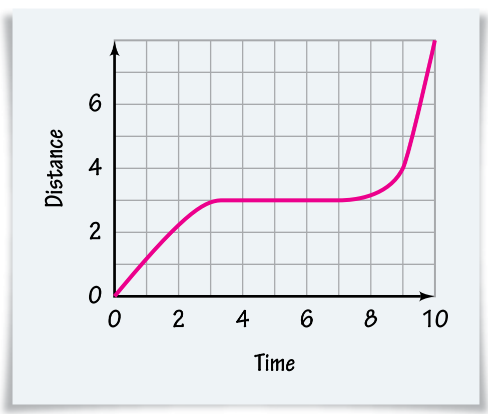




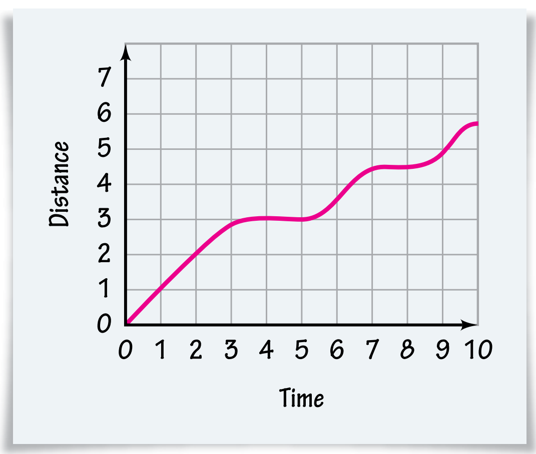
2. The graphs below show the pattern of time and distance traveled by two school buses. Make a copy of each graph. On copies of each graph mark the following intervals:

- when the bus is speeding up
- when the bus is slowing down
- when the bus is moving at a constant speed
- when the bus is stopped

a. Bus A

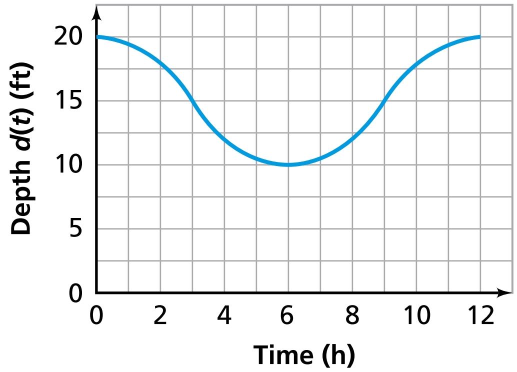


b. Bus B



3. Ocean water levels rise and fall with a tidal period of about 12 hours. The graph below shows water depth at the end of a pier in a seacoast city.

Water Depth Over Twelve Hours



The function $d(t)$ gives water depth at time t hours after midnight. Use the graph to complete the following sentences and explain what each sentence tells about water depth.

a. $d(0) = \blacksquare$

b. $d(2) = \blacksquare$

c. $d(4) = \blacksquare$

d. $d(6) = \blacksquare$

e. $d(9) = \blacksquare$

f. $d(\blacksquare) = 15$

4. What are the domain and range of the function for water depth shown above?

Complete the sentences to give correct statements.

5. $f(x) = -2x + 5$

a. $f(7) = \blacksquare$ b. $f(-3) = \blacksquare$ c. $f(\blacksquare) = 17$

6. $g(x) = x^2 + 5x$

a. $g(7) = \blacksquare$ b. $g(-3) = \blacksquare$ c. $g(\blacksquare) = 6$

7. $h(x) = 4(0.5)^x$

a. $h(2) = \blacksquare$ b. $h(-1) = \blacksquare$ c. $h(\blacksquare) = 4$

8. Describe the domain and range for the functions $f(x) = -2x + 5$, $g(x) = x^2 + 5x$, and $h(x) = 4(0.5^x)$.

9. For each function, sketch the graph and describe the domain and range.

a. $f(x) = 4x + 5$ **b.** $g(x) = x^2 + 2$ **c.** $h(x) = 2^x$ **d.** $j(x) = \frac{1}{x}$

10. Determine if the relationship in each table shows that y is a function of x.

a.

x	2	3	4	5	6
y	4	7	10	13	16

b.

x	3	4	1	−1	2
y	4	3	−1	1	2

c.

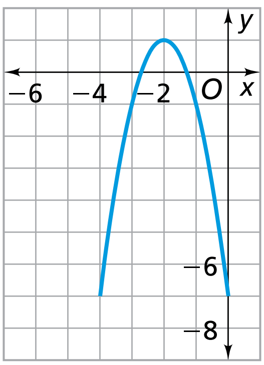
x	0	2	4	3	0
y	1	3	5	7	9

d.

