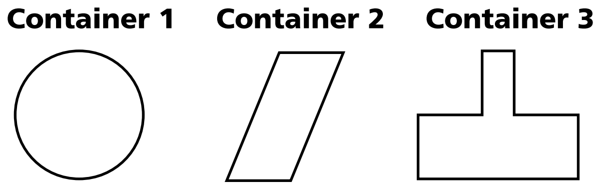
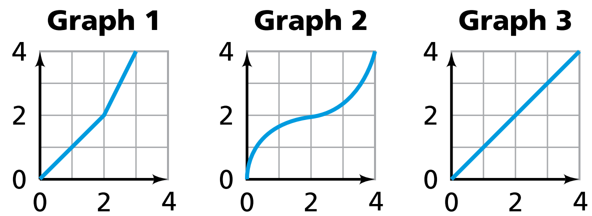
Applications

1. Suppose each container below is filled at a constant rate with water. Match each container with the graph that represents the relationship between the height of the water in the container and time.

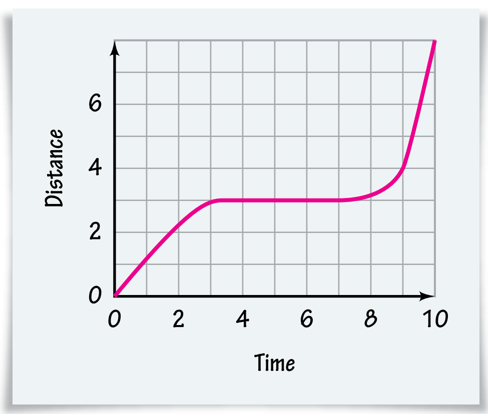




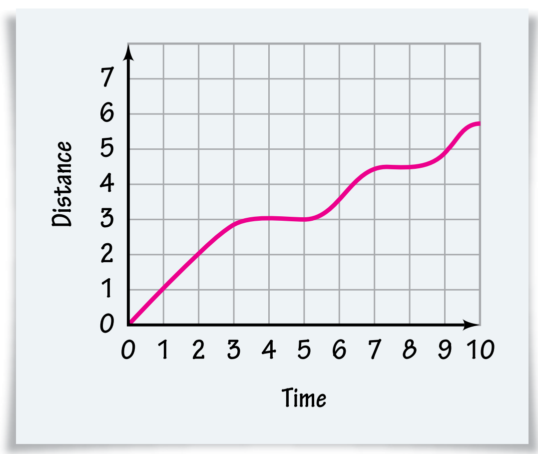
2. The graphs below show the pattern of time and distance traveled by two school buses. Make a copy of each graph. On copies of each graph mark the following intervals:

- when the bus is speeding up
- when the bus is slowing down
- when the bus is moving at a constant speed
- when the bus is stopped

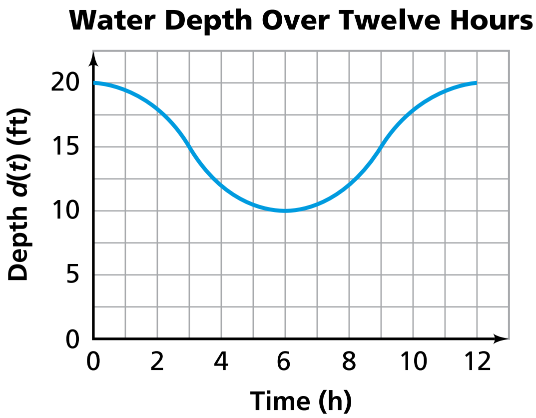
a. Bus A



b. Bus B



3. Ocean water levels rise and fall with a tidal period of about 12 hours. The graph below shows water depth at the end of a pier in a seacoast city.



The function $d(t)$ gives water depth at time t hours after midnight. Use the graph to complete the following sentences and explain what each sentence tells about water depth.

a. $d(0) = ■$

b. $d(2) = ■$

c. $d(4) = ■$

d. $d(6) = ■$

e. $d(9) = ■$

f. $d(■) = 15$

4. What are the domain and range of the function for water depth shown above?

Complete the sentences to give correct statements.

