Function Junction Concepts and Practice

SOLUTIONS

Solve the equation by factoring.

11.) $x^2+8x+16=0$

$x^2+4x+4x+16=0$

$x(x+4)+4(x+4)=0$

$(x+4)(x+4)=0$

$0=x+4$
$-4 \quad -4$
$-4=x$

$\boxed{x=-4}$

12.) $x^2-3x-10=0$

$(x-5)(x+2)=0$

$0=x-5$
$+5 \quad +5$
$5=x$

$x+2=0$
$-2 \quad -2$
$x=-2$

$\boxed{x=-2 \text{ and } 5}$

13.) $(2x+3)(x-6)=0$

$0=2x+3$
$-3 \quad -3$
$\frac{-3}{2}=\frac{2x}{2}$
$-\frac{3}{2}=x$

$x-6=0$
$+6 \quad +6$
$x=6$

$\boxed{x=-\frac{3}{2} \text{ and } 6}$

14.) $x^2-49=0$
$+49 \quad +49$
$\sqrt{x^2}=\sqrt{49}$
$x=\pm 7$

$\boxed{x=-7 \text{ and } 7}$

15.) $x^2-45=-12x$
$+12x \quad +12x$

$x^2+12x-45=0$

$x^2+15x-3x-45=0$

$x(x+15)-3(x+15)=0$

$(x-3)(x+15)=0$

$x-3=0$
$+3 \quad +3$
$x=3$

$x+15=0$
$-15 \quad -15$
$x=-15$

$\boxed{x=-15 \text{ and } 3}$

16.) $2x^2+11x+5=0$

$2x^2+10x+x+5=0$

$2x(x+5)+1(x+5)=0$

$(2x+1)(x+5)=0$

$2x+1=0$
$-1 \quad -1$
$\frac{2x}{2}=\frac{-1}{2}$

$x+5=0$
$-5 \quad -5$
$x=-5$

$\boxed{x=-5 \text{ and } -\frac{1}{2}}$

17.) $4x^2+7x+3=0$

$4x^2+4x+3x+3=0$

$4x(x+1)+3(x+1)=0$

$(4x+3)(x+1)=0$

$0=4x+3$
$-3 \quad -3$
$\frac{-3}{4}=\frac{4x}{4}$

$x+1=0$
$-1 \quad -1$
$x=-1$

$\boxed{x=-\frac{3}{4} \text{ and } -1}$

18.) $8x^2+2x-15=0$

$8x^2+12x-10x-15=0$

$4x(2x+3)-5(2x+3)=0$

$(4x-5)(2x+3)=0$

$4x-5=0$
$+5 \quad +5$
$\frac{4x}{4}=\frac{5}{4}$

$2x+3=0$
$-3 \quad -3$
$\frac{2x}{2}=\frac{-3}{2}$

$\boxed{x=\frac{5}{4} \text{ and } -\frac{3}{2}}$

23.) $y = x^2 + 6x - 16$

y-intercept: $\boxed{(0,-16)}$

x-intercepts:

$0 = (x+8)(x-2)$

$x+8=0 \quad x-2=0$

$x=-8 \quad x=2$

$\boxed{(2,0)\ (-8,0)}$

line of symmetry:

$\frac{2-8}{2} = -3$ $\quad \boxed{x=-3}$

vertex:

$y = (-3+8)(-3-2)$

$= -25$

$\boxed{(-3,-25)}$

up/down $\boxed{\text{UP}}$

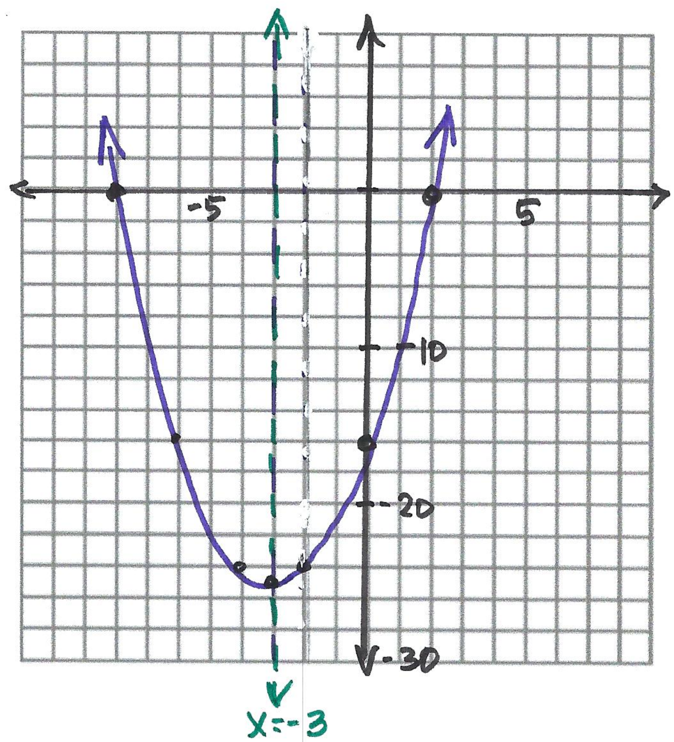
additional point:

