

10-3 Study Guide and Intervention *(continued)*

Solving Quadratic Equations by Completing the Square

Complete the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

- | | |
|---------------|---|
| Step 1 | Find $\frac{b}{2}$. |
| Step 2 | Find $\left(\frac{b}{2}\right)^2$. |
| Step 3 | Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$. |

Example

Solve $x^2 + 6x + 3 = 10$ by completing the square.

$$\begin{aligned}x^2 + 6x + 3 &= 10 && \text{Original equation} \\x^2 + 6x + 3 - 3 &= 10 - 3 && \text{Subtract 3 from each side.} \\x^2 + 6x &= 7 && \text{Simplify.} \\x^2 + 6x + 9 &= 7 + 9 && \text{Since } \left(\frac{6}{2}\right)^2 = 9, \text{ add 9 to each side.} \\(x + 3)^2 &= 16 && \text{Factor } x^2 + 6x + 9. \\x + 3 &= \pm 4 && \text{Take the square root of each side.} \\x &= -3 \pm 4 && \text{Simplify.} \\x &= -3 + 4 \quad \text{or} \quad x = -3 - 4 \\&= 1 && = -7\end{aligned}$$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

- | | | |
|--|---|---|
| 1. $t^2 - 4t + 3 = 0$
1, 3 | 2. $y^2 + 10y = -9$
-1, -9 | 3. $y^2 - 8y - 9 = 0$
-1, 9 |
| 4. $x^2 - 6x = 16$
-2, 8 | 5. $p^2 - 4p - 5 = 0$
-1, 5 | 6. $x^2 - 12x = 9$
-0.7, 12.7 |
| 7. $c^2 + 8c = 20$
-10, 2 | 8. $p^2 = 2p + 1$
-0.4, 2.4 | 9. $x^2 + 20x + 11 = -8$
-19, -1 |
| 10. $x^2 - 1 = 5x$
-0.2, 5.2 | 11. $a^2 = 22a + 23$
-1, 23 | 12. $m^2 - 8m = -7$
1, 7 |
| 13. $x^2 + 10x = 24$
-12, 2 | 14. $a^2 - 18a = 19$
-1, 19 | 15. $b^2 + 16b = -16$
-14.9, -1.1 |
| 16. $4x^2 = 24 + 4x$
-2, 3 | 17. $2m^2 + 4m + 2 = 8$
-3, 1 | 18. $4k^2 = 40k + 44$
-1, 11 |

Skills Practice***Solving Quadratic Equations by Completing the Square***

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1. $c^2 - 12c + 36 = 4$ **4, 8**

2. $w^2 - 10w + 25 = 16$ **1, 9**

3. $b^2 + 16b + 64 = 9$ **-11, -5**

4. $y^2 + 2y + 1 = 3$ **-2.7, 0.7**

5. $r^2 + 4r + 4 = 7$ **-4.6, 0.6**

6. $a^2 - 8a + 16 = 12$ **0.5, 7.5**

Find the value of c that makes each trinomial a perfect square.

7. $g^2 + 6g + c$ **9**

8. $y^2 + 4y + c$ **4**

9. $a^2 - 14a + c$ **49**

10. $n^2 - 2n + c$ **1**

11. $s^2 - 18s + c$ **81**

12. $p^2 + 20p + c$ **100**

Solve each equation by completing the square. Round to the nearest tenth if necessary.

13. $x^2 + 4x - 12 = 0$ **2, -6**

14. $v^2 - 8v + 15 = 0$ **3, 5**

15. $q^2 + 6q = 7$ **-7, 1**

16. $r^2 - 2r = 15$ **-3, 5**

17. $m^2 - 14m + 30 = 6$ **2, 12**

18. $b^2 + 12b + 21 = 10$ **-11, -1**

19. $z^2 - 4z + 1 = 0$ **0.3, 3.7**

20. $y^2 - 6y + 4 = 0$ **0.8, 5.2**

21. $r^2 - 8r + 10 = 0$ **1.6, 6.4**

22. $p^2 - 2p = 5$ **-1.4, 3.4**

23. $2a^2 + 20a = -2$ **-9.9, -0.1**

24. $0.5g^2 + 8g = -7$ **-15.1, -0.9**