5. $f(x) = -2x + 5$

a. $f(7) = ■$ **b.** $f(-3) = ■$ **c.** $f(■) = 17$

6. $g(x) = x^2 + 5x$

a. $g(7) = ■$ **b.** $g(-3) = ■$ **c.** $g(■) = 6$

7. $h(x) = 4(0.5)^x$

a. $h(2) = ■$ **b.** $h(-1) = ■$ **c.** $h(■) = 4$

8. Describe the domain and range for the functions $f(x) = -2x + 5$, $g(x) = x^2 + 5x$, and $h(x) = 4(0.5^x)$.

9. For each function, sketch the graph and describe the domain and range.

a. $f(x) = 4x + 5$ **b.** $g(x) = x^2 + 2$ **c.** $h(x) = 2^x$ **d.** $j(x) = \frac{1}{x}$

10. Determine if the relationship in each table shows that y is a function of x.

a.

x	2	3	4	5	6
y	4	7	10	13	16

b.

x	3	4	1	–1	2
y	4	3	–1	1	2

c.

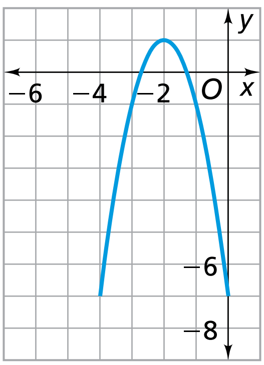
x	0	2	4	3	0
y	1	3	5	7	9

d.

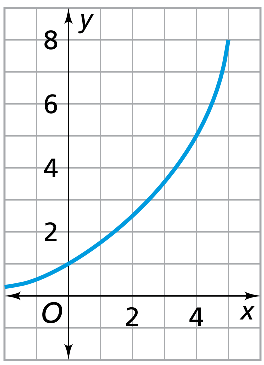
x	–4	–3	–1	1	2
y	0	1	1	1	0

For Exercises 11 and 12, use the graphs below.

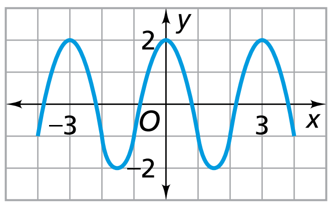
$f(x)$



$g(x)$



$h(x)$



11. Identify the domain and range of each function.

12. Use the graphs to complete these sentences that use function notation.

a. $f(-4) = \blacksquare$ **b.** $g(4) = \blacksquare$ **c.** $h(1) = \blacksquare$

d. $f(\blacksquare) = 1$ **e.** $g(\blacksquare) = 1$ **f.** $h(\blacksquare) = 2$

13. The fee for airport parking is shown below. For parts (a)–(f), calculate the cost of parking $c(t)$ for the given times.

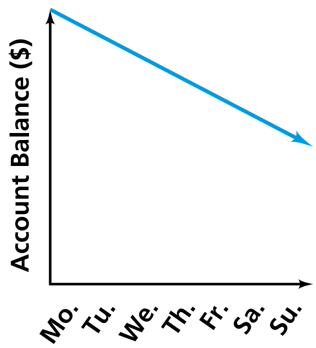
a. $c(0.5)$ **b.** $c(1.0)$ **c.** $c(2.5)$

d. $c(5.0)$ **e.** $c(5.5)$ **f.** $c(8.0)$

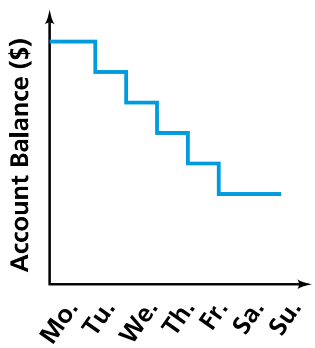
g. Draw a graph showing the charges for any time from 0 to 8 hours.