$y = (-2+8)(-2-2)$

$= (6)(-4)$

$= -24$

$\boxed{(-2,-24)}$



For questions 24-27, use the following information.

A stomp rocket is launched into the air from ground level. It's height *h* as a function of time *t* can be modeled by the equation $h = -16t^2 + 64t$.

24.) How high will the rocket be at 1.5 seconds?

$h = -16(1.5)^2 + 64(1.5)$

$= -16(2.25) + 96$

$= 60$

$\boxed{\text{60 feet}}$

25.) How long will it take for the rocket to hit the ground?

$0 = -16t^2 + 64t$

$0 = -16t(t-4)$

$-16t = 0 \quad t-4=0$

$t=0 \quad t=4$

$\boxed{\text{4 seconds}}$

26.) When will the rocket reach its maximum height?

LOS: $\frac{0+4}{2} = 2$

$\boxed{\text{It will reach max height in 2 seconds}}$

27.) The advertising on the package says "Can fly over 60 feet high!" Does this rocket exceed the height listed on the package?

$60 = -16t^2 + 64t$

$0 = -16t^2 + 64t - 60$

$b^2 - 4ac = (64)^2 - (4)(-16)(-60)$

$= 4096 - 3840 = 256$

$b^2 - 4ac > 0$

YES

It can fly over 60 feet high!

44.) Abraham throws a ball from a point 40 m above the ground. The height of the ball from the ground level after 't' seconds is defined by the function $h(t) = 40t - 5t^2$. How long will the ball take to hit the ground?

$h = -5t^2 + 40t + 40$ ← initial height

$0 = -5(t^2 - 8t - 8)$

$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-8)}}{2}$

$x = -0.9$ and 8.9

It will take 8.9 seconds for the ball to hit the ground.

45.) A rocket is launched into the air from ground level with an initial velocity of 120 feet per second.

a.) Write an equation that models how the height *h* of the rocket changes over time *t*.

$h = -16t^2 + 120t$

b.) How long will it take for the rocket to hit the ground?

$0 = -16t^2 + 120t$

$0 = -8t(2t - 15)$

$-8t = 0$, $t = 0$

$2t - 15 = 0$
$+15 \quad +15$
$\frac{2t}{2} = \frac{15}{2}$

The rocket will hit the ground in 7.5 seconds

c.) The advertising on the package says "Can fly over 250 feet high!" Does this rocket exceed the height listed on the package?

$250 = -16t^2 + 120t$
$-250 \qquad -250$
$0 = -16t^2 + 120t - 250$

$b^2 - 4ac = (120)^2 - 4(-16)(-250)$
$= 14400 - 16000$
$= -1600$

NO!

The rocket cannot even reach 250 ft!

d.) When is the rocket 160 feet off the ground?

$h = -16t^2 + 120t$

$160 = -16t^2 + 120t$

$0 = -16t^2 + 120t - 160$

$0 = -8(2t^2 - 15t + 20)$

$x = \frac{15 \pm \sqrt{(-15)^2 - 4(2)(20)}}{4}$

$x = 1.73$ and 5.77

The rocket is 160 ft off the ground after 1.73 sec. and 5.77 sec.

46.) A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by $h = -16t^2 + 64t + 80$.

a. What is the height of the rocket after 1.5 seconds?

$h = -16(1.5)^2 + 64(1.5) + 80$
$h = -16(2.25) + 96 + 80$
$h = 140$

$\boxed{140 \text{ feet}}$

b. What is the maximum height reached by the rocket?

$h = -16(t^2 - 4t - 5)$
$0 = -16(t-5)(t+1)$
$t-5=0 \quad t+1=0$
$t=5 \quad t=-1$

LOS: $\frac{5-1}{2} = 2$

$h = -16(2-5)(2+1)$
$h = -16(-3)(3)$
$h = 144$

$\boxed{144 \text{ feet}}$

c. How long will it take for the rocket to hit 128 feet?

