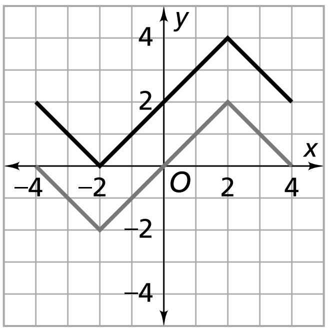
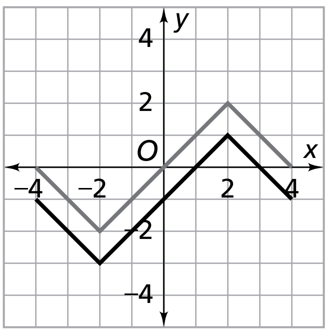
Applications

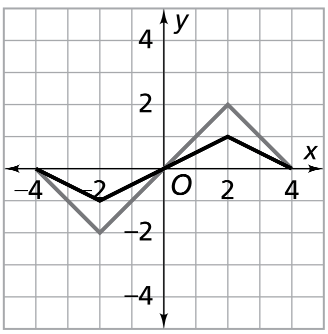
1. $g(x) = f(x) + 2$



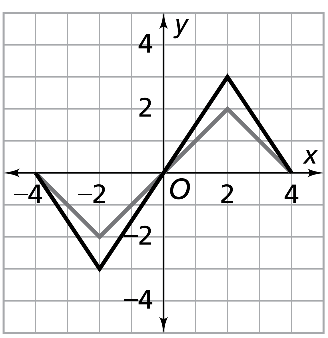
2. $h(x) = f(x) - 1$



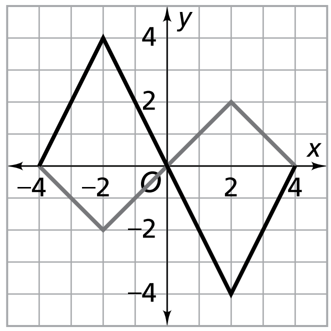
3. $j(x) = 0.5f(x)$



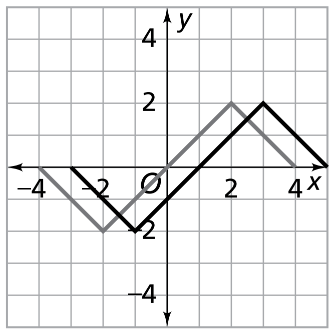
4. $k(x) = 1.5f(x)$



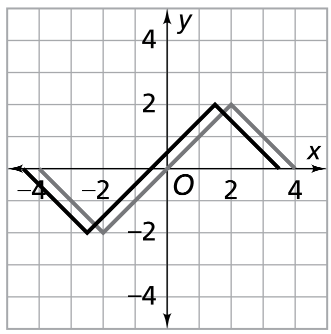
5. $m(x) = -2f(x)$



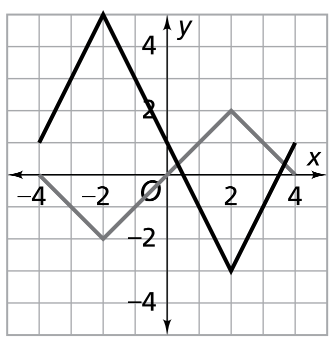
6. $n(x) = f(x - 1)$



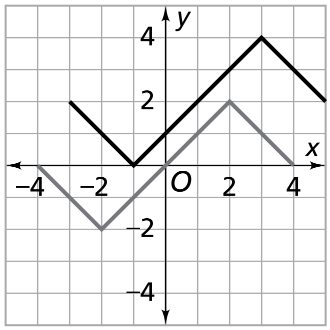
7. $p(x) = f(x + 0.5)$



8. $q(x) = -2f(x) + 1$



9. $r(x) = f(x - 1) + 2$



10. The table below summarizes the characteristics of the graphs and function rules. (See Figure 1.)

Connections

11. $(x, y) \rightarrow (x - 4, y - 2)$; translation

12. $(x, y) \rightarrow (-x, -y)$; flip over x-axis and over y-axis

13. $(x, y) \rightarrow (-x + 4, -y)$; flip over x-axis, flip over y-axis, and translation

