Frogs, Fleas, and Painted Cubes Topics and Practice WITH ANSWERS

- Factor binomials and trinomial expressions and equations
- Multiply a monomial times a binomial using the Distributive Property
- Multiply a binomial times a binomial using either the Box/Area method, the Vertical method, or the FOIL/Claw/Double Distributive Property method
- Identify the key features of a quadratic equation algebraically
- Graph a quadratic equation using the key features
- Given a real life situation identify key features to answer questions about the situation like how high an object travels and how long it is in the air
- Identify whether a relationship shown in a table is either linear, exponential, quadratic, or none by using the first and second differences
- Decide whether an equation represents a linear, exponential, or quadratic relationship

Resources:

- FF (Frogs, Fleas, and Painted Cubes) Problems 1.1 - 1.2
- FF Problems 2.2 - 2.4
- FF Problem 3.4
- All class work notes, handouts, and ACE Questions.

Expand the expression completely.

57.) $2x(x-5)$

$2x^2 - 10x$

58.) $(x+3)(x+10)$

$x^2+10x+3x+30$

$x^2+13x+30$

59.) $(x-7)(x-5)$

$x^2-5x-7x+35$

$x^2-12x+35$

60.) $(2x+3)(x-6)$

$2x^2-12x+3x-18$

$2x^2-9x-18$

61.) $(x+4)(2x^2+3x-1)$

$2x^3+3x^2-x+8x^2+12x-4$

$2x^3+11x^2+11x-4$

62.) $(2x^2-4x-3)(3x^2+2x-5)$

$6x^4+4x^3-10x^2-12x^3-8x^2+20x-9x^2-6x+15$

$6x^4-8x^3-27x^2+14x+15$

Factor the expression completely.

63.) $4x^2-20x$

$4x(x-5)$

64.) $x^2+7x+10$

$(x+5)(x+2)$

65.) x^2-x-30

$(x-6)(x+5)$

66.) $2x^2+13x+6$

$(2x+1)(x+6)$

Find the x-intercepts.