$128 = -16t^2 + 64t + 80$
$0 = -16t^2 + 64t - 48$
$0 = -16(t^2 - 4t + 3)$
$0 = -16(t-3)(t-1)$
$t-3=0 \quad t-1=0$
$t=3 \quad t=1$

After 1 second and after 3 seconds

d. After how many seconds will the rocket hit the lake?

5 seconds

from part b above

47.) After t seconds, a ball tossed in the air from the ground level reaches a height of H feet given by the equation $H(t) = 144t - 16t^2$.

a. What is the height of the ball after 3 seconds?

$h(3) = 144(3) - 16(3)^2 = 288$

$\boxed{288 \text{ feet}}$

b. What is H(4)?

$h(4) = 144(4) - 16(4)^2 = 576 - 256$
$= 320$

$\boxed{320 \text{ feet}}$

c. What is the maximum height the ball will reach?

$0 = 144t - 16t^2$
$0 = -16t(-9+t)$
$-16t=0 \quad -9+t=0$
$t=0 \quad t=9$

LOS: $\frac{0+9}{2} = 4.5$

$h = -16(4.5)(-9+4.5)$
$= -72(-4.5)$
$= 324$

$\boxed{324 \text{ feet}}$

d. Find the number of seconds the ball is in the air when it reaches a height of 224 feet.

$224 = 144t - 16t^2$
$0 = 144t - 16t^2 - 224$
$0 = -16(t^2 - 9t + 14)$
$0 = -16(t-7)(t-2)$
$t-7=0 \quad t-2=0$
$t=7 \quad t=2$

The ball will be at 224 feet after 2 seconds and 7 seconds.

94. How is the graph of g(x) = $(x + 12)^2$ related to the graph of $f(x) = x^2$?

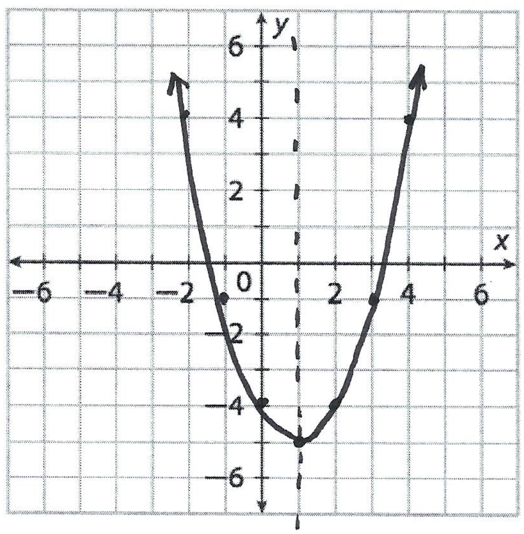
Translated 12 units left

95. How is the graph of g(x) = $(x - 10)^2$ related to the graph of $f(x) = x^2$?

Translated 10 units right.

For #'s 96-99, graph the quadratic function. Give the minimum or maximum value and the line of symmetry.

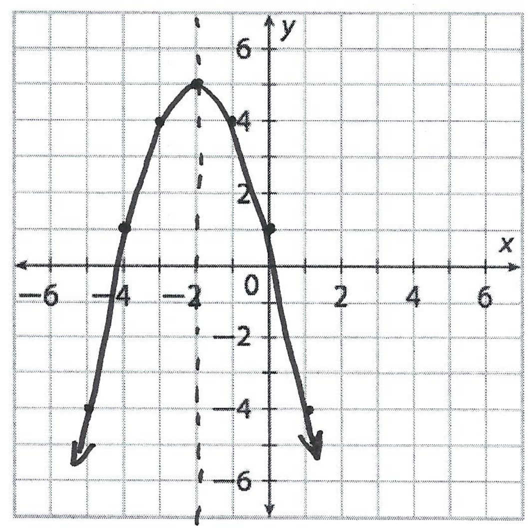
96. $g(x) = (x - 1)^2 - 5$



Minimum: (1,-5)