14. $(x, y) \rightarrow (x + 6, y)$; translation

15. $(x, y) \rightarrow (2x + 3, 2y)$; dilation and translation

Figure 1

Function Rule	Parabola	Line of Symmetry	Vertex
$a(x) = x^2$	3	$x = 0$	minimum point (0, 0)
$b(x) = (x - 2)^2$	5	$x = 2$	minimum point (2, 0)
$c(x) = -(x - 2)^2 - 2$	7	$x = 2$	maximum point (2, –2)
$d(x) = (x + 2)^2 - 2$	1	$x = -2$	minimum point (–2, –2)
$e(x) = 0.5x^2$	2	$x = 0$	minimum point (0, 0)
$f(x) = 1.5x^2$	4	$x = 0$	minimum point (0, 0)
$g(x) = -(x - 3)^2$	6	$x = 3$	maximum point (3, 0)
$h(x) = -x^2 - 2$	8	$x = 0$	maximum point (0, –2)

Extensions

16. The function $f(x) = x^2 - 4x - 5$ can also be expressed as $f(x) = (x - 5)(x + 1)$ and $f(x) = (x - 2)^2 - 9$.

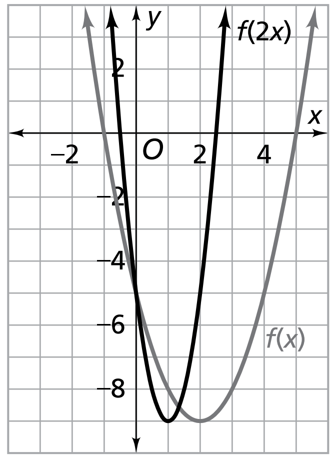
a. There are many ways to show equivalence of the three quadratic expressions. You can expand the factored and vertex forms as shown below.

$$\begin{aligned}
(x-5)(x+1) &= (x-5)(x) + (x-5)(1) && \text{(Distributive Property)}\\
&= x^2 - 5x + x - 5 && \text{(Distributive Property)}\\
&= x^2 + (-5x) + (1)x - 5 && \text{(Rewrite subtraction as equivalent addition.)}\\
&= x^2 + (-5x + 1x) - 5 && \text{(Associative Property)}\\
&= x^2 + (-5 + 1)x - 5 && \text{(Distributive Property)}\\
&= x^2 - 4x - 5 && \text{(Arithmetic)}\\
(x-2)^2 - 9 &= (x-2)(x-2) - 9 && \text{(Square of a Binomial)}\\
&= x^2 - 4x + 4 - 9 && \text{(Expand product of binomials as above.)}\\
&= x^2 - 4x - 5 && \text{(Arithmetic)}
\end{aligned}$$

b. The y-intercept point is $(0, -5)$. It is found from the standard polynomial form. The x-intercept points are $(5, 0)$ and $(-1, 0)$. These are found from the factored form and the principle that a product is zero when either factor is zero. The line of symmetry is $x = 2$ and the minimum point is $(2, -9)$. These are found from inspecting the vertex form, which tells us that the basic $y = x^2$ has been translated right 2 and down 9.

17. a. When $f(x) = x^2 - 4x - 5$, $f(2x) = 4x^2 - 8x - 5$.

b. The graphs of the two functions will be as shown here:



c. (See Figure 2.)

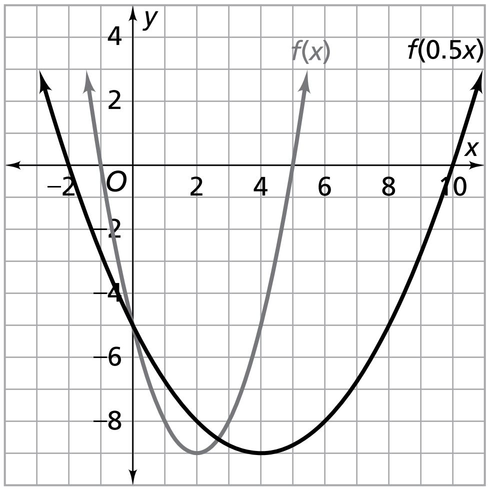
d. The effect of the substitution of $2x$ for x is a compression of the graph of $f(x)$ toward the y-axis by a factor of $\frac{1}{2}$. As a result, the x-intercepts (zeros) of the function and the line of symmetry x location are shrunk by a factor of 2. The y-intercept and minimum point are unchanged.