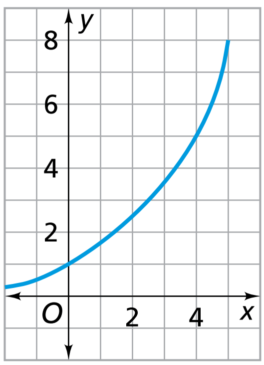
x	−4	−3	−1	1	2
y	0	1	1	1	0

For Exercises 11 and 12, use the graphs below.

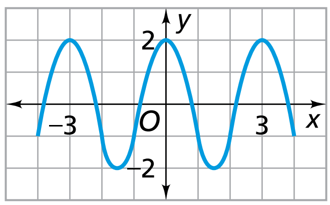
$f(x)$



$g(x)$



$h(x)$



11. Identify the domain and range of each function.

12. Use the graphs to complete these sentences that use function notation.

a. $f(-4) = $ ■ **b.** $g(4) = $ ■ **c.** $h(1) = $ ■

d. $f($■$) = 1$ **e.** $g($■$) = 1$ **f.** $h($■$) = 2$

13. The fee for airport parking is shown below. For parts (a)–(f), calculate the cost of parking $c(t)$ for the given times.

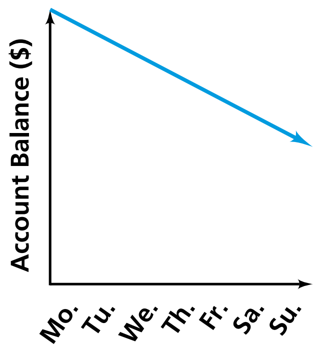
a. $c(0.5)$ **b.** $c(1.0)$ **c.** $c(2.5)$

d. $c(5.0)$ **e.** $c(5.5)$ **f.** $c(8.0)$

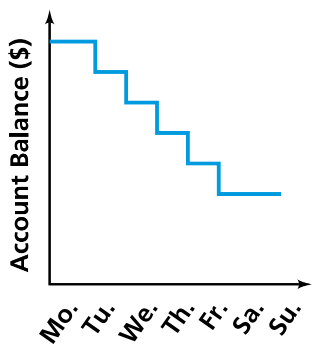
g. Draw a graph showing the charges for any time from 0 to 8 hours.

14. **Multiple Choice** At Akihito's school, lunches cost $1.25. Akihito starts the school year with a school lunch account balance of $100. Which graph best represents the pattern of change in Akihito's account balance during a typical week?

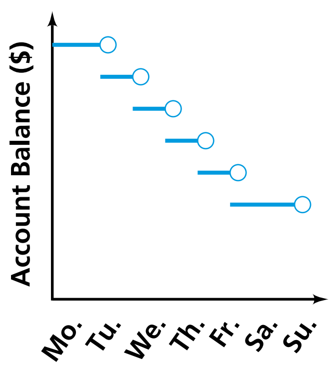
A.



B.



C.



D. None of these

15. Suppose $r(x)$ is the function that applies the standard rounding rule to numbers. Also, $c(x)$ applies the ceiling rule for rounding, and $f(x)$ applies the floor rule for rounding. Complete the following sentences to give correct statements.

a. $r(1.6) = \square$ **b.** $c(1.6) = \square$ **c.** $f(1.6) = \square$

d. $r(-1.6) = \square$ **e.** $c(-1.6) = \square$ **f.** $f(-1.6) = \square$

g. $r(-1.3) = \square$ **h.** $c(-1.3) = \square$ **i.** $f(-1.3) = \square$

j. $r(\square) = -2$ **k.** $c(\square) = -2$ **m.** $f(\square) = -2$

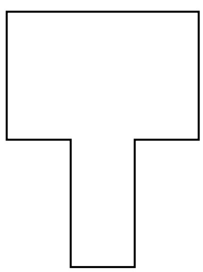
16. Graph each of the following piecewise functions.