L.O.S.: x = 1

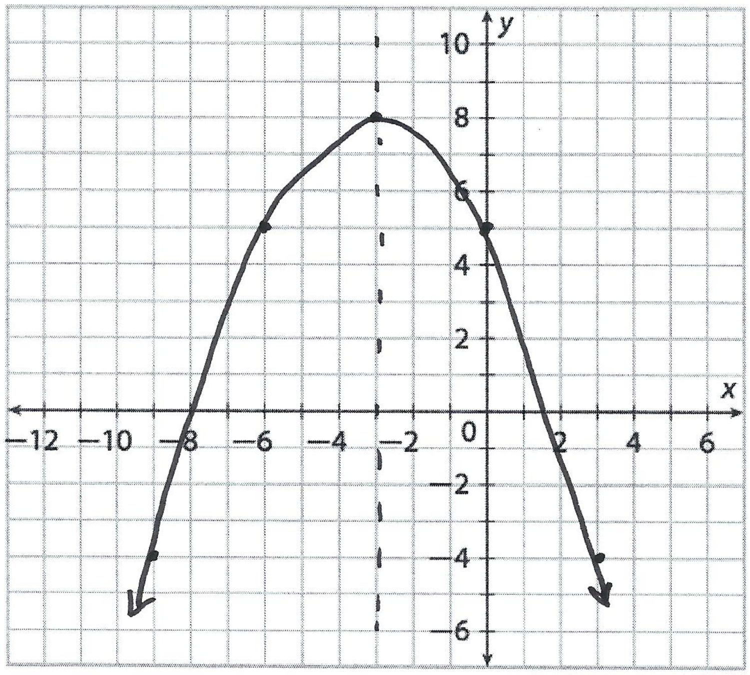
97. $g(x) = -(x + 2)^2 + 5$



Maximum: (-2,5)

LOS: X = -2

98. $g(x) = -\frac{1}{3}(x+3)^2 + 8$



Maximum: (-3, 8)

L.O.S.: X = -3

$g(0) = -\frac{1}{3}(0+3)^2+8$
$= -\frac{1}{3}(9)+8$
$= -3+8$
$= 5$

$g(3) = -\frac{1}{3}(3+3)^2+8$
$= -\frac{1}{3}(36)+8$
$= -12+8$
$= -4$

Solve each equation by taking the square root.

100. $a^2 + 1 = 2$
$-1 \quad -1$
$a^2 = 1$
$\boxed{a = \pm 1}$

101. $n^2 - 4 = 77$
$+4 \quad +4$
$n^2 = 81$
$\boxed{n = \pm 9}$

102. $m^2 + 7 = 6$
$-7 \quad -7$
$m^2 = -1$
$\boxed{\text{No solution}}$

103. $x^2 - 1 = 80$
$+1 \quad +1$
$x^2 = 81$
$\boxed{x = \pm 9}$

104. $4x^2 - 6 = 74$
$+6 \quad +6$
$\frac{4x^2}{4} = \frac{68}{4}$
$x^2 = 20$
$x = \sqrt{20}$
$\boxed{x = \pm 4.47}$

105. $3m^2 + 7 = 301$
$-7 \quad -7$
$\frac{3m^2}{3} = \frac{294}{3}$
$m^2 = 98$
$m = \pm\sqrt{98}$
$m = \pm 9.90$

106. $b^2 = -4b + 73$

$b^2+4b-73=0$

$x = \frac{-4 \pm \sqrt{(4)^2-4(1)(-73)}}{2}$

$= \frac{-4 \pm \sqrt{308}}{2}$

$\boxed{x = 6.77 \text{ and } -10.77}$

107. $n^2 - 16n = 12$

$n^2-16n-12=0$

$x = \frac{+16 \pm \sqrt{16^2-4(1)(-12)}}{2(1)}$

$= \frac{16 \pm \sqrt{304}}{2}$

$\boxed{x = 16.72 \text{ and } -0.72}$

Solve each equation by the method of your choice. All answers should be exact and simplified.

108. $6b^2 + 252 = -78b$

$6b^2+78b+252=0$

$6(b^2+13b+42)=0$

$6(b+6)(b+7)=0$

$b+6=0$, -6 -6, $b=-6$

$b+7=0$, -7 -7, $b=-7$

$\boxed{x = -6 \text{ and } -7}$

109. $v^2 = 1$

$\boxed{v = \pm 1}$

110. $2x = 4x^2 - 22$

$0 = 4x^2-2x-22$

$x = \frac{2 \pm \sqrt{(-2)^2-4(4)(-2)}}{2(4)}$

$= \frac{2 \pm \sqrt{356}}{8}$

$\boxed{x = 2.61 \text{ and } -2.11}$

111. $n^2 + 18n - 61 = 7$

$n^2+18n-68=0$

$x = \frac{-18 \pm \sqrt{18^2-4(1)(-68)}}{2(1)}$

$= \frac{-18 \pm \sqrt{596}}{2}$

$\boxed{x = -21.21 \text{ and } 3.21}$

112. $x^2 - 14 = 5x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$0=x-7$, $+7$ $+7$, $7=x$