Figure 2

	y-intercept	x-intercepts	Line of Symmetry	Minimum Point
$f(x)$	(0, –5)	(5, 0), (–1, 0)	$x = 2$	(2, –9)
$f(2x)$	(0, –5)	(2.5, 0), (–0.5, 0)	$x = 1$	(1, –9)

18. a. When $f(x) = x^2 - 4x - 5$,
$f(0.5x) = 0.25x^2 - 2x - 5$.

b. The graphs are shown here.

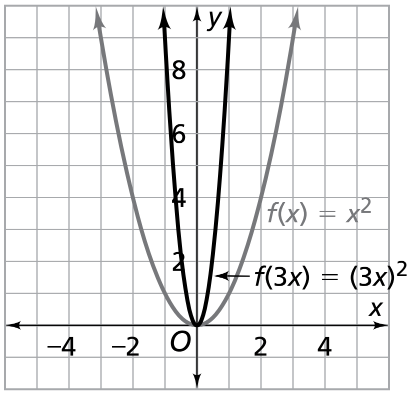


c. (See Figure 3.)

d. The effect of the substitution of $0.5x$ for x is a stretching of the graph of $f(x)$ away from the y-axis by a factor of 2. As a result, the x-intercepts or zeros of the function are doubled, as is the line of symmetry x location. Also the x-value of the minimum point is doubled. The y-intercept and the y-value of the minimum point are unchanged.

19. The effect of the substitution of kx for x is a compression or stretching of the graph of $f(x)$ toward or away from the y-axis by a factor of $\frac{1}{k}$. As a result, the x-intercepts or zeros of the function are changed by a factor of $\frac{1}{k}$ as is the line of symmetry location. But the y-intercept is unchanged. In general, the minimum or maximum point will have its x-coordinate changed by a factor of $\frac{1}{k}$ but its y-coordinate will be unchanged.

20. a. Because $f(x) = x^2$ has a graph that is symmetric about the line $x = 0$ and has only $(0, 0)$ as its x-intercept, $f(3x)$ has the same line of symmetry and x-intercepts. It also has the same minimum point and y-intercept.



b. In this case, $f(x)$ again has line of symmetry $x = 0$, so that is not changed by horizontal stretching or compression. However, the x-intercepts or zeros of $f(x)$ are $(-1, 0)$ and $(1, 0)$, and those of $f(0.5x)$ are $(-2, 0)$ and $(2, 0)$. The y-intercept and minimum point, both $(0, -1)$, are unchanged by the stretching.

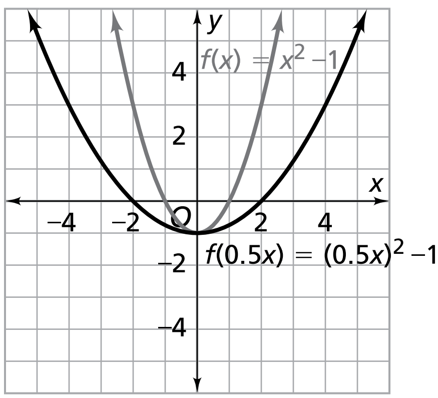
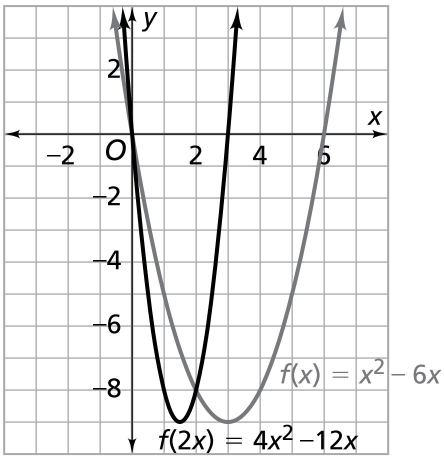


Figure 3

	y-intercept	x-intercepts	Line of Symmetry	Minimum Point
$f(x)$	(0, –5)	(5, 0), (–1, 0)	$x = 2$	(2, –9)
$f(0.5x)$	(0, –5)	(10, 0), (–2, 0)	$x = 4$	(4, –9)

c. $f(x) = x^2 - 6x$ has line of symmetry $x = 3$ and x-intercepts (0, 0) and (6, 0). $f(2x) = 4x^2 - 12x$ has line of symmetry $x = 1.5$ and x-intercepts (0, 0) and (3, 0). Both functions have y-intercept (0, 0). The minimum point of $f(x)$ is (3, −9), and the minimum point of $f(2x)$ is (1.5, −9).



These examples are all consistent with the conjecture described in Exercise 19.