14. Multiple Choice At Akihito's school, lunches cost $1.25. Akihito starts the school year with a school lunch account balance of $100. Which graph best represents the pattern of change in Akihito's account balance during a typical week?

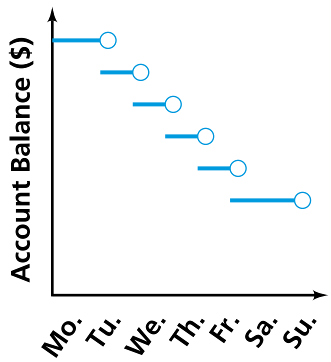
A.



B.



C.



D. None of these

15. Suppose $r(x)$ is the function that applies the standard rounding rule to numbers. Also, $c(x)$ applies the ceiling rule for rounding, and $f(x)$ applies the floor rule for rounding. Complete the following sentences to give correct statements.

a. $r(1.6) = $ ■ **b.** $c(1.6) = $ ■ **c.** $f(1.6) = $ ■

d. $r(-1.6) = $ ■ **e.** $c(-1.6) = $ ■ **f.** $f(-1.6) = $ ■

g. $r(-1.3) = $ ■ **h.** $c(-1.3) = $ ■ **i.** $f(-1.3) = $ ■

j. $r($■$) = -2$ **k.** $c($■$) = -2$ **m.** $f($■$) = -2$

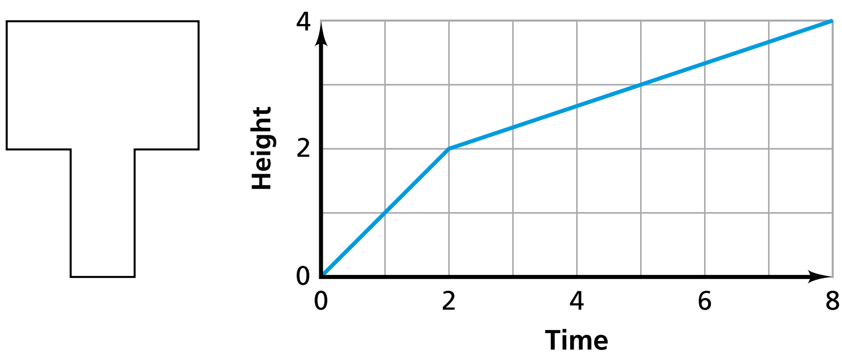
16. Graph each of the following piecewise functions.

a. $y = \begin{cases} x^2 \text{ if } x \leq 0 \\ 3x \text{ if } x > 0 \end{cases}$

b. $y = \begin{cases} \frac{1}{2}x + 4 \text{ if } x < 0 \\ -\frac{1}{2}x + 4 \text{ if } x \geq 0 \end{cases}$

c. $y = \begin{cases} 4 \text{ if } x < 2 \\ 2^x \text{ if } x \geq 2 \end{cases}$

17. The figure and graph below show the function for rate of filling for the container from Problem 1.4.

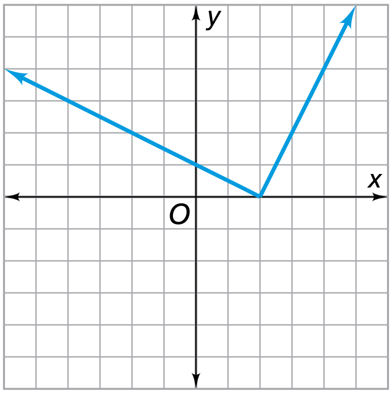


Suppose that the height of the water is measured in inches and the filling time in seconds. At what rate is water height rising during the following intervals?

a. the first two seconds

b. the time from 2 to 8 seconds

18. Desheng and Chelsea are trying to write a piecewise function rule for the following graph.



Desheng says the y-intercept is 1, the slope of the left part is $-\frac{1}{2}$, and the slope of the right part is 2. He writes the following function rule.