$0=x+2$, -2 -2, $-2=x$

$\boxed{x = -2 \text{ and } 7}$

113. $0 = -3x^2 - 48 - 24x$

$0 = -3x^2-24x-48$

$0 = -3(x^2+8x+16)$

$0 = -3(x+4)^2$

$(x+4)^2=0$

$x+4=0$, -4 -4

$x=-4$

$\boxed{x = -4}$

114. $n^2 - 10 = -20$

$+10$ $+10$

$n^2=-10$

$\boxed{\text{No Solution}}$

115. $x^2 = 35 + 12x$

$x^2-12x-35=0$

$x = \frac{12 \pm \sqrt{(12)^2-4(1)(-35)}}{2(1)}$

$x = \frac{12 \pm \sqrt{284}}{2}$

$\boxed{x = 14.43 \text{ and } -2.43}$

116. $n^2 + 9n + 13 = -7$

$n^2+9n+20=0$

$(n+4)(n+5)=0$

$0=n+4$, -4 -4, $-4=n$

$n+5=0$, -5 -5, $n=-5$

$\boxed{n = -4 \text{ and } -5}$

117. $8r^2 + 12r = 21$

$8r^2+12r-21=0$

$x = \frac{-12 \pm \sqrt{(12)^2-4(8)(-21)}}{2(8)}$

$= \frac{-12 \pm \sqrt{816}}{16}$

$\boxed{x = -2.54 \text{ and } 1.04}$

118. A football is punted into the air. Its height h, in meters, after t seconds is given by the equation $h=-4.9t^2+24.5t+1$

a) How high is the ball after 1 second?

$h=-4.9(1)^2+24.5(1)+1$
$=-4.9+24.5+1$
$=20.6$

The ball is 20.6 meters high after 1 second

b) Find the maximum height of the ball to one decimal place

$LOS = \frac{-b}{2a} = \frac{-24.5}{2(-4.9)} = 2.5$

$h=-4.9(2.5)^2+24.5(2.5)+1$
$=-30.625+61.25+1$
$=31.625$

Maximum height is 31.6 meters

c) When does the ball reach its maximum height?

Maximum height is reached after 2.5 seconds.

Line of symmetry gives the time of maximum height.

d) When does the ball hit the ground?

$0=-4.9t^2+24.5t+1$

$x=\frac{-24.5 \pm \sqrt{(24.5)^2-4(-4.9)(1)}}{2(-4.9)}$

$x=-0.04$ and 5.04

The ball will hit the ground after 5.04 sec.

119. A stone is thrown skyward from the top of a building. The distance in ft between the stone and the ground in t seconds is given by the function $d = -16t^2 - 4t + 442$. How long after the stone is thrown is it 430 ft from the ground?

$430=-16t^2-4t+442$
$-430 \qquad -430$

$0=-16t^2-4t+12$
$0=-4(4t^2+t-3)$
$0=-4(4t^2+4t-3t-3)$
$0=-4(4t(t+1)-3(t+1))$
$0=-4(4t-3)(t+1)$

$0=4t-3$
$+3 \quad +3$
$3/4=t$

$t+1=0$
$-1 \quad -1$
$t=-1$

The stone will be 430 ft from the ground after 3/4 seconds.

120. A rock is thrown from the top of a tall building. The distance, in feet, between the rock and the ground t seconds after it is thrown is given by **$d = -16t^2 - 4t + 382$.** How long after the rock is thrown is it 370 feet from the ground?

$370 = -16t^2 - 4t + 382$
$-370 \qquad\qquad -370$

$0 = -16t^2 - 4t + 12$
$0 = -4(4t^2 + t - 3)$
$0 = -4(4t-3)(t+1)$

$0 = 4t - 3$
$+3 \quad +3$
$3/4 = t$

$0 = t + 1$
$-1 = t$

The rock will be 370 feet from the ground after 3/4 seconds.

Find the discriminant of each quadratic equation then state the number of real and imaginary solutions.

$b^2 - 4ac$

121. $9n^2 - 3n - 8 = -10$
$+10 \quad +10$

$9n^2 - 3n + 2 = 0$

$(-3)^2 - 4(9)(2)$
$= 9 - 72$
$= -63$

No solutions

122. $-2x^2 - 8x - 14 = -6$
$+6 \quad +6$

$-2x^2 - 8x - 8 = 0$

$b^2 - 4ac : (-8)^2 - 4(-2)(-8)$
$= 64 - 64$
$= 0$

One Solution

123. $9m^2 + 6m + 6 = 5$
$-5 \quad -5$

$9m^2 + 6m + 1 = 0$

$b^2 - 4ac = (6)^2 - 4(9)(1)$
$= 36 - 36$
$= 0$

One Solution

124. $4a^2 = 8a - 4$

$4a^2 - 8a + 4 = 0$

$b^2 - 4ac = (-8) - 4(4)(4)$
$= 64 - 64$
$= 0$

One Solution