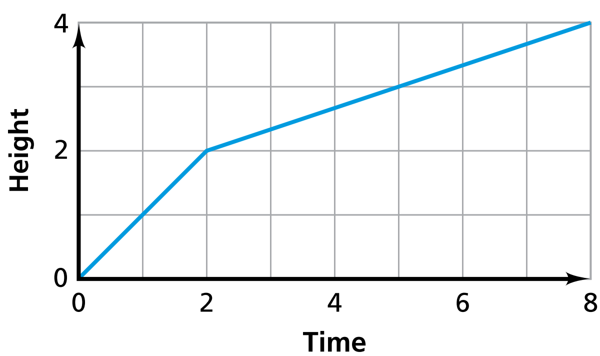
a. $y = \begin{cases} x^2 \text{ if } x \leq 0 \\ 3x \text{ if } x > 0 \end{cases}$

b. $y = \begin{cases} \frac{1}{2}x + 4 \text{ if } x < 0 \\ -\frac{1}{2}x + 4 \text{ if } x \geq 0 \end{cases}$

c. $y = \begin{cases} 4 \text{ if } x < 2 \\ 2^x \text{ if } x \geq 2 \end{cases}$

17. The figure and graph below show the function for rate of filling for the container from Problem 1.4.



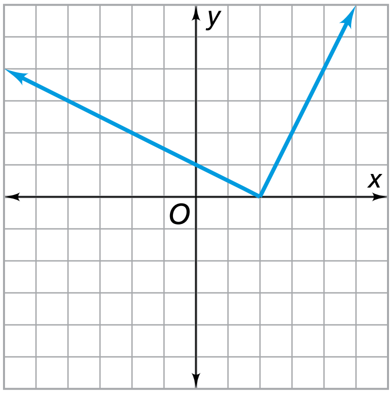


Suppose that the height of the water is measured in inches and the filling time in seconds. At what rate is water height rising during the following intervals?

a. the first two seconds

b. the time from 2 to 8 seconds

18. Desheng and Chelsea are trying to write a piecewise function rule for the following graph.



Desheng says the y-intercept is 1, the slope of the left part is $-\frac{1}{2}$, and the slope of the right part is 2. He writes the following function rule.

$$y = \begin{cases} -\frac{1}{2}x + 1 \text{ if } x < 2 \\ 2x + 1 \text{ if } x \geq 2 \end{cases}$$

Chelsea says the left part of the graph has slope $-\frac{1}{2}$ and y-intercept of 1. The right part of the graph has slope 2. It would have a y-intercept of -4 if the graph extended to intersect the y-axis. She writes the following function rule.

$$y = \begin{cases} -\frac{1}{2}x + 1 \text{ if } x \leq 2 \\ 2x - 4 \text{ if } x > 2 \end{cases}$$

Whose reasoning is correct? Explain.

19. Find inverses for these functions.

a. $f(x) = 6x$ **b.** $g(x) = x - 4$ **c.** $h(x) = 6x - 4$

d. $j(x) = \sqrt{x}$ **e.** $k(x) = 4x^2$ **f.** $m(x) = -\frac{3}{x}$

20. For each function in Exercise 19, do the following.

- Describe the domain and range of the function.
- Describe the domain and range of the inverse function.
- Explain any ways that the domain of the inverse function differs from the domain of original function. (In what ways must the domain of the inverse function be limited?)

Connections

21. For $f(x) = \sqrt{x}$, evaluate each of the following.

a. $f(121)$ **b.** $f\left(\frac{1}{4}\right)$ **c.** $f\left(m^2\right)$ **d.** $f\left(4q^2\right)$

22. For $f(x) = 4^x$, complete the following sentences.

a. $f(4) = \blacksquare$ **b.** $f(-2) = \blacksquare$ **c.** $f(\blacksquare) = 1$

d. $f(\blacksquare) = 2$ **e.** $f(a) = \blacksquare$ **f.** $f(b + 2) = \blacksquare$

23. For $g(x) = x^2$, evaluate each of the following expressions.

a. $g(3)$ **b.** $g\left(\frac{1}{2}\right)$ **c.** $g(-d)$ **d.** $g\left(\frac{n}{2m}\right)$

24. For $j(x) = \frac{1}{2}x$, evaluate each of the following expressions.

a. $j(-7)$ **b.** $j(0)$ **c.** $j(2s)$ **d.** $j\left(\frac{r}{t}\right)$

25. Linear, quadratic, and exponential functions all have as their domains the set of real numbers.

a. Why is the domain of $r(x) = \sqrt{x}$ not all real numbers?

b. Why is the domain of $s(x) = \frac{1}{x}$ not all real numbers?