67.) $x^2 + 8x + 16 = 0$

$(x+4)(x+4)=0$

$x+4=0$

$-4\ -4$

$x=-4$

68.) $x^2 - 3x - 10 = 0$

$(x-5)(x+2)=0$

$x-5=0$ $\quad$ $x+2=0$

$+5\ +5$ $\quad$ $-2\ -2$

$x=5$ or $x=-2$

69.) $2x^2 - 9x - 18 = 0$

$2x+3=0$ $\quad$ $x-6=0$

$-3\ -3$ $\quad$ $+6\ +6$

$\frac{2x}{2} = \frac{-3}{2}$ $\quad$ $x=6$

$x=-\frac{3}{2}$ or

70.) $x^2 - 49 = 0$

$(x+7)(x-7)=0$

$x+7=0$ $\quad$ $x-7=0$

$-7\ -7$ $\quad$ $+7\ +7$

$x=-7$ or $x=7$

71.) $x^2 + 12x - 45 = 0$

$(x+15)(x-3)=0$

$x+15=0$ $\quad$ $x-3=0$

$-15\ -15$ $\quad$ $+3\ +3$

$x=-15$ or $x=3$

$4x^2+4x+3x+3$

$4x(x+1)+3(x+1)=0$

$(4x+3)(x+1)=0$

$4x+3=0$ $\quad$ $x+1=0$

$-3\ -3$ $\quad$ $-1\ -1$

72.) $2x^2 + 11x + 5 = 0$

$(2x+1)(x+5)=0$

$2x+1=0$ $\quad$ $x+5=0$

$-1\ -1$ $\quad$ $-5\ -5$

$\frac{2x}{2} = \frac{-1}{2}$ $\quad$ $x=-5$

$x=-\frac{1}{2}$ or

73.) $4x^2 + 7x + 3 = 0$

$4x^2+4x+3x+3$

$4x(x+1)+3(x+1)=0$

$(4x+3)(x+1)=0$

$4x+3=0$ $\quad -3\ -3$

$\frac{4x}{4} = \frac{-3}{4}$

$x = -\frac{3}{4}$

$x+1=0$ $\quad -1\ -1$

$x=-1$

$x=-\frac{3}{4}$ or $x=-1$

74.) $8x^2 + 2x - 15 = 0$

$(4x-5)(2x+3)=0$

$4x-5=0$ $\quad +5\ +5$

$\frac{4x}{4} = \frac{5}{4}$

$x=\frac{5}{4}$ or

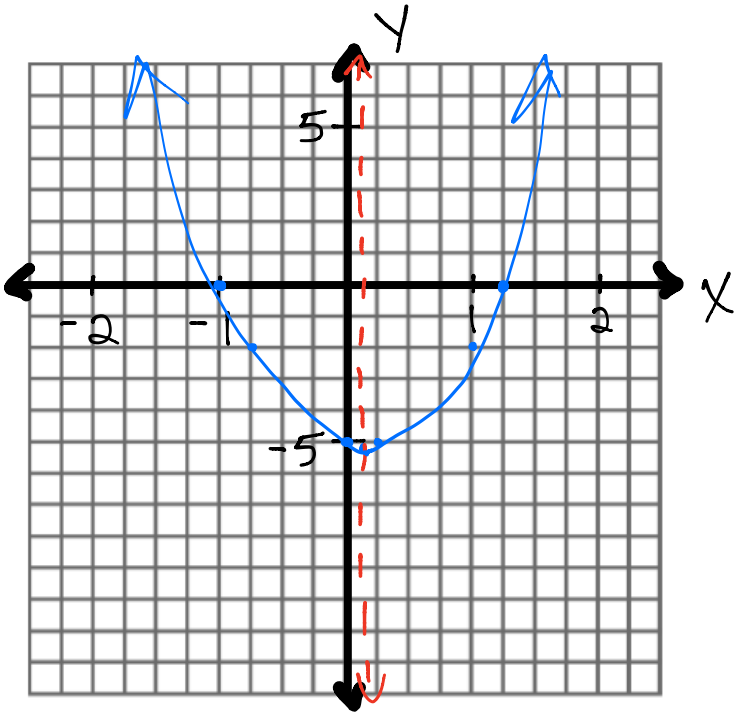
$2x+3=0$ $\quad -3\ -3$

$\frac{2x}{2} = \frac{-3}{2}$

$x=-\frac{3}{2}$

For 75 through 79, identify the key features and graph the equation.

75.) $y = 4x^2 - x - 5$ → $y=(4x-5)(x+1)$



y-intercept:

$(0,-5)$

x-intercepts:

$(4x-5)(x+1)=0$ $\quad (1.25,0),(-1,0)$

$4x-5=0$ $\quad +5\ +5$

$\frac{4x}{4} = \frac{5}{4}$

$x=\frac{5}{4}$ or 1.25

$x+1=0$ $\quad -1\ -1$

$x=-1$

line of symmetry:

$\frac{-1+\frac{5}{4}}{2} = .125$ or $\frac{1}{8}$

$x = .125$

vertex:

$y=(4(.125)-5)(.125+1)$ $\quad (.125,-5.0625)$

$y=(-4.5)(1.125)$

$y=-5.0625$

up/down:

$a=4$

additional point:

$y=(4(1)-5)(1+1)$

$y=(-1)(2)$ $\quad (1,-2)$

$y=-2$

76.) $y = 6x^2 - 15x - 21$

$y = 3(2x^2 - 5x - 7)$

$y = 3(2x - 7)(x + 1)$

y-intercept:

$(0, -21)$

x-intercepts:

$0 = (2x - 7)(x + 1)$ $\quad (3.5, 0), (-1, 0)$

$2x - 7 = 0$ $\quad x + 1 = 0$

$+7 \; +7$ $\quad -1 \; -1$

$\frac{2x}{2} = \frac{7}{2}$ $\quad x = -1$

$x = 3.5$

line of symmetry:

$\frac{3.5 + (-1)}{2} = 1.25$ $\quad x = 1.25$

vertex:

$y = 3(2(1.25) - 7)(1.25 + 1)$ $\quad (1.25, -30.375)$

$y = 3(-4.5)(2.25)$

$y = -30.375$

up/down:

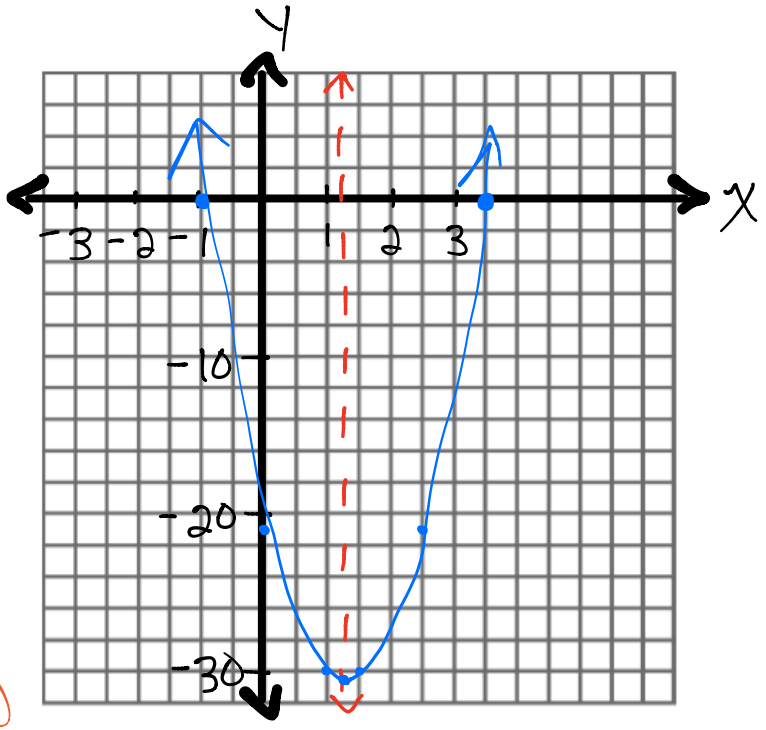
$a = 6$

additional point:

$y = 3(2(1) - 7)(1 + 1)$ $\quad (1, -30)$

$y = 3(-5)(2)$

$y = -30$



77.) $y = x^2 - 4x - 45$

$y = (x - 9)(x + 5)$

y-intercept:

$(0, -45)$

x-intercepts:

$0 = (x - 9)(x + 5)$ $\quad (9, 0), (-5, 0)$

$x - 9 = 0$ $\quad x + 5 = 0$

$+9 \; +9$ $\quad -5 \; -5$

$x = 9$ $\quad x = -5$

line of symmetry:

$\frac{9 + (-5)}{2} = 2$ $\quad x = 2$

vertex:

$y = (2 - 9)(2 + 5)$

$y = (-7)(7)$ $\quad (2, -49)$

$y = -49$

up/down:

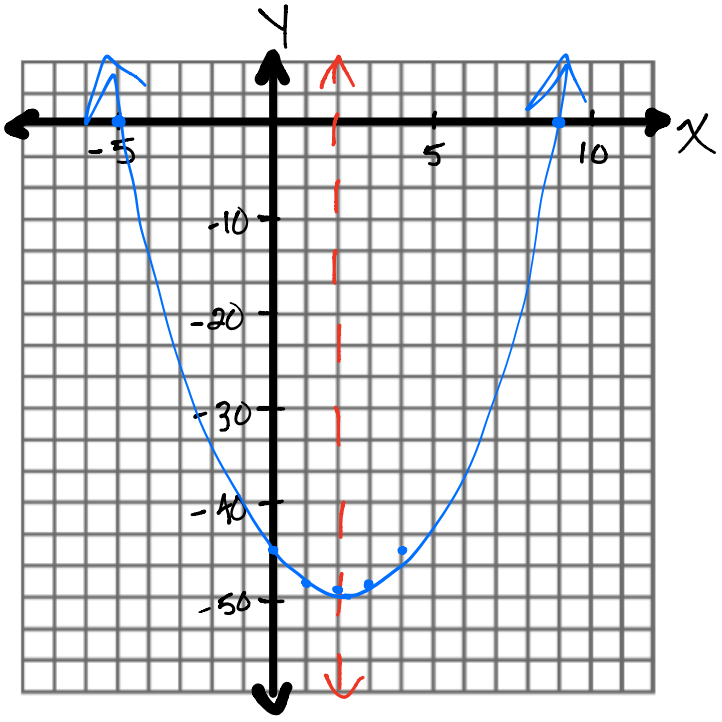
$a = 1$

additional point:

$y = (1 - 9)(1 + 5)$

$y = (-8)(6)$ $\quad (1, -48)$

$y = -48$



78. $y = 2x^2 + 9x + 10$

$y = (2x+5)(x+2)$

y-intercept:

$(0, 10)$

x-intercepts:

$0 = (2x+5)(x+2)$ $\quad (-2.5, 0), (-2, 0)$

$2x+5=0$ $\quad x+2=0$

$-5\ -5$ $\quad -2\ -2$

$\frac{2x}{2} = \frac{-5}{2}$ $\quad x = -2$

$x = -2.5$

line of symmetry:

$\frac{-2.5 + (-2)}{2} = -2.25$ $\quad x = -2.25$

vertex:

$y = (2(-2.25)+5)(-2.25+2)$

$y = (.5)(-.25)$ $\quad (-2.25, -.125)$

$y = -.125$

up/down

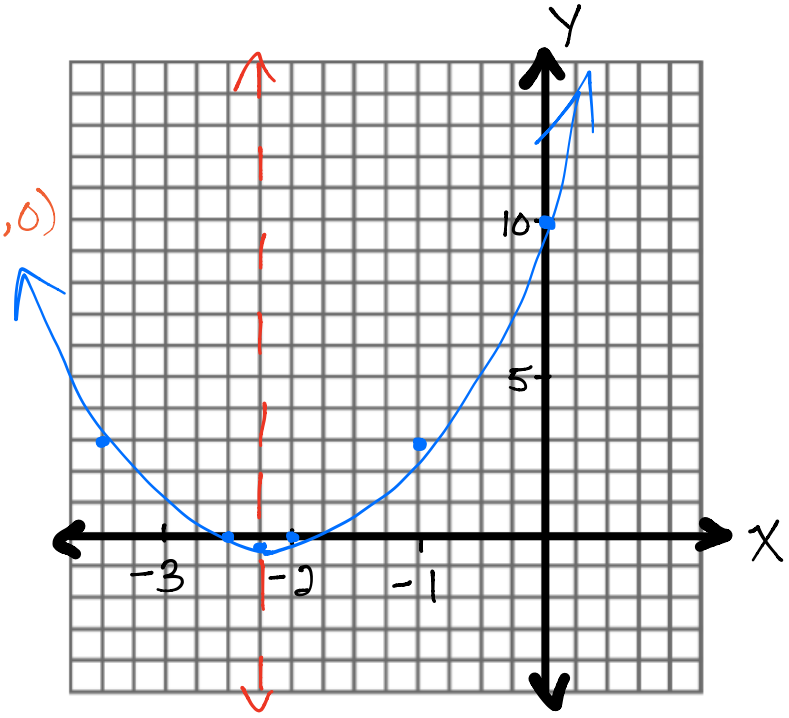
$a = 2$

additional point:

$y = (2(-1)+5)(-1+2)$

$y = (3)(1)$

$y = 3$ $\quad (-1, 3)$



79. $y = x^2 + 6x - 16$

$y = (x+8)(x-2)$

y-intercept:

$(0, -16)$

x-intercepts:

$0 = (x+8)(x-2)$ $\quad (-8, 0), (2, 0)$

$x+8=0$ $\quad x-2=0$

$-8\ -8$ $\quad +2\ +2$

$x = -8$ $\quad x = 2$

line of symmetry:

$\frac{-8+2}{2} = -3$ $\quad x = -3$

vertex:

$y = (-3+8)(-3-2)$

$y = (5)(-5)$ $\quad (-3, -25)$

$y = -25$

up/down

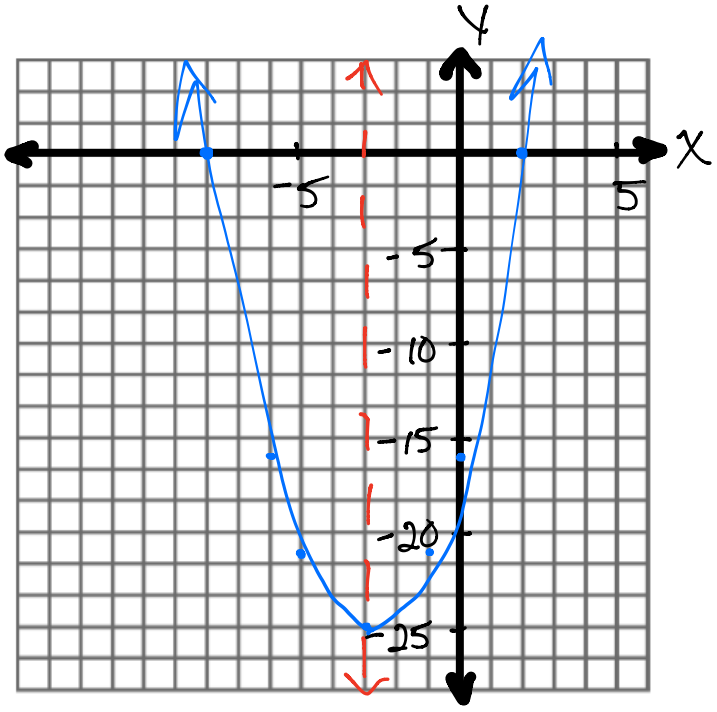
$a = 1$

additional point:

$y = (-1+8)(-1-2)$

$y = (7)(-3)$

$y = -21$ $\quad (-1, -21)$



80. A stomp rocket is launched into the air from ground level. It's height *h* as a function of time *t* can be modeled by the equation $h = -16t^2 + 64t$.

a. How high will the rocket be at 1.5 seconds?

$h = -16(1.5)^2 + 64(1.5)$

$h = 60$ feet

b. How long will it take for the rocket to hit the ground? → $h = 0$

$0 = -16t^2 + 64t$

$0 = -16t(t - 4)$ 4 seconds

$\frac{-16t}{-16} = \frac{0}{-16}$ $\quad t - 4 = 0$, $+4 \; +4$

$t = 0$ $\quad t = 4$

c. When will the rocket reach its maximum height?

$\frac{0 + 4}{2} = 2$ 2 seconds

d. The advertising on the package says "Can fly over 60 feet high!" Does this rocket exceed the height listed on the package?

$h = -16(2)^2 + 64(2)$ Yes!

$h = -16(4) + 128$

$h = -64 + 128$

$h = 64$ feet

81. Abraham throws a ball from a point 40 m above the ground. The height of the ball from the ground level after 't' seconds is defined by the function $h(t) = 40t - 5t^2$. How long will the ball take to hit the ground?

$0 = 40t - 5t^2$

$0 = -5t(-8 + t)$ 8 seconds

$\frac{-5t}{-5} = \frac{0}{-5}$ $\quad -8 + t = 0$, $+8 \quad +8$

$t = 0$ $\quad t = 8$

82. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by $\mathbf{h = -16t^2 + 64t + 80}$.

a. What is the height of the rocket after 1.5 seconds?

$h = -16(1.5)^2 + 64(1.5) + 80$

$h = -16(2.25) + 96 + 80$

$h = 140$ feet

b. What is the maximum height reached by the rocket?