$$y = \begin{cases} -\frac{1}{2}x + 1 \text{ } if\, x < 2 \\ 2x + 1 \text{ } if\, x \geq 2 \end{cases}$$

Chelsea says the left part of the graph has slope $-\frac{1}{2}$ and y-intercept of 1. The right part of the graph has slope 2. It would have a y-intercept of -4 if the graph extended to intersect the y-axis. She writes the following function rule.

$$y = \begin{cases} -\frac{1}{2}x + 1 \text{ } if\, x \leq 2 \\ 2x - 4 \text{ } if\, x > 2 \end{cases}$$

Whose reasoning is correct? Explain.

19. Find inverses for these functions.

a. $f(x) = 6x$ **b.** $g(x) = x - 4$ **c.** $h(x) = 6x - 4$

d. $j(x) = \sqrt{x}$ **e.** $k(x) = 4x^2$ **f.** $m(x) = -\frac{3}{x}$

20. For each function in Exercise 19, do the following.

- Describe the domain and range of the function.
- Describe the domain and range of the inverse function.
- Explain any ways that the domain of the inverse function differs from the domain of original function. (In what ways must the domain of the inverse function be limited?)

Connections

21. For $f(x) = \sqrt{x}$, evaluate each of the following.

a. $f(121)$ **b.** $f\left(\frac{1}{4}\right)$ **c.** $f\left(m^2\right)$ **d.** $f\left(4q^2\right)$

22. For $f(x) = 4^x$, complete the following sentences.

a. $f(4) = \blacksquare$ **b.** $f(-2) = \blacksquare$ **c.** $f(\blacksquare) = 1$

d. $f(\blacksquare) = 2$ **e.** $f(a) = \blacksquare$ **f.** $f(b + 2) = \blacksquare$

23. For $g(x) = x^2$, evaluate each of the following expressions.

a. $g(3)$ **b.** $g\left(\frac{1}{2}\right)$ **c.** $g(-d)$ **d.** $g\left(\frac{n}{2m}\right)$

24. For $j(x) = \frac{1}{2}x$, evaluate each of the following expressions.

a. $j(-7)$ **b.** $j(0)$ **c.** $j(2s)$ **d.** $j\left(\frac{r}{t}\right)$

25. Linear, quadratic, and exponential functions all have as their domains the set of real numbers.

a. Why is the domain of $r(x) = \sqrt{x}$ not all real numbers?

b. Why is the domain of $s(x) = \frac{1}{x}$ not all real numbers?

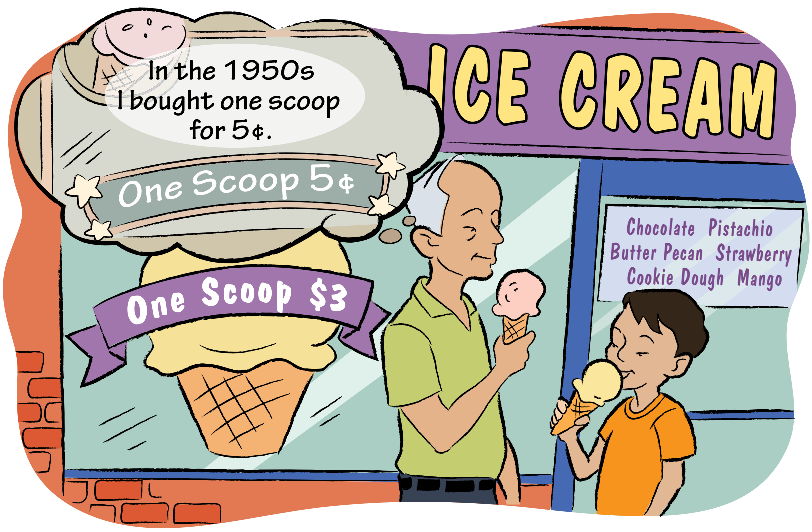
26. Many variables in your life change as time passes. Tell whether any of the following changes shows a pattern like a step function. **Hint:** More than one pattern may be a step function.

a. your height

b. the price of a one-scoop cone of ice cream

c. your age in years

d. the number of questions left to answer as you work on homework



27. For each of the following numeric equations, write the other equations in the fact family.

a. $7 + 12 = 19$ **b.** $4 \times 3 = 12$

28. Addition and subtraction are inverse operations. Multiplication and division are also inverse operations. How do those inverse operation labels relate to the inverse functions in Problem 1.5?