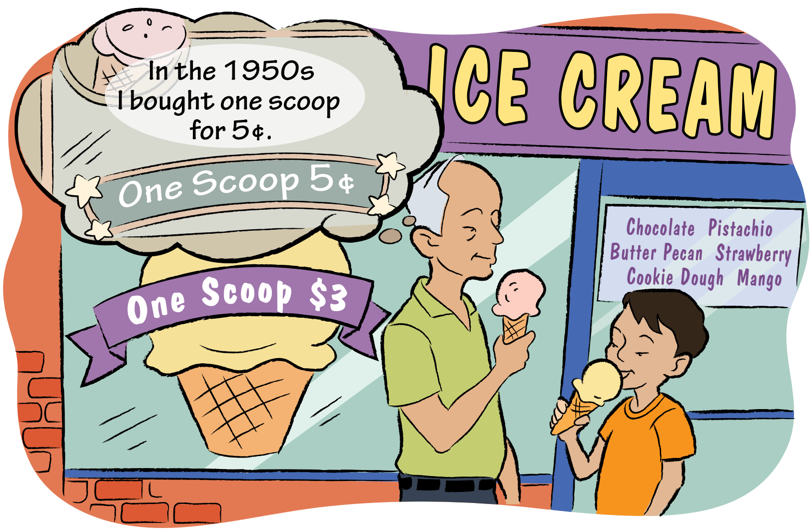
26. Many variables in your life change as time passes. Tell whether any of the following changes shows a pattern like a step function. **Hint:** More than one pattern may be a step function.

a. your height

b. the price of a one-scoop cone of ice cream

c. your age in years

d. the number of questions left to answer as you work on homework



27. For each of the following numeric equations, write the other equations in the fact family.

a. $7 + 12 = 19$ **b.** $4 \times 3 = 12$

28. Addition and subtraction are inverse operations. Multiplication and division are also inverse operations. How do those inverse operation labels relate to the inverse functions in Problem 1.5?

29. Solve each of these equations using ideas about fact families and inverse operations.

a. $x + 7 = 12$ **b.** $5x = 35$ **c.** $5x + 7 = 82$

d. $\frac{7}{x} = 12$ **e.** $\frac{9}{x-2} = 3$ **f.** $\frac{5}{x} + 7 = 8$

Extensions

30. **a.** Copy and complete the following table of values.

Variations of the Ceiling Rounding Function *c*(*x*)

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$c(x) - x$	■	■	■	■	■	■	■	■	■	■	■
$x - c(x)$	■	■	■	■	■	■	■	■	■	■	■

b. Using data in the table, draw a graph of $c(x) - x$ from $x = 0$ to $x = 5$.

c. Using data in the table, draw a graph of $x - c(x)$ from $x = 0$ to $x = 5$.

d. What can you conclude from the two graphs?

31. **a.** Copy and complete the following table of values.

Variations of the Floor Rounding Function *f*(*x*)

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$f(x) - x$	■	■	■	■	■	■	■	■	■	■	■
$x - f(x)$	■	■	■	■	■	■	■	■	■	■	■

b. Using data in the table, draw a graph of $f(x) - x$ from $x = 0$ to $x = 5$.

c. Using data in the table, draw a graph of $x - f(x)$ from $x = 0$ to $x = 5$.

d. What can you conclude from the two graphs?

32. What are the domain and range of the ceiling function $c(x)$?

33. Sketch graphs for each of these pairs of functions for $x = 0$ to $x = 5$. Draw the line $y = x$ on each graph. Then describe the relationship of the pair of inverse function graphs to the $y = x$ line.

a. $f(x) = 2x$ and $g(x) = 0.5x$

b. $h(x) = x^2$ and $j(x) = \sqrt{x}$

34. Describe functions with these domains and ranges.

a. domain: all real numbers
range: all real numbers greater than or equal to zero

b. domain: all real numbers
range: all integers

c. domain: all nonnegative real numbers
range: all nonpositive real numbers

35. Connie and Margaret are given the following extra credit problem: For $g(x) = 2x + 4$, find $g(g(1))$.

Connie's Method		Margaret's Method
$g(g(1))$ is the same as $(g(1))^2$ So, $g(1) = 2(1) + 4 = 6$ which means that $(g(1))^2 = 6^2 = 36$.	OR	$g(1) = 2(1) + 4 = 6$ This means that $g(g(1)) = g(6)$ $g(6) = 2(6) + 4 = 16$

Which of these methods is correct? Explain.

36. a. Suppose $f(x) = 5x + 35$ and $g(x) = 5(2^x)$. Find the point where the graphs of $y = f(x)$ and $y = g(x)$ intersect.

b. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.

37. Scott and Jim are driving from Gilbertville to Rivertown. The cities are 30 miles apart. Halfway between them is an intersection with the road east to Delmore City. You can see Scott and Jim's route on the diagram below. They are traveling at 60 mph (miles per hour).

a. Suppose that Scott and Jim measure distance in miles along the roads shown and time in minutes. Write a piecewise function rule for the function relating distance d from Delmore City to time t.

b. Draw a graph that shows how far they are from Delmore City at any time in their trip from Gilbertville to Rivertown.

c. Suppose that you measure distance "as the crow flies," rather than along the roads that are shown. How would that change the function rule and graph?