$h = -16(t^2 - 4t - 5)$

$0 = -16(t-5)(t+1)$

$t - 5 = 0$ $\quad +5 \; +5$ $\quad t = 5$

$t + 1 = 0$ $\quad -1 \; -1$ $\quad t = -1$

$\frac{5 + (-1)}{2} = 2$

$h = -16(2)^2 + 64(2) + 80$

$h = -16(4) + 128 + 80$

$h = 144$ feet

c. How long will it take for the rocket to hit 128 feet? **CHALLENGE!**

$128 = -16t^2 + 64t + 80$

$-128 \qquad -128$

$0 = -16t^2 + 64t - 48$

$0 = -16(t^2 - 4t + 3)$

$0 = -16(t-3)(t-1)$

$t - 3 = 0$ $\quad +3 \; +3$ $\quad t = 3$

$t - 1 = 0$ $\quad +1 \; +1$ $\quad t = 1$

At 1 second and 3 seconds.

d. After how many seconds will the rocket hit the lake?

5 seconds.

83. After t seconds, a ball tossed in the air from the ground level reaches a height of H feet given by the equation $\mathbf{H(t) = 144t - 16t^2}$.

a. What is the height of the ball after 3 seconds?

$H(3) = 144(3) - 16(3)^2$

$H(3) = 288$ feet

b. What is H(4)?

$H(4) = 144(4) - 16(4)^2$

$H(4) = 320$ feet

c. What is the maximum height the ball will reach?

$0 = 144t - 16t^2$

$0 = -16t(-9 + t)$

$-16t = 0$ $\quad -16 \; -16$ $\quad t = 0$

$-9 + t = 0$ $\quad +9 \; +9$ $\quad t = 9$

$\frac{0 + 9}{2} = 4.5$

$H(4.5) = 144(4.5) - 16(4.5)^2$

$H(4.5) = 324$

324 feet

State whether each equation below represents a linear, exponential, or quadratic relationship and **explain how you know.**

101. $y = 3x^2 - 12$

Quadratic. 2 is highest power on x in expanded form.

102. $y = 2(3.5)^x$

Exponential. x is the exponent.

103. $y = (x - 4)(2x + 8)$

Quadratic. Two linear factors.

104. $y = -x + 14$

Linear. Follows $y = mx + b$.

105. $y = 6(x - 2) + 7(x + 1)$

Linear. Simplified is $y = 13x - 5$.

106. $y = .25^x$

Exponential. x is the exponent.

107. $y = \frac{1}{2}x^2 - 15x + 11$

Quadratic. 2 is highest power on x in expanded form.

108. $y = 3(2x + 12)(-3x + 2)$

Quadratic. Two linear factors.

For 109 - 114, decide whether the following relationships are linear, exponential, quadratic, or none of these by using their first and second differences. Explain your reasoning. If it is one of those three, calculate the next 2 values in the table.

109.

x	y
-3	33
-2	26
-1	19
0	12
1	5
2	-2
3	-9
4	-16

1st Differences: >-7, >-7, >-7, >-7

2nd Differences: 0

Type of Relationship: Linear. (1st diff. same.)

110.

x	y
0	3
1	12
2	48
3	192
4	768
5	3072
6	12288
7	49152

1st Differences: >9, >36, >144, >576

2nd Differences: >27, >108, >432

Type of Relationship: Exponential. Diff. not the same, but are all being multiplied by 4.

111.

x	y
1	469
2	513
3	557
4	601
5	645
6	689
7	733
8	777

1st Differences: >44, >44, >44, >44

2nd Differences: 0

Type of Relationship: Linear. (1st diff. same.)

112.

x	y
-1	9
0	0
1	-7
2	-12
3	-15
4	-16
5	-15
6	-12

1st Differences: -9, -7, -5, -3, -1, +1, +3

2nd Differences: 2, 2, 2, 2, 2, 2

Type of Relationship: Quadratic. 2nd diff same.

113.

x	y
2	12
3	33
4	62
5	99
6	144
7	197
8	258
9	327

1st Differences: 21, 29, 37, 45, 53, 61, 69

2nd Differences: 8, 8, 8, 8, 8, 8

Type of Relationship: Quadratic. 2nd diff same.

114.

x	y
-3	-5
-2	-1
-1	-6
0	0
1	-7

1st Differences: 4, -5, 6, -7

2nd Differences: -9, +11, -13

Type of Relationship: None. 1st or 2nd diff. aren't the same and there isn't any multiplicative pattern.