29. Solve each of these equations using ideas about fact families and inverse operations.

a. $x + 7 = 12$ **b.** $5x = 35$ **c.** $5x + 7 = 82$

d. $\frac{7}{x} = 12$ **e.** $\frac{9}{x-2} = 3$ **f.** $\frac{5}{x} + 7 = 8$

Extensions

30. **a.** Copy and complete the following table of values.

Variations of the Ceiling Rounding Function *c*(*x*)

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$c(x) - x$	■	■	■	■	■	■	■	■	■	■	■
$x - c(x)$	■	■	■	■	■	■	■	■	■	■	■

b. Using data in the table, draw a graph of $c(x) - x$ from $x = 0$ to $x = 5$.

c. Using data in the table, draw a graph of $x - c(x)$ from $x = 0$ to $x = 5$.

d. What can you conclude from the two graphs?

31. **a.** Copy and complete the following table of values.

Variations of the Floor Rounding Function *f*(*x*)

x	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25	2.5
$f(x) - x$	■	■	■	■	■	■	■	■	■	■	■
$x - f(x)$	■	■	■	■	■	■	■	■	■	■	■

b. Using data in the table, draw a graph of $f(x) - x$ from $x = 0$ to $x = 5$.

c. Using data in the table, draw a graph of $x - f(x)$ from $x = 0$ to $x = 5$.

d. What can you conclude from the two graphs?

32. What are the domain and range of the ceiling function $c(x)$?

33. Sketch graphs for each of these pairs of functions for $x = 0$ to $x = 5$. Draw the line $y = x$ on each graph. Then describe the relationship of the pair of inverse function graphs to the $y = x$ line.

a. $f(x) = 2x$ and $g(x) = 0.5x$

b. $h(x) = x^2$ and $j(x) = \sqrt{x}$

34. Describe functions with these domains and ranges.

a. domain: all real numbers
range: all real numbers greater than or equal to zero

b. domain: all real numbers
range: all integers

c. domain: all nonnegative real numbers
range: all nonpositive real numbers

35. Connie and Margaret are given the following extra credit problem: For $g(x) = 2x + 4$, find $g(g(1))$.

Connie's Method	OR	Margaret's Method
$g(g(1))$ is the same as $(g(1))^2$ So, $g(1) = 2(1) + 4 = 6$ which means that $(g(1))^2 = 6^2 = 36$.		$g(1) = 2(1) + 4 = 6$ This means that $g(g(1)) = g(6)$ $g(6) = 2(6) + 4 = 16$

Which of these methods is correct? Explain.

36. **a.** Suppose $f(x) = 5x + 35$ and $g(x) = 5(2^x)$. Find the point where the graphs of $y = f(x)$ and $y = g(x)$ intersect.

b. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$.

37. Scott and Jim are driving from Gilbertville to Rivertown. The cities are 30 miles apart. Halfway between them is an intersection with the road east to Delmore City. You can see Scott and Jim's route on the diagram below. They are traveling at 60 mph (miles per hour).

a. Suppose that Scott and Jim measure distance in miles along the roads shown and time in minutes. Write a piecewise function rule for the function relating distance d from Delmore City to time t.

b. Draw a graph that shows how far they are from Delmore City at any time in their trip from Gilbertville to Rivertown.

c. Suppose that you measure distance "as the crow flies," rather than along the roads that are shown. How would that change the function rule and graph?